# King Fahd University of Petroleum and Minerals

**EE-306 – Electromechanical Devices - Semester 191**

## Due Date Oct. 16th 2019

## Homework 3 Solution

Question 1)

A single-phase, 10 kVA, 2200/220V 60 Hz transformer has:
$R_1$= 1 Ω, $L_1$= 7.96 mH, $R_2$= 0.1 Ω and $L_2$= 0.796 mH.
A load of 6 kW, 0.8 pf lagging is connected to its secondary side. Using transformer approximated equivalent circuit without excitation branch shown in Figure 1, find:

1- Transformer input current.
2- Input power factor
3- Draw the phasor diagram

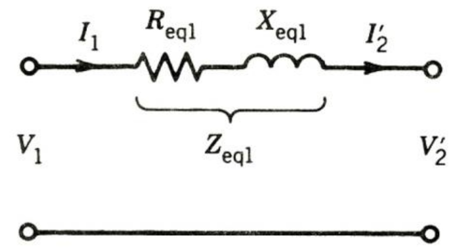



*Figure 1: Approximated equivalent circuit*

Solution:

1) transformer input current

$$Z_1 = R_1 + j2\pi f L_1 = 1 + j2\pi * 60 * 7.96 * 10^{-3} = 1 + j3\,\Omega$$

$$Z_2 = R_2 + j2\pi f L_2 = 0.1 + j2\pi * 60 * 0.796 * 10^{-3} = 0.1 + j0.3\,\Omega$$

So, $Z_2' = a^2 Z_2 = 10 + j30\,\Omega$

Connecting the 6 kW load and referring to primary side, then

$$I_2' = \frac{P}{V_2'\,pf} = \frac{6{,}000}{2200 * 0.8} = 3.4\,A$$

As the magnetization branch is neglected here, the transformer input current equals the $I_2' = 3.4\,A$

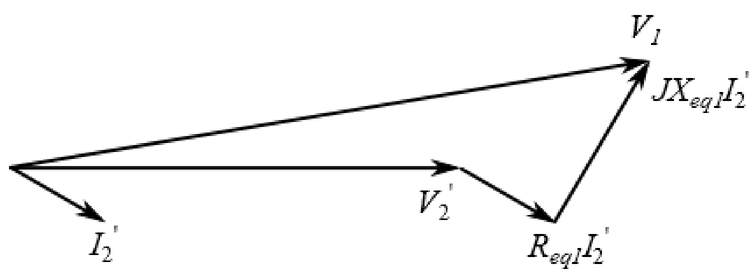
2) input power factor:

The power factor is calculated by cosine the angle between the input voltage and current vectors. Therefore,

$$V_1 = V_2' + I_2' Z_{eq-1} = 2200\angle 0 + 3.4\angle -\cos^{-1}(0.8) * (11 + j33) = 2298\angle 1.6^o\,V$$

Then, the input power factor equals $\cos(1.6 - (-\cos^{-1} 0.8)) = 0.78\ Lagging$.

3) Phasor diagram:



Question 2)

A single-phase, 20 kVA, 2200/220V 60 Hz transformer has equivalent impedance $5 + j25\ \Omega$ referred to its primary side. Find the voltage regulation if the transformer is loaded by:

1- Full load, 0.8 lagging pf.
2- 70% of its full load with 0.8 leading pf.

Solution:

$$VR\ \% = \frac{|V_1| - |V_2'|}{|V_2'|} * 100.$$

1) Full load means that the current and the apparent power are rated values. Therefore,

$$I_2' = \frac{S}{V_2'} = \frac{20{,}000}{2200} = 9\ A$$

$$V_1 = V_2' + I_2' Z_{eq-1} = 2200\angle 0 + 9\angle - \cos^{-1}(0.8) * (5 + j25) = \ 2377\angle 3.7^o\ V$$

$$VR\ \% = \frac{2377 - 2200}{2200} * 100 = 8.08\%$$

2) at 70% of rated, the current is 0.7*9=6.36 A, power factor is 0.8 leading

$$V_1 = V_2' + I_2' Z_{eq-1} = 2200\angle 0 + 6.36\angle \cos^{-1}(0.8) * (5 + j25) = \ 2135\angle 3.93^o\ V$$

$$VR\ \% = \frac{2135 - 2200}{2200} * 100 = -2.95\ \%$$

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Question 3)

A 10 kVA, 2300/230 V, single-phase transformer has been tested while keeping the meters are connected at the HV side. The tests data are as follows:

- OCT at rated voltage: 0.45 A and 70 W (Measurements at HV side)
- SCT at rated current: 120 V, 240 W (Measurements at HV side)

1- Determine the approximate equivalent with  excitation branch circuit referred to HV side.

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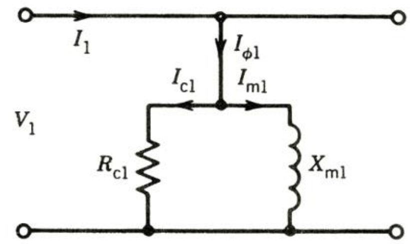
2- Determine the transformer efficiency at full load, 0.8 pf lagging

Solution:

Since the meters are connected in the HV side, the parameters can be calculated referred to HV side without transferring.

1) Approximate equivalent with excitation branch referred to HV side.



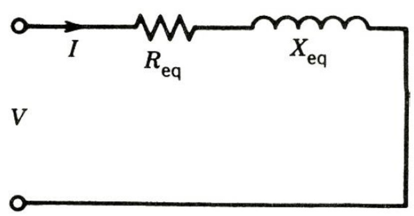

→From the open circuit test,

$$R_c = \frac{V_{oc}^2}{P_{oc}} = \frac{2300^2}{70} = 75{,}571.4\ \Omega$$

$$I_c = \frac{V_{oc}}{R_c} = \frac{2300}{75{,}571.4} = 0.03\ A \quad and\ I_m = \sqrt{(I_{oc}^2 - I_c^2)} = 0.448\ A$$

Then, $X_m = \frac{V_{oc}}{I_m} = \frac{2300}{0.448} = 5{,}123\ \Omega$

→For the short circuit test, the current is rated which is $I_2' = \frac{S}{V_2'} = \frac{10{,}000}{2300} = 4.3478\ A$




$$R_{eq-1} = \frac{P_{sc}}{I_{sc}^2} = \frac{240}{4.3478^2} = 12.696\ \Omega$$

$$Z_{eq-1} = \frac{V_{sc}}{I_{sc}} = \frac{120}{4.3478} = 27.6\ \Omega$$

$$X_{eq-1} = \sqrt{\left(Z_{eq-1}^2 - R_{eq-1}^2\right)} = 24.5\ \Omega$$

2) Transformer efficiency at full load, 0.8 pf lagging ($X$ = 1 for full load)

$$\eta = \frac{X.S_{rat}.pf}{P_{core} + X^2 P_{cu-rated} + X.S_{rat}.pf}$$

$$\eta_{\%} = \frac{10{,}000 * 0.8}{70 + 240 + 10{,}000 * 0.8} * 100 = 96.26\ \%$$

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Question 4)

For the transformer of Question 3,

1- Determine the percentage loading of the transformer at which the efficiency is a maximum at a load pf of 0.8
2- Calculate the maximum efficiency

Solution:

For maximum efficiency at a definite power factor, the copper loss equals core losses. Hence,

1) The loading factor $X$ is

$$X = \sqrt{\left(\frac{P_c}{P_{cu-rate}}\right)} = \sqrt{\left(\frac{70}{240}\right)} = 0.54$$

2)

$$\eta_{\%-max} = \frac{X.S_{rat}.pf}{2.P_{core} + X.S_{rat}.pf} = \frac{0.54 * 10{,}000 * 0.8}{70 * 2 + 0.54 * 10{,}000 * 0.8} * 100 = 96.86\ \%$$

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Question 5)

The efficiency of a 400 kVA, 60 Hz single-phase transformer is 98.77% when delivering full load current of 0.8 pf, and 99.13 % while delivering half rated current at unity power factor. Find:

1- core losses
2- full-load copper losses

Solution:

$$\eta = \frac{X.S_{rat}.pf}{P_{core} + X^2 P_{cu-rated} + X.S_{rat}.pf}$$

Case-1: $0.9877 = \frac{1*400{,}000*0.8}{P_{core}+1^2*P_{cu-rated}+1*400{,}000*0.8}$ → $P_{core} + P_{cu-rated} = 3936\ W$ (Eq-1)

Case-2: $0.9913 = \frac{0.5*400{,}000}{P_{core}+0.5^2*P_{cu-rated}+0.5*400{,}000}$ → $P_{core} + {}^1/_4\, P_{cu-rated} = 1740\ W$ (Eq-2)

Solving Eq-1 and Eq-2,

$$P_{core} = 1008\ W\ \ and\ \ P_{cu-rated} = 2928\ W$$

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Best Regards