# KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
# ELECTRICAL ENGINEERING DEPARTMENT

## EE 306 – Term 181

**HW # 6: Induction Motors**

**Due Date: (UT-Classes, Dec. 11th, 2018; MW-Classes, Dec. 12th, 2018)**

**Key Solutions**

**Problem # 1:**

A three-phase, four-pole, 30-hp, 220-V, 60-Hz, Y-connected induction motor draws a current of 77-A from the supply line at a power factor of 0.88. At this operating condition, the motor losses are known to be the following:

Stator copper losses = 1033 W ; Rotor copper losses = 1299 W

Core loss = 485 W ; Mechanical losses (friction & windage) = 540 W

Determine:

a) the power transferred across the air gap

b) the internally developed torque in Newton-meters

c) the slip expressed in percentage

d) the mechanical power developed in watts

e) the horsepower output

f) the motor speed in rpm

g) the torque at the output shaft

h) the efficiency of operation

$P_{cu1} = 1033\ W$

$P_{cu2} = 1299\ W$

$P_{core} = 485\ W$

$P_{F\&W} = 540\ W$

$V_1 = 220/\sqrt{3} = 127\ V$

$P_{in} = 3 V_1 I_1 \cos \widehat{V_1 I_1} = 3 \times 127 \times 77 \times 0.88$

$\boxed{P_{in} = 25.8\ KW}$

$P_g = P_{in} - P_{cu1} - P_{core} = 25.8 \times 10^3 - 1033 - 485$

a) $\boxed{P_g = 24.3\ KW}$

(b) $T_d = \frac{P_g}{\omega_s}$ , $\omega_s = \frac{2\pi n_s}{60}$ , $n_s = \frac{120 \times f}{P} = \frac{120 \times 60}{4} = 1800$ rpm

$$\omega_s = \frac{2\pi \times 1800}{60} = 188.5 \text{ rad/sec}$$

$$T_d = \frac{24.3 \times 10^3}{188.5}$$

$$\boxed{T_d = 128.9 \text{ N-m}}$$

(c) $S = \frac{P_{cu2}}{P_g} = \frac{1299}{24.3 \times 10^3} \Rightarrow S = 0.0534$

$$\boxed{S = 5.34\%}$$

(d) $P_d = (1-S) \times P_g = (1 - 0.0534) \times 24.3 \times 10^3$

$$\boxed{P_d = 23.0 \text{ KW}}$$

(e) $P_o = P_d - P_{F\&W} = 23 \times 10^3 - 540 = 22.46$ KW

$P_o = \frac{22.46 \times 10^3}{746} \Rightarrow \boxed{P_o = 30 \text{ hp}}$

(f) $n_m = (1-S) \times n_s = (1 - 0.0534) \times 1800 \Rightarrow \boxed{n_s = 1704 \text{ rpm}}$

(g) $T_o = \frac{P_o}{\omega} = \frac{22.46 \times 10^3}{\times 1704} \Rightarrow \boxed{T_o = 126 \text{ N-m}}$ (h) $\eta = \frac{P_o}{P_{in}} = \frac{22.46}{25.8}$

**Problem # 2:**

A 3-phase, Y-connected, 220-V, 10-HP, 60-Hz, 6-pole induction motor has the following parameters in ohms.

$R_1 = 0.294\ \Omega$, $R_2 = 0.144\ \Omega$, $X_1 = 0.503\ \Omega$, $X_2 = 0.209\ \Omega$, $X_m = 13.25\ \Omega$

The total friction, windage and core losses may be assumed to be constant at 403 W, independent of load. For a slip of 3%, determine:

(a) the rotor current, developed torque and developed power.

(b) the maximum developed torque and the corresponding speed.

(c) the starting torque and starting current.

(d) How much resistance must be inserted in the rotor circuit to bring the motor speed at maximum torque down to 900 rpm?

(e) the new starting torque.

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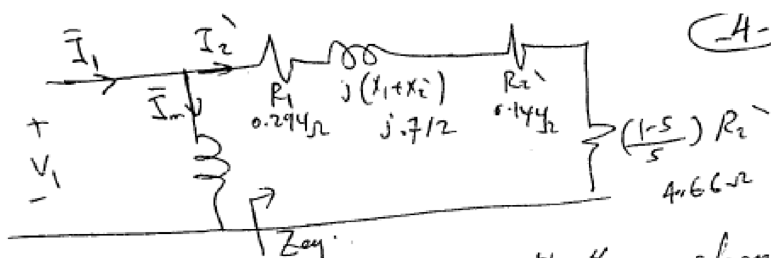


$P_{Rot} = 403$ W

$n_s = 1200$ rpm

$V_1 = \frac{220}{\sqrt{3}} = 127$ V

Since the core loss is lumped with the mechanical losses, the core resistance will be removed from the equivalent circuit.

$$S = 0.03 \Rightarrow \frac{1-S}{S} \times R_2' = \frac{1-0.03}{0.03} \times 0.144 = 4.66\ \Omega$$

$$Z_{eq} = \left(R_1 + \frac{R_2'}{S}\right) + j(X_1 + X_2') = \left(0.294 + \frac{0.144}{0.03}\right) + j(0.503 + .209)$$

$$\boxed{Z_{eq} = 5.14 \angle 7.95^\circ\ \Omega}$$

$$V_1 = 220/\sqrt{3} = 127\ V$$

(a) $$\bar{I}_2' = \frac{V_1}{Z_{eq}} = \frac{127 \angle 0}{5.14 \angle 7.95^\circ} \Rightarrow \boxed{\bar{I}_2' = 24.7 \angle -7.95^\circ\ A}$$

$$P_d = 3 I_2'^2 \frac{(1-S)}{S} R_2' = 3(24.7)^2 (4.66) \Rightarrow P_d = 8.52 \text{ kW}$$

$$n = (1-S) n_s = (1-0.03) * 1200 \Rightarrow n = 1164 \text{ rpm}$$

$$T_d = \frac{P_d}{\omega_m} = \frac{P_d}{\frac{2\pi n}{60}} = \frac{8.52 * 10^3}{\frac{2\pi(1164)}{60}} \Rightarrow \boxed{T_d = 69.9 \text{ N-m}}$$

(b) $$T_{max} = \frac{3V_1^2}{2\omega_s} \cdot \frac{1}{\left[R_1 + \sqrt{R_1^2 + (X_1+X_2')^2}\right]} = \frac{3*(127)^2}{2\left(\frac{2\pi * 1200}{60}\right)} * \frac{1}{\left[0.294 + \sqrt{(0.294)^2 + (0.712)^2}\right]}$$

$$\boxed{T_{max} = 181 \text{ N-m}}$$

$$S_{max} = \frac{R_2'}{\sqrt{R_1^2 + (X_1+X_2')^2}} = \frac{0.144}{\sqrt{(0.294)^2 + (0.712)^2}} = 0.187$$



(c) $$|I_{2\,start}'| = \frac{V_1}{\sqrt{(R_1+R_2')^2 + (X_1+X_2')^2}} = \frac{127}{\sqrt{(0.438)^2 + (0.712)^2}}$$

$$\theta = \tan^{-1}\frac{(X_1+X_2')}{R_1+R_2'} = 58.5^\circ$$

$$\boxed{\bar{I}_{2\,start}' = 151.9 \angle{-58.5^\circ} \text{ A}}$$

$$\bar{I}_m = \frac{\bar{V}_1}{jX_m} = \frac{127\angle 0}{13.25\angle 90} = -j9.6 \text{ A}$$

$$\bar{I}_{1\,start} = \bar{I}_m + \bar{I}_2' = 9.6\angle{-90} + 151.9\angle{-58.5^\circ}$$

$$\boxed{\bar{I}_{1\,start} = 159.6 \angle{-60.3^\circ} \text{ A}}$$

$$T_{d\,starting} = \frac{3 I_{2\,start}'^2 \cdot R_2'}{\omega_s} = \frac{3V_1^2}{\omega_s} \cdot \frac{R_2'}{(R_1+R_2')^2 + (X_1+X_2')^2}$$

$$T_{d\,starting} = \frac{3(127)^2 * (0.144)}{(0.438)^2 + (0.712)^2} * \frac{1}{\frac{2\pi * 1200}{60}}.$$

$$\boxed{T_{d\,starting} = 79.\ \text{N-m}}$$

(d) $n_{T_{max}} = 900$ rpm $\Rightarrow S_{max} = \frac{n_s - n}{n_s} = \frac{1200-900}{1200} = 0.25$

$\Rightarrow$ max. Torque $\Rightarrow S_{max} = \frac{R'_{2new}}{\sqrt{R_1^2 + (X_1 + X'_2)^2}} = 0.25$

$R'_{2new} = 0.25 * \sqrt{[(0.294)^2 + (0.712)^2]} = 0.193\,\Omega$

$R_{2\,added} = R'_{2new} - R'_2 = 0.193 - 0.144$

$$\boxed{R_2 \text{ added} = 0.0486\,\Omega}$$

$\Rightarrow \boxed{T_{starting\,new} = 100\ \text{N-m}}$