KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

ELECTRICAL ENGINEERING DEPARTMENT

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EE 520 (131)

MID-TERM EXAM

October 26th 2013

12:30 - 2:30 pm

Key Solutions

Student Name:

Student I.D.#:

Student Serial #:

Question # 1 (20)	
Question # 2 (30)	
Question # 3 (25)	
Question # 4 (25)	
Total	

QUESTION # 1-a (10 Marks)

Find the L U_1 factorization of the following symmetric matrix:

$$A = \begin{bmatrix} 2.0 & 1.0 & 3.0 \\ 1.0 & 5.0 & 4.0 \\ 3.0 & 4.0 & 7.0 \end{bmatrix}$$

Solution:

$$L = \begin{bmatrix} 2.0 & 0 & 0 \\ 1.0 & 4.5 & 0 \\ 3.0 & 2.5 & 1.1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 0.5 & 1.5 \\ 0 & 1 & 0.55 \\ 0 & 0 & 1 \end{bmatrix}$$

QUESTION # 1-b (10 Marks)

Store the non-zero entries only using the row-column entry method before and after the factorization (consider the arbitrary fashion case).

Solution:

Before the factorization; the non-zero entries of the matrix will be

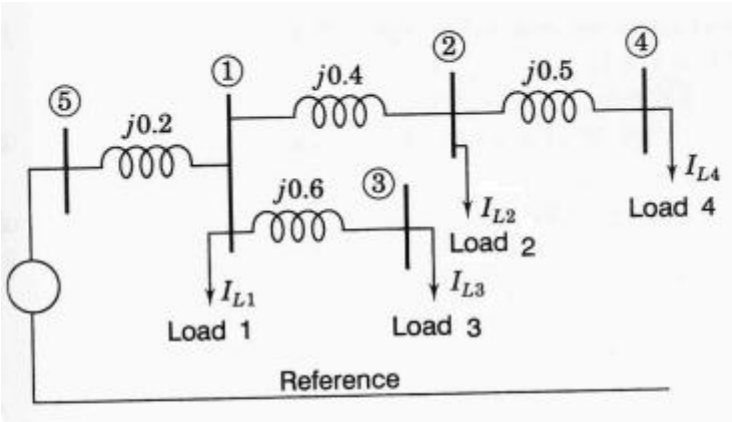
Counter	STO	IR	IC
1	2.0	1	1
2	1.0	1	2
3	3.0	1	3
4	5.0	2	2
5	4.0	2	3
6	7.0	3	3

After the factorization; the non-zero entries of the matrix will be

Counter	STO	IR	IC
1	2.0	1	1
2	0.5	1	2
3	1.5	1	3
4	1.0	2	1
5	4.5	2	2
6	0.55	2	3
7	3.0	3	1
8	2.5	3	2
9	1.1	3	3

QUESTION # 2-a (15 Marks)

Consider the 5-bus system shown below. The voltage source is $1.2 \angle 0^o$ and the load currents are $I_{L1} = -j0.1$; $I_{L2} = -j0.2$; $I_{L3} = -j0.1$; $I_{L4} = -j0.2$. (all values are in per-unit).



The bus impedance matrix after converting the voltage source to a current source (by ignoring bus 5) is

$$Z_{BUS} = j \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.6 & 0.2 & 0.6 \\ 0.2 & 0.2 & 0.8 & 0.2 \\ 0.2 & 0.6 & 0.2 & 1.1 \end{bmatrix}$$

Find the four bus voltages V_1 , V_2 , V_3 , V_4 .

Solution:

The bus voltages are

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = Z_{BUS}\, I = j \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.6 & 0.2 & 0.6 \\ 0.2 & 0.2 & 0.8 & 0.2 \\ 0.2 & 0.6 & 0.2 & 1.1 \end{bmatrix} \; j \begin{bmatrix} -5.9 \\ 0.2 \\ 0.1 \\ 0.2 \end{bmatrix} = \begin{bmatrix} 1.08 \\ 0.92 \\ 1.02 \\ 0.82 \end{bmatrix} \quad \text{per-unit}$$

<u>QUESTION # 2-b</u> (15 Marks)

Modify Z_{BUS} (using building algorithm) to include a capacitor bank of reactance 5.4 per-unit connected from bus 4 to the reference and then recalculate the bus voltages.

Solution:

The loop matrix after adding the capacitor bank is

$$Z_{LOOP} = j\begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.6 & 0.2 & 0.6 & 0.6 \\ 0.2 & 0.2 & 0.8 & 0.2 & 0.2 \\ 0.2 & 0.6 & 0.2 & 1.1 & 1.1 \\ 0.2 & 0.6 & 0.2 & 1.1 & -4.3 \end{bmatrix}$$

After Kron reduction, the modified Z_{BUS} is

$$Z_{BUS} = j\begin{bmatrix} 0.20930 & 0.22791 & 0.20930 & 0.25116 \\ 0.22791 & 0.68372 & 0.22791 & 0.75349 \\ 0.20930 & 0.22791 & 0.80930 & 0.25116 \\ 0.25116 & 0.75349 & 0.25116 & 1.38140 \end{bmatrix}$$

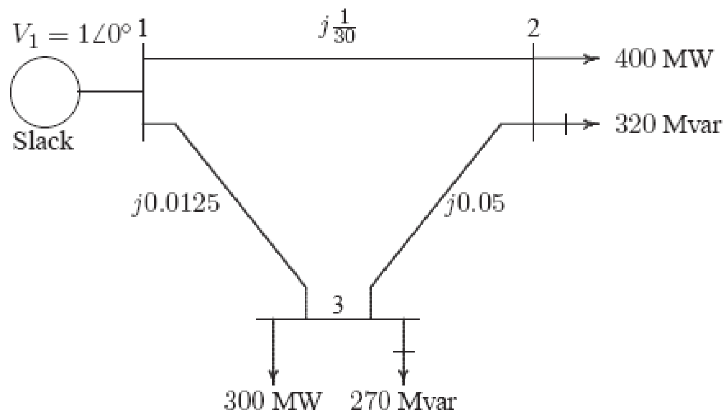
The bus voltages after adding the capacitor bank are

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = Z_{BUS}\, I = j\begin{bmatrix} 0.20930 & 0.22791 & 0.20930 & 0.25116 \\ 0.22791 & 0.68372 & 0.22791 & 0.75349 \\ 0.20930 & 0.22791 & 0.80930 & 0.25116 \\ 0.25116 & 0.75349 & 0.25116 & 1.38140 \end{bmatrix} j\begin{bmatrix} -5.9 \\ 0.2 \\ 0.1 \\ 0.2 \end{bmatrix}$$

$$= \begin{bmatrix} 1.118 \\ 1.034 \\ 1.058 \\ 1.030 \end{bmatrix} \text{ per-unit}$$

QUESTION # 3 (25 Marks)

Figure below shows the one-line diagram of a simple three-bus power system with generation at bus 1. Line impedances are marked in per unit on a 100 MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.



(a) Using Gauss-Seidel method and initial flat values for the bus voltages, determine V_2 and V_3. Perform two iterations.
(b) If after several iterations the bus voltages converge to
V_2 = 0.90 - j 0.10 pu
V_3 = 0.95 - j 0.05 pu
Determine the slack bus real and reactive power.

Solution:

(a) Line impedances are converted to admittances

$$y_{12} = -j30$$
$$y_{13} = \frac{1}{j0.0125} = -j80$$
$$y_{23} = \frac{1}{j0.05} = -j20$$

At the P-Q buses, the complex loads expressed in per units are

$$S_2^{sch} = -\frac{(400 + j320)}{100} = -4.0 - j3.2 \text{ pu}$$
$$S_3^{sch} = -\frac{(300 + j270)}{100} = -3.0 - j2.7 \text{ pu}$$

For hand calculation, we use (6.28). Bus 1 is taken as reference bus (slack bus). Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.0 + j0.0$, V_2 and V_3 are computed from (6.28) as follows

$$V_2^{(1)} = \frac{\frac{S_2^{sch*}}{V_2^{(0)*}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}}$$
$$= \frac{\frac{-4.0+j3.2}{1.0-j0} + (-j30)(1.0 + j0) + (-j20)(1.0 + j0)}{-j50}$$
$$= 0.936 - j0.08$$

and

$$V_3^{(1)} = \frac{\frac{S_3^{sch*}}{V_3^{(0)*}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}}$$
$$= \frac{\frac{-3.0+j2.7}{1-j0} + (-j80)(1.0 + j0) + (-j20)(0.936 - j0.08)}{-j100}$$
$$= 0.9602 - j0.046$$

For the second iteration we have

$$V_2^{(2)} = \frac{\frac{-4.0+j3.2}{0.936+j0.08} + (-j30)(1.0 + j0) + (-j20)(0.9602 - j0.046)}{-j50}$$
$$= 0.9089 - j0.0974$$

and

$$V_3^{(2)} = \frac{\frac{-3.0+j2.7}{0.9602+j0.046} + (-j80)(1.0 + j0) + (-j20)(0.9089 - j0.0974)}{(-j100)}$$
$$= 0.9522 - j0.0493$$

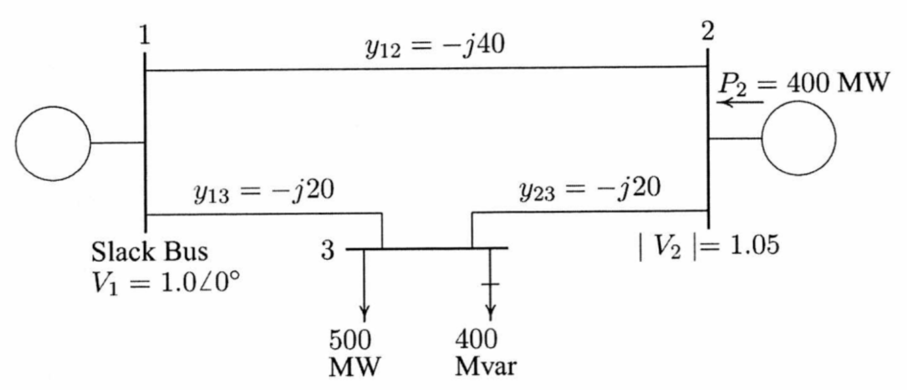
(b) With the knowledge of all bus voltages, the slack bus power is obtained from (6.27)

$$P_1 - jQ_1 = V_1^*[V_1(y_{12} + y_{13}) - (y_{12}V_2 + y_{13}V_3)]$$
$$= 1.0[1.0(-j30 - j80) - (-j30)(0.9 - j0.1) -$$
$$(-j80)(0.95 - j0.05)]$$
$$= 7.0 - j7.0$$

or the slack bus real and reactive powers are $P_1 = 7.0$ pu = 700 MW and $Q_1 = 7.0$ pu = 700 Mvar.

QUESTION # 4 (25 Marks)

Consider the three-bus power system shown below. Line admittances are marked in per unit on a 100 MVA base. Obtain the power flow solution using the fast decoupled algorithm. Perform two iterations.



Solution:

(a) In this system, bus 1 is the slack bus and the corresponding bus susceptance matrix for evaluation of phase angles $\Delta\delta_2$ and $\Delta\delta_3$ form the bus admittance matrix in Problem 6.12 is

$$B' = \begin{bmatrix} -60 & 20 \\ 20 & -40 \end{bmatrix}$$

The inverse of the above matrix is

$$[B']^{-1} = \begin{bmatrix} -0.02 & -0.01 \\ -0.01 & -0.03 \end{bmatrix}$$

The expressions for real power at bus 2 and 3 and the reactive power at bus 3 are given in Problem 6.12. The slack bus voltage is $V_1 = 1.0\angle 0$ pu, and the bus 2 voltage magnitude is $|V_3| = 1.05$ pu. Starting with an initial estimate of $|V_3^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, and $\delta_3^{(0)} = 0.0$, the power residuals are computed from (6.63) and (6.64)

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = 4 - (0) = 4$$
$$\Delta P_3^{(0)} = P_3^{sch} - P_3^{(0)} = -5 - (0) = -5$$
$$\Delta Q_3^{(0)} = Q_3^{sch} - Q_3^{(0)} = -4 - (-1) = -3$$

The fast decoupled power flow algorithm given by (6.77) becomes

$$\begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \end{bmatrix} = -\begin{bmatrix} -0.02 & -0.0 \\ -0.01 & -0.03 \end{bmatrix} \begin{bmatrix} \frac{4}{1.05} \\ \frac{-5}{1.0} \end{bmatrix} = \begin{bmatrix} .0262 \\ -0.1119 \end{bmatrix}$$

Since bus 2 is a regulated bus, the corresponding row and column of $\boldsymbol{B'}$ are eliminated and we get

$$B'' = [-40]$$

From (6.78), we have

$$\Delta|V_3| = -\left[\frac{-1}{40}\right]\left[\frac{-3}{1.0}\right] = -0.075$$

The new bus voltages in the first iteration are

$$\begin{aligned}
\Delta\delta_2^{(0)} &= 0.0262 \qquad \delta_2^{(1)} = 0 + (0.0262) = 0.0262 \;\text{ radian} = 1.5006^\circ \\
\Delta\delta_3^{(0)} &= -0.1119 \quad \delta_3^{(1)} = 0 + (-0.1119) = -0.1119 \;\text{ radian} = -6.4117^\circ \\
\Delta|V_3^{(0)}| &= -0.075 \quad |V_2^{(1)}| = 1 + (-0.075) = 0.925 \;\text{ pu}
\end{aligned}$$

For the second iteration, the power residuals are

$$\begin{aligned}
\Delta P_2^{(1)} &= P_2^{sch} - P_2^{(1)} = 4 - (3.7739) = 0.2261 \\
\Delta P_3^{(1)} &= P_3^{sch} - P_3^{(1)} = -5 - (-4.7399) = -.2601 \\
\Delta Q_3^{(1)} &= Q_3^{sch} - Q_3^{(1)} = -4 - (-3.3994) = -0.6006
\end{aligned}$$

$$\begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta\delta_3^{(1)} \end{bmatrix} = -\begin{bmatrix} -0.02 & -0.0 \\ -0.01 & -0.03 \end{bmatrix}\begin{bmatrix} \frac{0.2261}{1.05} \\ \frac{-0.2601}{0.925} \end{bmatrix} = \begin{bmatrix} 0.0015 \\ -0.0063 \end{bmatrix}$$

From (6.78), we have

$$\Delta|V_3| = -\left[\frac{-1}{40}\right]\left[\frac{-0.6006}{0.925}\right] = -0.0162$$

The new bus voltages in the second iteration are

$$\begin{aligned}
\Delta\delta_2^{(1)} &= 0.0015 \qquad \delta_2^{(2)} = 0.0262 + (0.0015) = 0.0277 \;\text{ radian} = 1.5863^\circ \\
\Delta\delta_3^{(1)} &= -0.0063 \quad \delta_3^{(2)} = -0.1119 + (-0.0063) = -0.1182 \;\text{radian} = -6.7716^\circ \\
\Delta|V_3^{(1)}| &= -0.0162 \quad |V_3^{(2)}| = 0.925 + (-0.0162) = 0.9088 \;\text{ pu}
\end{aligned}$$