KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

ELECTRICAL ENGINEERING DEPARTMENT

EE 306 – Term 182

HW # 5: Synchronous Machines

Key Solutions

Problem # 1:

During a short-circuit test, a Y-connected synchronous generator produces 100 A of short-circuit armature current per phase at a field current of 2.5 A. At the same field current, the open-circuit line voltage is measured to be 440 V.

(a) Calculate the saturated synchronous reactance under these conditions.

(b) If the armature resistance is 0.3 Ω per phase, and the generator supplies 60 A to a purely resistive Y-connected load of 3 Ω per phase at this field current setting, determine the voltage regulation under these load conditions.

SOLUTION

(a) The saturated synchronous reactance at a field current of 2.5 A can be found from the information supplied in the problem. The open circuit line voltage at I_F = 2.5 A is 440 V, and the short-circuit current is 100 A. Since this generator is Y-connected, the corresponding phase voltage is $V_\phi = 440 \text{ V}/\sqrt{3} = 254 \text{ V}$ and the armature current is $I_A = 100 \text{ A}$. Therefore, the *saturated* synchronous reactance is

$$X_S = \frac{254 \text{ V}}{100 \text{ A}} = 2.54 \ \Omega$$

(b) Assume that the desired line voltage is 440 V, which means that the phase voltage $\mathbf{V}_\phi = 254\angle 0^\circ \text{ V}$. The armature current is $\mathbf{I}_A = 60\angle 0^\circ \text{ A}$, so the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$
$$\mathbf{E}_A = 254\angle 0^\circ + (0.30\ \Omega)(60\angle 0^\circ \text{ A}) + j(2.54\ \Omega)(60\angle 0^\circ \text{ A})$$
$$\mathbf{E}_A = 312\angle 29.3^\circ \text{ V}$$

This is also the phase voltage at no load conditions. The corresponding line voltage at no load conditions would be $V_{L,\text{nl}} = (312 \text{ V})(\sqrt{3}) = 540 \text{ V}$. The voltage regulation is

$$\text{VR} = \frac{V_{T,\text{nl}} - V_{T,\text{fl}}}{V_{T,\text{fl}}} \times 100\% = \frac{540 - 440}{440} \times 100\% = 22.7\%$$

Problem # 2:

The internal generated voltage E_A of a **2-pole, Δ-connected, 60 Hz,** three phase synchronous generator is 14.4 kV, and the terminal voltage V_T is 12.8 kV. The synchronous reactance of this machine is 4 Ω, and the armature resistance can be ignored.

(a) If the torque angle of the generator $\delta = 18°$, how much power is being supplied by this generator at the current time?

(b) What is the power factor of the generator at this time?

(c) Sketch the phasor diagram under these circumstances.

(d) Ignoring losses in this generator, what torque must be applied to its shaft by the prime mover at these conditions?

SOLUTION

(a) If resistance is ignored, the output power from this generator is given by

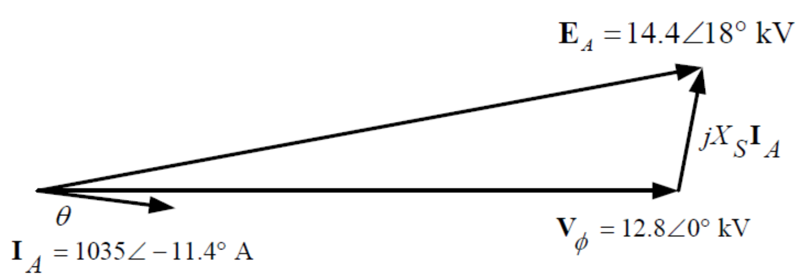
$$P = \frac{3V_\phi E_A}{X_S}\sin\delta = \frac{3(12.8\text{ kV})(14.4\text{ kV})}{4\ \Omega}\sin 18° = 42.7\text{ MW}$$

(b) The phase current flowing in this generator can be calculated from

$$\mathbf{I}_A = \frac{14.4\angle 18°\text{ kV} - 12.8\angle 0°\text{ kV}}{j4\ \Omega} = 1135\angle -11.4°\text{ A}$$

Therefore the impedance angle $\theta = 11.4°$, and the power factor is $\cos(11.4°) = 0.98$ lagging.

(c) The phasor diagram is



(d) The induced torque is given by the equation

$$P_{\text{conv}} = \tau_{\text{ind}}\omega_m$$

With no losses,

$$\tau_{\text{app}} = \tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} = \frac{42.7\text{ MW}}{2\pi(60\text{ hz})} = 113{,}300\text{ N•m}$$

Problem # 3:

A 480-V, 60 Hz, 400-hp 0.8-PF-leading eight-pole Δ-connected synchronous motor has a synchronous reactance of 0.6 Ω and negligible armature resistance. Ignore its friction, windage, and core losses for the purposes of this problem. Assume that $|\mathbf{E}_A|$ is directly proportional to the field current I_F (in other words, assume that the motor operates in the linear part of the magnetization curve), and that $|\mathbf{E}_A|$ = 480 V when I_F = 4 A.

(a) What is the speed of this motor?

(b) If this motor is initially supplying 400 hp at 0.8 PF lagging, what are the magnitudes and angles of $\mathbf{E}_A$ and $\mathbf{I}_A$?

(c) How much torque is this motor producing? What is the torque angle δ? How near is this value to the maximum possible induced torque of the motor for this field current setting?

(d) If $|\mathbf{E}_A|$ is increased by 30 percent, what is the new magnitude of the armature current? What is the motor's new power factor?

SOLUTION

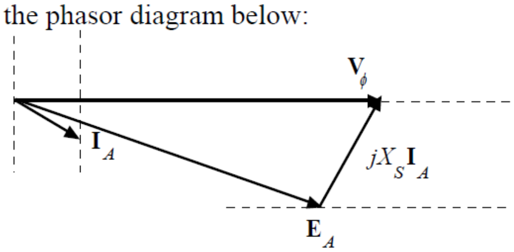
(a) The speed of this motor is given by

$$n_m = \frac{120 f_{se}}{P} = \frac{120(60\text{ Hz})}{8} = 900\text{ r/min}$$

(b) If losses are being ignored, the output power is equal to the input power, so the input power will be

$$P_{\text{IN}} = (400\text{ hp})(746\text{ W/hp}) = 298.4\text{ kW}$$

This situation is shown in the phasor diagram below:



The line current flow under these circumstances is

$$I_L = \frac{P}{\sqrt{3}\, V_T\ \text{PF}} = \frac{298.4\text{ kW}}{\sqrt{3}(480\text{ V})(0.8)} = 449\text{ A}$$

Because the motor is Δ-connected, the corresponding phase current is $I_A = 449/\sqrt{3} = 259\text{ A}$. The angle of the current is $-\cos^{-1}(0.80) = -36.87°$, so $\mathbf{I}_A = 259\angle -36.87°\text{ A}$. The internal generated voltage $\mathbf{E}_A$ is

$$\mathbf{E}_A = \mathbf{V}_\phi - jX_S\mathbf{I}_A$$

$$\mathbf{E}_A = (480\angle 0°\text{ V}) - j(0.6\ \Omega)(259\angle -36.87°\text{ A}) = 406\angle -17.8°\text{ V}$$

(c) This motor has 6 poles and an electrical frequency of 60 Hz, so its rotation speed is n_m = 1200 r/min. The induced torque is

$$\tau_{\text{ind}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{298.4 \text{ kW}}{(900 \text{ r/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)} = 3166 \text{ N}\cdot\text{m}$$

The maximum possible induced torque for the motor at this field setting is the maximum possible power divided by ω_m

$$\tau_{\text{ind,max}} = \frac{3V_\phi E_A}{\omega_m X_S} = \frac{3(480 \text{ V})(406 \text{ V})}{(900 \text{ r/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)(0.6\ \Omega)} = 10{,}340 \text{ N}\cdot\text{m}$$

The current operating torque is about 1/3 of the maximum possible torque.

(d) If the magnitude of the internal generated voltage E_A is increased by 30%, the new torque angle can be found from the fact that $E_A \sin\delta \propto P = \text{constant}$.

$$E_{A2} = 1.30\ E_{A1} = 1.30(406 \text{ V}) = 487.2 \text{ V}$$

$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_1\right) = \sin^{-1}\left(\frac{406 \text{ V}}{487.2 \text{ V}}\sin(-17.8^\circ)\right) = -14.8^\circ$$

The new armature current is

$$\mathbf{I}_{A2} = \frac{\mathbf{V}_\phi - \mathbf{E}_{A2}}{jX_S} = \frac{480\angle 0^\circ \text{ V} - 487.2\angle -14.8^\circ \text{ V}}{j0.6\ \Omega} = 208\angle -4.1^\circ \text{ A}$$

The magnitude of the armature current is 208 A, and the power factor is cos (-24.1°) = 0.913 lagging.

Problem # 4:

A 3-phase, 5 kVA, 208 V, four-pole, 60 Hz, star-connected synchronous machine has negligible stator winding resistance and a synchronous reactance of 8 Ω per phase at rated terminal voltage. This synchronous machine is operated as a synchronous motor from the 3-phase, 208 V, 60 Hz power supply. The field excitation is adjusted so that the power factor is unity when the machine draws 3 kW from the supply.

(a) Find the excitation voltage and the power angle. Draw the phasor diagram for this condition,

(b) If the field excitation is held constant and the shaft load is slowly increased, determine the maximum torque (i.e., pull-out torque) that the motor can deliver.

Solution:

(a) $3V_t I_a \cos\phi = 3\text{ kW} = 3V_t I_a$ for $\cos\phi = 1$.

$$I_a = \frac{3000}{3 \times 120} = 8.33\text{ A}$$

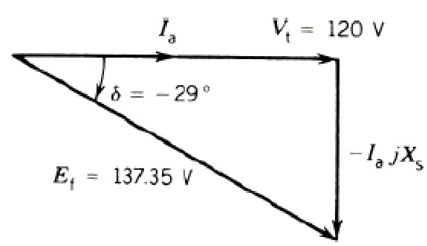
$$E_f = V_t - I_a jX_s$$
$$= 120\angle 0^\circ - 8.33\angle 0^\circ \cdot 8\angle 90^\circ$$
$$= 137.35\angle -29^\circ$$

Excitation voltage $E_f = 137.35$ V/phase

Power angle $\delta = -29^\circ$

Note that because of motor action, the power angle is negative.

$$E_f = \sqrt{|V_t|^2 + |I_a X_s|^2} = \sqrt{120^2 + (8.33 \times 8)^2}$$
$$= 137.35\text{ V/phase}$$
$$\tan\delta = \frac{|I_a X_s|}{|V_t|} = \frac{8.33 \times 8}{120} = 0.555$$
$$|\delta| = 29^\circ$$
$$\delta = -29^\circ$$



(b)

Maximum torque will be developed at $\delta = 90^o$, and since $P_{max} = \dfrac{3|V_t||E_f|}{|X_s|}$

Therefore, $P_{max} = \dfrac{3 \times 137.35 \times 120}{8}$, and

$$T_{max} = \frac{P_{max}}{\omega_{syn}} = \frac{6180.75}{1800/60 \times 2\pi} = 32.8\text{ N. m}$$