HW3 Due UT Classes Feb. 19th 2019; MW Classes Feb. 20th 2019

Key Solution

Problem 1:

A 480/240V, 4.8kV A, 60Hz, single-phase transformer is used to supply a 4.8kV A load with a 0.8 lagging power factor, at rated voltage (240V)

1. If the transformer were ideal, what would be the magnitude of the current on the primary (480V) side?

2. What is the impedance of the load under the ideal assumption?

3. Again, if the transformer is ideal, what would the impedance be as viewed from the primary side?

Solution:

a. The complex power at the load is

$$\bar{S}_2 = 4.8[0.8 + j\sin(\arccos(0.8))]\ kVA = 4.8\angle 36.87\ kVA$$

Assuming an ideal (lossless) transformer, where the input power is equal to the output power ($\bar{S}_1 = \bar{S}_2$), the primary side current is

$$\bar{I}_1 = (\frac{\bar{S}_1}{\bar{V}_1})^* = \frac{4.8\angle -36.87\ kVA}{480\angle 0\ V} = 10\angle -36.87\ A$$

b. With the primary side voltage angle as the reference angle:

$$\bar{Z}_2 = \frac{\bar{V}_2}{\bar{I}_2} = \frac{\bar{V}_2\bar{V}_2^*}{\bar{S}_2^*} = \frac{240^2}{4800\angle -36.87} = 12\angle 36.87\ \Omega$$

c. With the primary side voltage angle as the reference angle:

$$\bar{Z}_1 = \frac{\bar{V}_1}{\bar{I}_1} = \frac{480\angle 0}{10\angle -36.87} = 48\angle 36.87\ \Omega$$

Problem 2:

A 15-kVA 8000/230-V distribution transformer has an impedance referred to the primary of 80 + j300Ω. The components of the excitation branch referred to the primary side are R_C = 350 kΩ and X_M = 70 kΩ.

(a) If the primary voltage is 7967 V and the load impedance is Z_L = 3.0 +j1.5 Ω, what is the secondary voltage of the transformer? What is the voltage regulation of the transformer?

(b) If the load is disconnected and a capacitor of –j4.0 Ω is connected in its place, what is the secondary voltage of the transformer? What is its voltage regulation under these conditions?

Solution:

(a) The turns ratio is

$$a = 8000/230 = 34.78$$

The load impedance referred to the primary side is

$$Z_L^P = a^2 Z_L = (34.78)^2 (3.0 + j1.5) = 3629 + j1815\,\Omega$$

The referred secondary current is

$$I_S^P = \frac{V_P}{Z_i^P + Z_L^P} = \frac{7967\angle 0^\circ}{(80 + j300) + (3629 + j1815)} = 1.87\angle -29.7^\circ \text{ A}$$

The referred secondary voltage is

$$V_S^P = I_S^P Z_L^P = (1.87\angle -29.7^\circ)(3629 + j1815) = 7588\angle -3.1^\circ \text{ V}$$

The actual secondary voltage is

$$V_S = \frac{V_S^P}{a} = \frac{7588\angle -3.1^\circ}{34.78} = 218.2\angle -3.1^\circ \text{ V}$$

The voltage regulation is

$$VR = \frac{V_P - V_S^P}{V_S^P} = \frac{7967 - 7588}{7588} \times 100\% = 4.99\%$$

(b) The turns ratio is

$$a = 8000/230 = 34.78$$

The load impedance referred to the primary side is

$$Z_L^P = a^2 Z_L = (34.78)^2(-j4.0) = -j4839\,\Omega$$

The referred secondary current is

$$I_S^P = \frac{V_P}{Z_i^P + Z_L^P} = \frac{7967\angle 0^\circ}{(80 + j300) + (-j4839)} = 1.75\angle 89.0^\circ \text{ A}$$

The referred secondary voltage is

$$V_S^P = I_S^P Z_L^P = (1.75\angle 89.0^\circ)(-j4839) = 8468\angle -1.0^\circ \text{ V}$$

The actual secondary voltage is

$$V_S = \frac{V_S^P}{a} = \frac{8468\angle -1.0^\circ}{34.78} = 243.2\angle -1.0^\circ \text{ V}$$

The voltage regulation is

$$VR = \frac{V_P - V_S^P}{V_S^P} = \frac{7967 - 8468}{8468} \times 100\% = -5.92\%$$

Problem 3:

A 250 kVA, 3600/240 V, single-phase transformer has the following test data:

	Voltage (V)	Current (A)	Power (W)
O/C Test	240	57.85	4985
S/C Test	187	69.45	4823

Find:

a) The approximate equivalent circuit referred to HV and LV side.

a) The voltage regulation and efficiency when the load takes 1100 A at 220 V and 0.6 lag pf. (NOTE: this is not rated load).

b) The voltage regulation and efficiency at rated load conditions and 0.8 lag pf.

Solution:

From the OC Test:

$$pf_{oc} = \frac{4985}{240 \times 57.85} = 0.359 \text{ lag} \quad \text{then:} \quad \theta_{oc} = \cos^{-1} 0.359 = 69^0$$

$$I_{oc} = 57.85\angle{-69} = 20.77 - j54 \text{ A}$$

$$R_c = \frac{240}{20.77} = 11.55\,\Omega \quad \text{and:} \quad X_m = \frac{240}{54} = 4.445\,\Omega$$

These values are referred to the *lv* side. Referring to the *hv* side gives:

$R_c = 2600\ \Omega$ and $X_m = 1000\ \Omega$

From the SC Test:

$$pf_{sc} = \frac{4823}{187 \times 69.45} = 0.3714 \text{ lag} \quad \text{then:} \quad \theta_{sc} = \cos^{-1} 0.3713 = 68.2^0$$

$$I'_2 = 69.45\angle{-68.2} \text{ A}$$

$$Z = \frac{187\angle 0}{69.45\angle{-68.2}} = 2.693\angle 68.2\ \Omega \quad \text{then convert to rectangular to get:}$$

$R = 1.0\ \Omega$ and $X = 2.5\ \Omega$, which are referred to the *hv* side

a) $\boldsymbol{I_2} = 1100\angle{-53.1}$ A $\therefore$ $\boldsymbol{I'_2} = 73⅓\angle{-53.1}$ A $\quad$ $\boldsymbol{V_2} = 220\angle 0$ V $\quad$ $\boldsymbol{V'_2} = 3300\angle 0^0$

$\boldsymbol{V_p} = 3300\angle 0^0 + (1.0 + j2.5) \times 73⅓\angle{-53.1^0} = 3491\angle 0.8^0$ V

$$VR = \frac{3491 - 3300}{3300} x100\% = 5.79\%$$

$$I_p = I_2' + \frac{V_p}{R_c} + \frac{V_p}{jX_m} = 73.33\angle -53.1 + \frac{3491\angle 0.8}{2600} + \frac{3491\angle 0.8}{j1000} = 76.95\angle{-53.9^0} \text{ A}$$

$P_{in} = \text{Re}\{3491\angle 0.8 \times 76.95\angle{+53.9}\} = 155.3$ kW

and $\quad P_{out} = 220 \times 1100 \times 0.6 = 145.2$ kW

$$\eta = \frac{145.2}{155.3} x100\% = 93.5\%$$

b) $\boldsymbol{I_2} = \dfrac{250 \times 10^3}{240} \angle -\cos^{-1} 0.8 = 1042\angle -36.9^0$ A $\quad \therefore \quad \boldsymbol{I'_2} = \dfrac{I_2}{a} = 69.44\angle -36.9^0$ A

$\boldsymbol{V_p} = 3600\angle 0^0 + (1.0 + j2.5) \times 69.44\angle{-36.9^0} = 3761\angle 1.5^0$ V

$$VR = \frac{3761 - 3600}{3600} x100\% = 4.47\%$$

$$I_p = I_2' + \frac{V_p}{R_c} + \frac{V_p}{jX_m} = 69.44\angle -36.9 + \frac{3761\angle 1.5}{2600} + \frac{3761\angle 1.5}{j1000} = 72.94\angle{-38.5^0} \text{ A}$$

$P_{in} = \text{Re}\{3761\angle 1.5 \times 72.94\angle{+38.5^0}\} = 210.3$ kW

and $\quad P_{out} = 250 \times 10^3 \times 0.8 = 200$ kW

$$\eta = \frac{200}{210.3} x100\% = 95.1\%$$

Problem 4:

A 1φ, 25 kVA, 2300=230 V transformer has the following parameters:

$$Z_{eq,H} = 4.0 + j5.0\ \Omega$$
$$R_{c,L} = 450\ \Omega$$
$$X_{m,L} = 300\ \Omega$$

(a) The transformer is connected to a load whose power factor varies. Determine the worst-case voltage regulation for full-load output, and draw the phasor diagram of this case.

(b) Determine efficiency when the transformer delivers full load at rated voltage and 0.85 power factor lagging.

(c) Determine the percentage loading of the transformer at which the efficiency is a maximum and calculate this efficiency if the power factor is 0.85 and load voltage is 230 V.

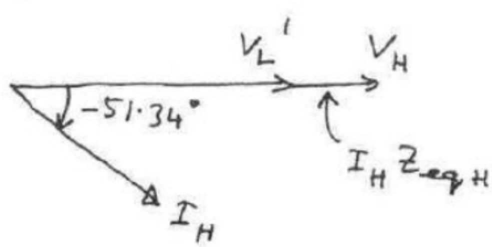
Solution:

a.

$\theta_{eq} = \tan^{-1}\frac{5}{4} = 51.34°$

For worst case VR → $\theta_2 = -51.34°$

and V_L' and V_H are in phase.



$I_H = \frac{25000}{2300} = 10.87\text{A}$

$Z_{eq\,H} = \sqrt{4^2 + 5^2} = 6.4\ \Omega$

$I_H Z_{eq\,H} = 10.87 \times 6.4 = 69.6\text{ V}$

$VR = \frac{69.6}{2300} \times 100\% = 3.03\%$

b.

$$P_{out} = 25 \times 0.85 = 21.25\,kW$$

$$P_{cu} = I_H^2 R_{eq\,H} = 10.87^2 \times 4 = 472.63\,W$$

$$P_{core} = \frac{230^2}{450} = 117.56\,W$$

$$Eff = \frac{21{,}250}{21{,}250 + 472.63 + 117.56} \times 100\% = 97.3\%$$

c.

$$X = \sqrt{\frac{117.56}{472.63}} = 0.499$$

$$P_{cu} = P_{core} = 117.56\,W$$

$$P_{out} = 25 \times 0.499 = 12.475\,kW$$

$$Eff = \frac{12475}{12475 + 117.56 + 117.56} \times 100\% = 98.15\%$$