KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
ELECTRICAL ENGINEERING DEPARTMENT

EE 306 – Term 192

HW # 1: Three-Phase Circuits

Due Date: (Feb. 2nd for UT Classes and Feb. 3rd for MW Classes)

Key Solutions

Problem # 1:

Given the number $A_1 = 5\angle 30^o$(in polar form) and $A_2 = -3 + j4$ (in rectangular form). Calculate the following, given the answers in both rectangular and polar forms:

a. $A_1 + A_2$
b. $A_1 * A_2$
c. $A_1/(A_2)^*$

Solution:

$$A_1 = 5\angle 30^\circ = 4.33 + j2.50$$
$$A_2 = -3 + j4 = 5\angle 126.9^\circ$$

(a) $A_1 + A_2 = 1.33 + j6.5 = 6.63\angle 78.4^\circ$

(b) $A_1 A_2 = 25\angle 156.9^\circ = -23 + j9.8$

(c) $A_1/A_2^* = 1\angle 156.9^\circ = -0.92 + j0.39$

Problem # 2:

A load with an impedance of $Z = 25\angle 53.1°\,\Omega$ is fed from a single-phase source of 220V.

a. Find the resistance and reactance of the load.
b. Find the real (active) and imaginary (reactive) power of the load.
c. Find the power factor of the load, and state whether it is lagging or leading.

Solution:

(a) $Z = 15 + j20$

$R = 15\ \Omega$

$X = 20\ \Omega$

(b) $I = \dfrac{V}{Z} = \dfrac{220\angle 0°}{25\angle 53.1°} = 8.8\angle -53.1°$

(b) $S = VI^* = (220\angle 0°)(8.8\angle -53.1°)^* = 1936\angle 53.1° = 1162 + j\,1549\ \text{VA}$

$P = 1162\ \text{W}$

$Q = 1549\ \text{VAR}$

(c) $PF = \cos 53.1° = 0.6$ lagging

Problem # 3:

Prove that the line voltage of a Y-connected generator with an *acb* phase sequence lags the corresponding phase voltage by 30°. Draw a phasor diagram showing the phase and line voltages for this generator.

Solution:

If the generator has an *acb* phase sequence, then the three phase voltages will be

$$\mathbf{V}_{an} = V_\phi \angle 0°$$

$$\mathbf{V}_{bn} = V_\phi \angle -240°$$

$$\mathbf{V}_{cn} = V_\phi \angle -120°$$

The relationship between line voltage and phase voltage is derived below. By Kirchhoff's voltage law, the line-to-line voltage $\mathbf{V}_{ab}$ is given by

$$\mathbf{V}_{ab} = \mathbf{V}_a - \mathbf{V}_b$$

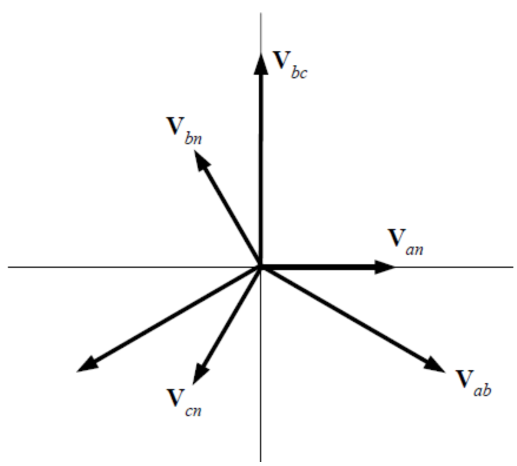
$$\mathbf{V}_{ab} = V_\phi \angle 0° - V_\phi \angle -240°$$

$$\mathbf{V}_{ab} = V_\phi - \left(-\frac{1}{2}V_\phi + j\frac{\sqrt{3}}{2}V_\phi \right) = \frac{3}{2}V_\phi - j\frac{\sqrt{3}}{2}V_\phi$$

$$\mathbf{V}_{ab} = \sqrt{3}V_\phi \left(\frac{\sqrt{3}}{2} - j\frac{1}{2} \right)$$

$$\mathbf{V}_{ab} = \sqrt{3}V_\phi \angle -30°$$

Thus the line voltage *lags* the corresponding phase voltage by 30°. The phasor diagram for this connection is shown below.



Problem # 4:

A balanced 3-phase Y-connected load with phase impedance of $20+j15\ \Omega$ is connected to a 400-V, 3-phase, 50-Hz supply. Calculate:

a. the line current.
b. the real and reactive power supplied.

If a 3-phase Δ-connected capacitor bank is connected parallel to the above load, calculate the capacitance per phase to obtain a resultant power factor of 0.95 lagging.

Solution:

$Z_{ph} = 20 + j15\ \Omega = 25\angle 36.87$

$V_{ph} = \frac{400}{\sqrt{3}}\angle 0 = 231\angle 0$

a) $I_{ph} = I_L = \frac{V_{ph}}{Z_{ph}} = \frac{231\angle 0}{25\angle 36.87} = 9.24\angle -36.87^\circ$

b) $P = \sqrt{3} * V_L I_L \cos 36.87 = \sqrt{3} * 400 * 9.24 * 0.8$

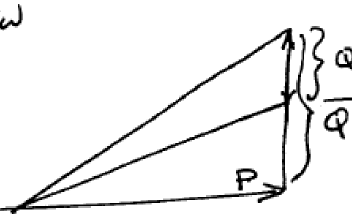
$= 5120\ W$

$Q = \sqrt{3}\, V_L I_L \sin 36.87 = \sqrt{3} * 400 * 9.24 * 0.6$

$= 3840\ VAR$

When the capacitor bank is connected

$Pf_{new} = 0.95$; $\phi_{new} = 18.19^\circ$; $\tan\phi_{new} = 0.3286$



real Power remains Constant

$Q_{new} = P \tan\phi_{new} = 1682\ VAR$

$Q_c = Q - Q_{new}$

$Q_c = 3840 - 1682 = 2158\ VAR$

$Q_{cph} = \frac{2158}{3} = 719.3\ VAR$

$Q_{cph} = \frac{V_L^2}{X_C} \longrightarrow X_c = \frac{V_L^2}{Q_{cph}} = \frac{(400)^2}{719.3} = 222.44\ \Omega = \frac{1}{\omega C}$

$C = \frac{1}{2\pi * 50 * 222.44} = 14.31\ \mu F$

Problem # 5:

A balanced 3-phase, 173-V, 60-Hz source supplies the two following loads:

- A Δ-connected load with a phase impedance of (18+*j*24) Ω,
- A Y-connected load with a phase impedance of 10∠53.13° Ω.

Find:

a. The power factor of the entire load.
b. The total line current supplied.
c. The total real, reactive, and apparent powers.

Solution:

- Convert Δ to Y $$Z_y = \frac{18 + j\,24}{3} = 6 + j\,8$$
- Parallel combination of the 2 loads (per phase)

$$Z_T = \frac{(6 = j\,8)(10\angle 53.1^0)}{6 + j\,8 + 10\angle 53.1^0} = 5\angle 53.1^0$$

a. Power factor= cos(53.1^0) = 0.6 lag

b. $$I_L = I_{ph} = \frac{173/\sqrt{3}\angle 0^0}{5\angle 53.1^0} = 20\angle -53.1^0 A$$

c.

$$P_T = \sqrt{3}x173x\,20x\,0.6 = 3.596kW$$

$$Q_T = \sqrt{3}x173x\,20x\,0.8 = 4.794kVAR$$

$$|S_T| = \sqrt{3}x173x\,20 = 5.993kVA$$ P