# KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

# ELECTRICAL ENGINEERING DEPARTMENT

## EE 306 – Term 192

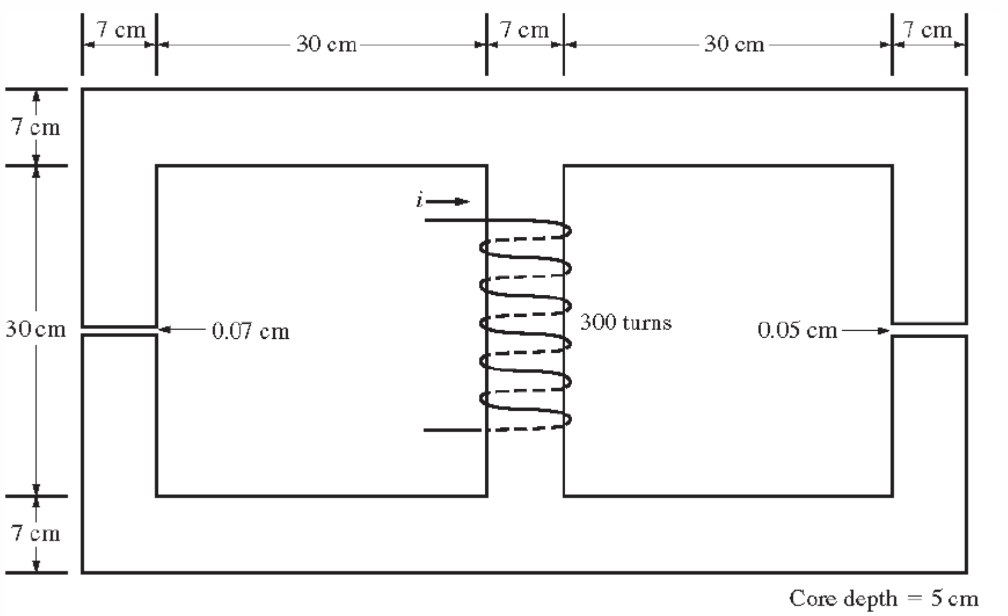
### HW # 2: Magnetic Circuits

**Due Date: (Feb. 16th for UT Classes and Feb. 17th for MW Classes)**

**Key Solutions**

**Problem # 1:**

A ferromagnetic core with a relative permeability of 1500 is shown below. The dimensions are as shown in the diagram, and the depth of the core is 5 cm. The air gaps on the left and right sides of the core are 0.050 and 0.070 cm, respectively. Because of fringing effects, the effective area of the air gaps is 5 percent larger than their physical size. If there are 300 turns in the coil wrapped around the center leg of the core and if the current in the coil is 1.0 A, what is the flux in each of the left, center, and right legs of the core? What is the flux density in each air gap?




SOLUTION This core can be divided up into five regions. Let $\mathcal{R}_1$ be the reluctance of the left-hand portion of the core, $\mathcal{R}_2$ be the reluctance of the left-hand air gap, $\mathcal{R}_3$ be the reluctance of the right-hand portion of the core, $\mathcal{R}_4$ be the reluctance of the right-hand air gap, and $\mathcal{R}_5$ be the reluctance of the center leg of the core. Then the total reluctance of the core is

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_5 + \frac{(\mathcal{R}_1 + \mathcal{R}_2)(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}$$

$$\mathcal{R}_1 = \frac{l_1}{\mu_r \mu_0 A_1} = \frac{1.11 \text{ m}}{(1500)(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.05 \text{ m})} = 168 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_2 = \frac{l_2}{\mu_0 A_2} = \frac{0.0007 \text{ m}}{(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.05 \text{ m})(1.05)} = 152 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_3 = \frac{l_3}{\mu_r \mu_0 A_3} = \frac{1.11 \text{ m}}{(1500)(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.05 \text{ m})} = 168 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_4 = \frac{l_4}{\mu_0 A_4} = \frac{0.0005 \text{ m}}{(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.05 \text{ m})(1.05)} = 108 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_5 = \frac{l_5}{\mu_r \mu_0 A_5} = \frac{0.37 \text{ m}}{(1500)(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.05 \text{ m})} = 56.1 \text{ kA} \cdot \text{t/Wb}$$

The total reluctance is

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_5 + \frac{(\mathcal{R}_1 + \mathcal{R}_2)(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} = 56.1 + \frac{(168+152)(168+108)}{168+152+168+108} = 204 \text{ kA} \cdot \text{t/Wb}$$

The total flux in the core is equal to the flux in the center leg:

$$\phi_{\text{center}} = \phi_{\text{TOT}} = \frac{\mathcal{F}}{\mathcal{R}_{\text{TOT}}} = \frac{(300 \text{ t})(1.0 \text{ A})}{204 \text{ kA} \cdot \text{t/Wb}} = 0.00147 \text{ Wb}$$

The fluxes in the left and right legs can be found by the "flux divider rule", which is analogous to the current divider rule.

$$\phi_{\text{left}} = \frac{(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}\phi_{\text{TOT}} = \frac{(168+108)}{168+152+168+108}(0.00147 \text{ Wb}) = 0.00068 \text{ Wb}$$

$$\phi_{\text{right}} = \frac{(\mathcal{R}_1 + \mathcal{R}_2)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}\phi_{\text{TOT}} = \frac{(168+152)}{168+152+168+108}(0.00147 \text{ Wb}) = 0.00079 \text{ Wb}$$

The flux density in the air gaps can be determined from the equation $\phi = BA$:

$$B_{\text{left}} = \frac{\phi_{\text{left}}}{A_{\text{eff}}} = \frac{0.00068 \text{ Wb}}{(0.07 \text{ cm})(0.05 \text{ cm})(1.05)} = 0.185 \text{ T}$$

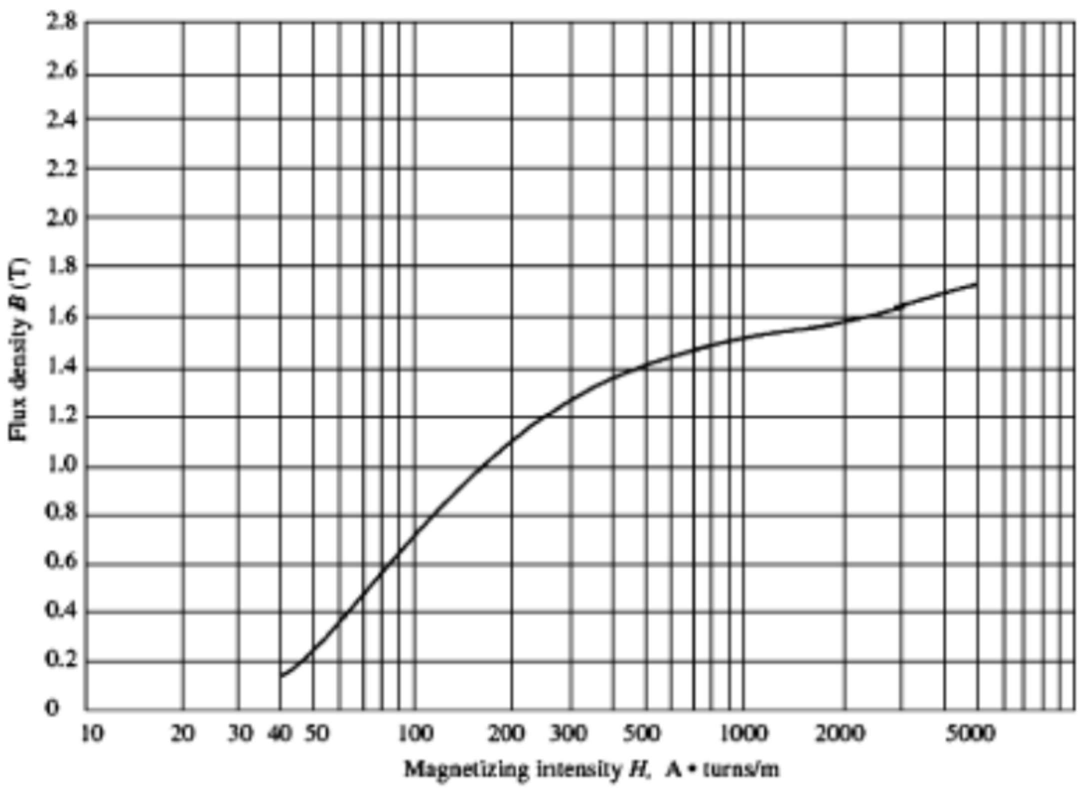
$$B_{\text{right}} = \frac{\phi_{\text{right}}}{A_{\text{eff}}} = \frac{0.00079 \text{ Wb}}{(0.07 \text{ cm})(0.05 \text{ cm})(1.05)} = 0.215 \text{ T}$$

**Problem # 2:**

A square magnetic core has a mean path length of 55 cm and a cross-sectional area of 150 $cm^2$ . A 200-turn coil of wire is wrapped around one leg of the core. The magnetization curve of the core material is shown in the figure below.

**(a)** How much current is required to produce 12 mWb of flux in the core?
**(b)** What is the relative permeability of the core at that level of current?
**(c)** What is its reluctance?
**(d)** Repeat part (a) if an air-gap of length 1 mm is cut across the core.

Assume a 5% increase in the effective air-gap area to account for fringing.



**Solution:**

$$B = \frac{\Phi}{A} = \frac{0.012}{0.015} = 0.8\ T$$

From the magnetization curve, the corresponding H

$$H \cong 115\ AT/m$$

Hence, $F = Hl = 115 * 0.55 = 63.25\ AT$

So,

a) $I = \frac{F}{N} = \frac{63.25}{200} = 0.316\ A$

b) $\mu = \frac{B}{H} = \frac{0.8}{115} = 0.00696\ H/m$

$$\mu_r = \frac{\mu}{\mu_o} = \frac{0.00696}{4\pi * 10^{-7}} = 5540$$

c) $\mathcal{R} = \frac{F}{\Phi} = \frac{63.25}{0.012} = 5270\ AT/Wb$

or, $\mathcal{R} = \frac{l}{\mu A} = \frac{0.55}{5540 * 4\pi * 10^{-7} * 150 * 10^{-4}} = 5270\ AT/Wb$

d) With the same flux $\Phi = 12\ mWb$, $B_c = 0.8\ T$

$H_c = 115\ AT/m$, $l_c = 0.55\ m$ (Neglect $l_g$; $l_g \ll l_c$)

$$B_g = \frac{\Phi}{A_g} = \frac{0.012}{1.05 * 150 * 10^{-4}} = 0.762\ T$$

$$H_g = \frac{B_g}{\mu_o} = \frac{0.762}{4\pi * 10^{-7}} = 0.061 * 10^{7}\ AT/m, \quad F_g = H_g l_g$$

$$F_{total} = F_c + F_g = 63.25 + 0.061 * 10^{7} * 1 * 10^{-3} = 673.25$$

$$I = \frac{F_{tot}}{N} = \frac{673.25}{200} = 3.366\ A$$

**Problem # 3:**

**(a)** The total iron loss in the core of a transformer having volume 0.16 $cm^3$ is 2170 W when excited at 50 Hz. The hysteresis loop of the core material, taken to some maximum flux density $B_{max}$, has an area of 9 $cm^2$ when drawn to scales of:

1 cm = 0.1 $Wb/m^2$ and 1 cm = 250 AT/m

Calculate the hysteresis and eddy-current losses of the core in watts?

**(b)** Repeat **Part (a),** if the same core of the transformer is excited to the same maximum flux density as in Part (a) but at the frequency of 60 Hz. Also, what will be total iron losses in the transformer core at 60 Hz?

**Solution:**

(a) $W_h = xy \times$ (area of hysteresis loop)

where $x$ & $y$ are scale factors

$W_h = 9 \times 0.1 \times 250 = 225$ $J/m^3/cycle$.

At $f = 50Hz$

Hysteresis loss $= V_{core} \times W_h \times f$

$P_h = 0.16 \times 225 \times 50$ $\boxed{P_h = 1800W}$

(b) Similarly

At $f = 60Hz$

$\boxed{P_h = 1800 \times 60/50 = 2160 W}$

$\boxed{P_e = 370 \times (60/50)^2 = 533 W}$

Total Iron losses $= 2160 + 533$

$\boxed{\text{Total Iron losses} = 2690 W}$