KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
ELECTRICAL ENGINEERING DEPARTMENT

EE 306 – Term 191

HW # 1: Three-Phase Circuits
Due Date: (Sep. 16th, 2019)
Key Solutions

Problem # 1:

Determine the phase sequence of the set of voltages

$$v_{an} = 200\cos(\omega t + 10^\circ)$$

$$v_{bn} = 200\cos(\omega t - 110^\circ)\ , \quad v_{cn} = 200\cos(\omega t - 230^\circ)$$

Solution:

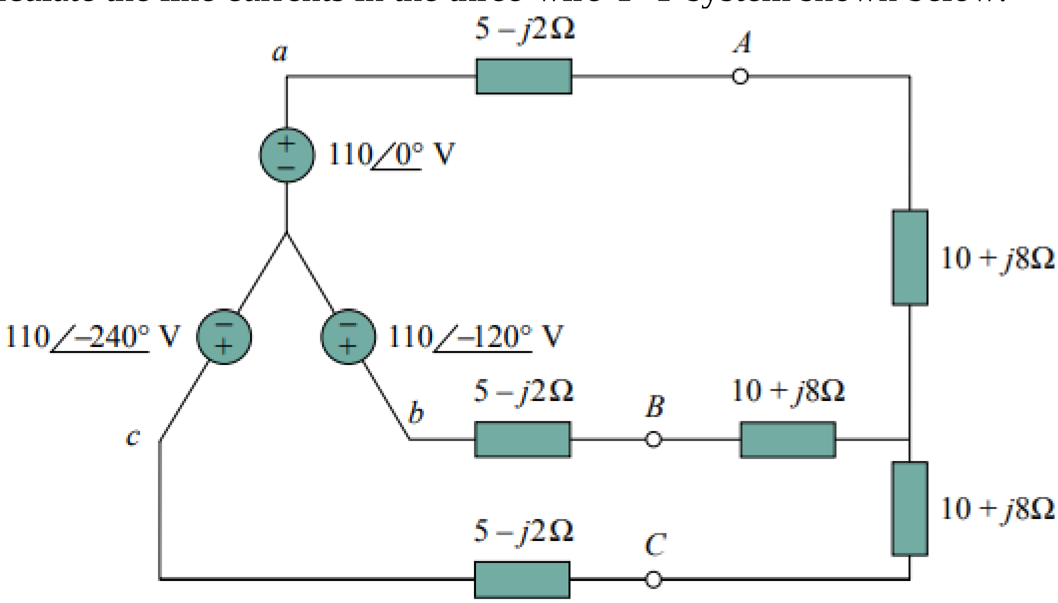
The voltages can be expressed in phasor form as

$$\mathbf{V}_{an} = 200\angle 10^\circ, \qquad \mathbf{V}_{bn} = 200\angle -110^\circ\ , \qquad \mathbf{V}_{cn} = 200\angle -230^\circ$$

We notice that $\mathbf{V}_{an}$ leads $\mathbf{V}_{bn}$ by 120° and $\mathbf{V}_{bn}$ in turn leads $\mathbf{V}_{cn}$ by 120°. Hence, we have an $a\,b\,c$ sequence.

Problem # 2:

Calculate the line currents in the three-wire Y-Y system shown below.



Solution:

The three-phase circuit is balanced. This can be replaced by its single-phase equivalent circuit such. I_a from the single-phase analysis as

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

where $\mathbf{Z}_Y = (5 - j2) + (10 + j8) = 15 + j6 = 16.155\underline{/21.8^\circ}$. Hence,

$$\mathbf{I}_a = \frac{110\underline{/0^\circ}}{16.155\underline{/21.8^\circ}} = 6.81\underline{/-21.8^\circ}\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ} = 6.81\underline{/-141.8^\circ}\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\underline{/-240^\circ} = 6.81\underline{/-261.8^\circ}\ \text{A} = 6.81\underline{/98.2^\circ}\ \text{A}$$

Problem # 3:

A balanced *abc*-sequence Y-connected source with $\mathbf{V}_{an} = 100\underline{/10^\circ}$ V is connected to a Δ-connected balanced load $(8 + j4)\ \Omega$ per phase.

Calculate the phase and line currents at the load side.

Solution:

This can be solved in two ways.

Method 1:

The load impedance is

$$\mathbf{Z}_\Delta = 8 + j4 = 8.944\underline{/26.57^\circ}\ \Omega$$

If the phase voltage $\mathbf{V}_{an} = 100\underline{/10^\circ}$, then the line voltage is

$$\mathbf{V}_{ab} = \mathbf{V}_{an}\sqrt{3}\underline{/30^\circ} = 100\sqrt{3}\underline{/10^\circ + 30^\circ} = \mathbf{V}_{AB}$$

or

$$\mathbf{V}_{AB} = 173.2\underline{/40^\circ}\ \text{V}$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{173.2\angle 40^\circ}{8.944\angle 26.57^\circ} = 19.36\angle 13.43^\circ \text{ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120^\circ = 19.36\angle -106.57^\circ \text{ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle +120^\circ = 19.36\angle 133.43^\circ \text{ A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ = \sqrt{3}(19.36)\angle 13.43^\circ - 30^\circ$$

$$= 33.53\angle -16.57^\circ \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 33.53\angle -136.57^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ = 33.53\angle 103.43^\circ \text{ A}$$

Method 2:

Use Delta-Y transformation

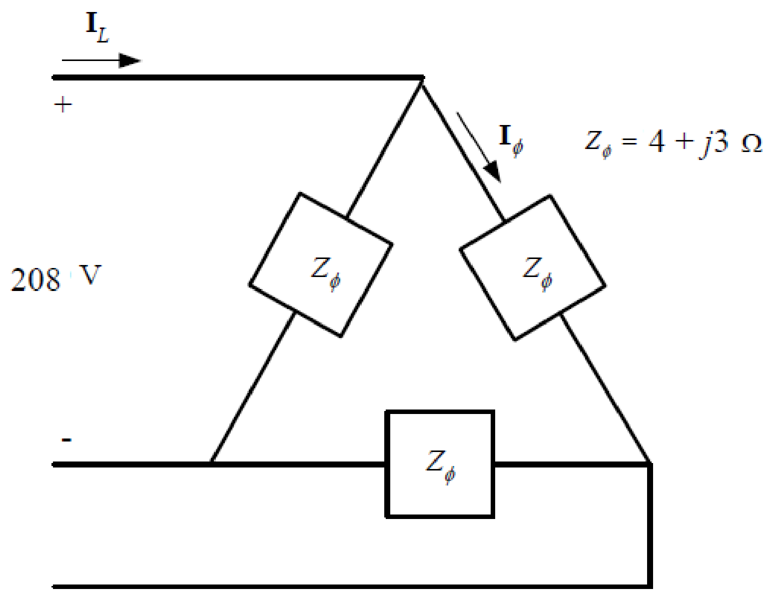
$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_\Delta/3} = \frac{100\angle 10^\circ}{2.981\angle 26.57^\circ} = 33.54\angle -16.57^\circ \text{ A}$$

as above. Other line currents are obtained using the *abc* phase sequence.

Problem # 4:

Three impedances of $4 + j3\ \Omega$ are Δ-connected and tied to a three-phase 208-V power line. Find I_ϕ, I_L, P, Q, S, and the power factor of this load.

Solution:



$V_L = V_\phi = 208\ \text{V}$, and $Z_\phi = 4 + j3\ \Omega = 5\angle 36.87°\ \Omega$, so

$$I_\phi = \frac{V_\phi}{Z_\phi} = \frac{208\ \text{V}}{5\ \Omega} = 41.6\ \text{A}$$

$$I_L = \sqrt{3} I_\phi = \sqrt{3}(41.6\ \text{A}) = 72.05\ \text{A}$$

$$P = 3\frac{V_\phi^2}{Z}\cos\theta = 3\frac{(208\ \text{V})^2}{5\ \Omega}\cos\ 36.87° = 20.77\ \text{kW}$$

$$Q = 3\frac{V_\phi^2}{Z}\sin\theta = 3\frac{(208\ \text{V})^2}{5\ \Omega}\sin\ 36.87° = 15.58\ \text{kvar}$$

$$S = \sqrt{P^2 + Q^2} = 25.96\ \text{kVA}$$

$$\text{PF} = \cos\ \theta = 0.8\ \text{lagging}$$

Problem # 5:

Figure below shows a one-line diagram of a simple power system containing a single 480 V generator and three loads. Assume that the transmission lines in this power system are lossless, and answer the following questions.

(a) Assume that Load 1 is Y-connected. What are the phase voltage and currents in that load?

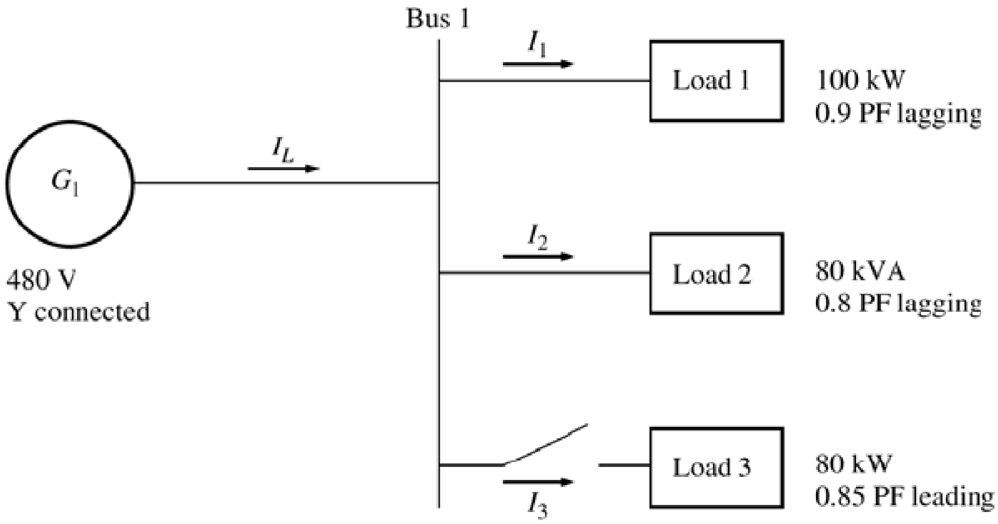
(b) Assume that Load 2 is Δ-connected. What are the phase voltage and currents in that load?

(c) What real, reactive, and apparent power does the generator supply when the switch is open?

(d) What is the total line current I_L when the switch is open?

(e) What real, reactive, and apparent power does the generator supply when the switch is closed?

(f) What is the total line current I_L when the switch is closed?



SOLUTION Since the transmission lines are lossless in this power system, the full voltage generated by G_1 will be present at each of the loads.

(a) Since this load is Y-connected, the phase voltage is

$$V_{\phi 1} = \frac{480 \text{ V}}{\sqrt{3}} = 277 \text{ V}$$

The phase current can be derived from the equation $P = 3V_\phi I_\phi \cos\theta$ as follows:

$$I_{\phi 1} = \frac{P}{3V_\phi \cos\theta} = \frac{100 \text{ kW}}{3(277 \text{ V})(0.9)} = 133.7 \text{ A}$$

(b) Since this load is Δ-connected, the phase voltage is

$$V_{\phi 2} = 480 \text{ V}$$

The phase current can be derived from the equation $S = 3V_\phi I_\phi$ as follows:

$$I_{\phi 2} = \frac{S}{3V_\phi} = \frac{80 \text{ kVA}}{3(480 \text{ V})} = 55.56 \text{ A}$$

(c) The real and reactive power supplied by the generator when the switch is open is just the sum of the real and reactive powers of Loads 1 and 2.

$$P_1 = 100 \text{ kW}$$

$$Q_1 = P\tan\theta = P\tan\left(\cos^{-1}\text{PF}\right) = (100 \text{ kW})(\tan 25.84°) = 48.4 \text{ kvar}$$

$$P_2 = S\cos\theta = (80 \text{ kVA})(0.8) = 64 \text{ kW}$$

$$Q_2 = S\sin\theta = (80 \text{ kVA})(0.6) = 48 \text{ kvar}$$

$$P_G = P_1 + P_2 = 100 \text{ kW} + 64 \text{ kW} = 164 \text{ kW}$$

$$Q_G = Q_1 + Q_2 = 48.4 \text{ kvar} + 48 \text{ kvar} = 96.4 \text{ kvar}$$

(d) The line current when the switch is open is given by $I_L = \dfrac{P}{\sqrt{3}\ V_L \cos\theta}$, where $\theta = \tan^{-1}\dfrac{Q_G}{P_G}$.

$$\theta = \tan^{-1}\frac{Q_G}{P_G} = \tan^{-1}\frac{96.4 \text{ kvar}}{164 \text{ kW}} = 30.45°$$

$$I_L = \frac{P}{\sqrt{3}\ V_L \cos\theta} = \frac{164 \text{ kW}}{\sqrt{3}(480 \text{ V})\cos(30.45°)} = 228.8 \text{ A}$$

(e) The real and reactive power supplied by the generator when the switch is closed is just the sum of the real and reactive powers of Loads 1, 2, and 3. The powers of Loads 1 and 2 have already been calculated. The real and reactive power of Load 3 are:

$$P_3 = 80 \text{ kW}$$

$$Q_3 = P\tan\theta = P\tan\left(\cos^{-1}\text{PF}\right) = (80 \text{ kW})\left[\tan(-31.79°)\right] = -49.6 \text{ kvar}$$

$$P_G = P_1 + P_2 + P_3 = 100 \text{ kW} + 64 \text{ kW} + 80 \text{ kW} = 244 \text{ kW}$$

$$Q_G = Q_1 + Q_2 + Q_3 = 48.4 \text{ kvar} + 48 \text{ kvar} - 49.6 \text{ kvar} = 46.8 \text{ kvar}$$

(f) The line current when the switch is closed is given by $I_L = \dfrac{P}{\sqrt{3}\ V_L \cos\theta}$, where $\theta = \tan^{-1}\dfrac{Q_G}{P_G}$.

$$\theta = \tan^{-1}\frac{Q_G}{P_G} = \tan^{-1}\frac{46.8 \text{ kvar}}{244 \text{ kW}} = 10.86°$$

$$I_L = \frac{P}{\sqrt{3}\ V_L \cos\theta} = \frac{244 \text{ kW}}{\sqrt{3}(480 \text{ V})\cos(10.86°)} = 298.8 \text{ A}$$