# KING FAHD UNIVERSITY OF PETROLEUM \& MINERALS ELECTRICAL ENGINEERING DEPARTMENT 

## EE 306 - Term 181

HW \# 2: Magnetic Circuits
Key Solutions

## Problem \# 1:

In the figure below, the flux density at cross section $\mathrm{A}_{1}$ is $B_{1}=0.4 \mathrm{~T}$, determine $B_{2}$.


Solution $\quad \Phi=B_{1} \times A_{1}=(0.4 \mathrm{~T})\left(2 \times 10^{-2} \mathrm{~m}^{2}\right)=0.8 \times 10^{-2} \mathrm{~Wb}$. Since all flux is confined to the core, the flux at cross section 2 is the same as at cross section 1 . Therefore,

$$
B_{2}=\Phi / A_{2}=\left(0.8 \times 10^{-2} \mathrm{~Wb}\right) /\left(1 \times 10^{-2} \mathrm{~m}^{2}\right)=0.8 \mathrm{~T}
$$

## Problem \# 2:

A 3-legs core and its $B-H$ curve are shown below. All dimensions are in mm. Calculate the required current to be passed in the coil having 200 turns in order to establish a flux of 1.28 mWb in the air gap. Neglect fringing effect and leakage flux.



## Solution:

The equivalent magnetic circuit is

$\phi_{g}=\phi_{2}=1.28 \times 10^{-3}$
Cross sectional area of central $\operatorname{limb} A_{2}=16 \times 10-4 \mathrm{~m}^{2}$

$$
\begin{gathered}
\text { Flux density } B_{g}=B_{2}=\frac{1.28 \times 10^{-3}}{16 \times 10^{-4}} \mathrm{~T}=0.8 \mathrm{~T} \\
\therefore, H_{g}=\frac{B_{g}}{\mu_{0}}=\frac{0.8}{4 \pi \times 10^{-7}} \mathrm{AT} / \mathrm{m}=63.66 \times 10^{4} \mathrm{AT} / \mathrm{m}
\end{gathered}
$$

mmf required for gap $H_{g} l_{g}=63.66 \times 10^{4} \times 1 \times 10^{-4} \mathrm{AT}=63.66 \mathrm{AT}$
flux density, $B_{2}=0.8 \mathrm{~T} \because$ fringing \& leakage neglected
corresponding H from graph, $H_{2} \approx 500 \mathrm{AT} / \mathrm{m}$
Mean iron length, $l_{2}=(440-0.1) \mathrm{mm} \approx 0.44 \mathrm{~m}$
mmf required for iron portion, $\mathrm{H}_{2} l_{2}=220 \mathrm{AT}$
Total mmf required for iron \& air gap, $\quad=\quad(220+63.66) \mathrm{AT}$

$$
m m f_{\mathrm{CD}}=283.66 \mathrm{AT} .
$$

Due to parallel connection, mmf acting across path 1 is same as mmf acting across path 2 . Our intention here, will be to calculate $\phi_{1}$ in path 1 .
mean length of the path, $l_{1}=l_{D E}+l_{E F}+l_{F C}=2 \times 170+440 \mathrm{~mm}=0.78 \mathrm{~m}$

$$
\because H_{1}=\frac{283.66}{0.78}=363.67 \mathrm{AT} / \mathrm{m}
$$

corresponding flux density from graph, $B_{1} \approx 0.39 \mathrm{~T}$

$$
\therefore \text { flux, } \phi_{1}=B_{1} A_{1}=0.39 \times 24 \times 10^{-4}=0.94 \times 10^{-3} \mathrm{~Wb}
$$

calculate the mmf necessary to drive $\phi_{3}$ in path 3 as follows.

$$
\text { flux in path } 3, \phi_{3}=\phi_{1}+\phi_{2}=2.22 \times 10^{-3} \mathrm{~Wb}
$$

$$
\text { flux density, } B_{3}=\frac{\phi_{3}}{\mathrm{~A}_{3}}=\frac{2.22 \times 10^{-3}}{24 \times 10^{-4}}=0.925 \mathrm{~T}
$$

corresponding H from graph, $H_{3} \approx 562.5 \mathrm{AT} / \mathrm{m}$
total mmf required for path $3=H_{3} l_{3}=562.5 \times 0.78 \mathrm{AT}=438.7 \mathrm{AT}$
$\therefore \mathrm{mmf}$ to be supplied by the coil, $N I=283.66+438.7 \mathrm{AT}$

$$
\text { or } 200 I=722.36 \mathrm{AT}
$$

$\therefore$ exciting current needed, $I=\frac{722.36}{200} \mathrm{~A}=3.61 \mathrm{~A}$

Problem \# 3:
For the magnetic circuit and the corresponding magnetizing-curve ( $B-H$ curves) shown below

(mean length)

a. Find the value of $I$ required to develop a magnetic flux of $\Phi=6 \times 10^{-4} \mathrm{~Wb}$.
b. Determine $\mu$ and $\mu_{r}$ for the material under these conditions.

## Solution:

The magnetic circuit can be represented as

a. The flux density is

$$
B=\frac{\Phi}{A}=\frac{6 \times 10^{-4} \mathrm{~Wb}}{2 \times 10^{-3} \mathrm{~m}^{2}}=0.3 \mathrm{~T}
$$

Using the cast-steel $B-H$ curve

$$
H=200 \mathrm{At} / \mathrm{m} \quad ; \mathrm{I}=(200 \times 0.16) / 400=0.08 \mathrm{~A}
$$

b. The permeability of the material is

$$
\mu=\frac{B}{H}=1.5 \times 10^{-3} \mathrm{~Wb} / \mathbf{A} \cdot \mathrm{m}
$$

The relative permeability is

$$
\mu_{r}=\frac{\mu}{\mu_{o}}=1193.7
$$

## Problem \# 4:

An electromagnet has a magnetic-circuit that can be regarded as comprising three parts in series, each of uniform cross-sectional area.
(i) a length of 8 cm and cross-section area of $0.511 \mathrm{~cm}^{2}$.
(ii) a length of 6 cm and cross-sectional area of $0.9 \mathrm{~cm}^{2}$.
(iii) an air-gap of length 0.5 mm and cross-sectional area of $1.5 \mathrm{~cm}^{2}$.

Parts (i) and (ii) are of a material having a magnetic characteristic given by the following table:

| $H(\mathrm{AT} / \mathrm{m})$ | 100 | 210 | 340 | 500 | 800 | 1500 |
| :--- | ---: | ---: | ---: | ---: | ---: | :--- |
| $B\left(\mathrm{wb} / \mathrm{m}^{2}\right)$ | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 |

Determine the current necessary in a coil of 4000 turns wound on part (ii) to produce in the air-gap a flux density of $0.3 \mathrm{wb} / \mathrm{m}^{2}$.

## Solution:

$$
\begin{aligned}
& \begin{array}{cccccc}
i & 8 & 0.511 & 45 & 0.88 & 620 \\
i j & 6 & 0.9 & 45 & 0.5 & 275 \\
\text { dep }_{\text {ep }} & 0.05 & 1.5 & 45 & .3 & 124 \times 10^{6}
\end{array} \\
& \phi_{y}=.3 \times 1.5 \times 10^{-4}=45 \mu \text {, all part are in spices } \\
& A T=\Sigma H \ell=620 * 0.08+275 * 0.06 \\
& +.2387 \times 10^{6} \times 0.05 \times 10^{-2}=185.5 \\
& I=A T / 4000=46.4 \mathrm{~mA} .
\end{aligned}
$$

## Problem \# 5:

The magnetic circuit shown has an infinitely permeable magnetic core.

$\mathrm{g}_{1}=5 \mathrm{~mm}$
$A_{1}=5 \mathrm{~cm}^{2}$
$\mathrm{g}_{2}=5 \mathrm{~mm}$
$\mathrm{g}_{3}=10 \mathrm{~mm}$
$\mathrm{A}_{2}=5 \mathrm{~cm}^{2}$
$\mathrm{~A}_{3}=10 \mathrm{~cm}^{2}$
$\mathrm{N}_{1}=80$ turns
$\mathrm{N}_{2}=100$ turns
$\mathrm{N}_{3}=125$ turns

A current of 12 A flows in the first coil $\mathrm{N}_{1}$. The second and third coils $\mathrm{N}_{2}$ and $\mathrm{N}_{3}$ respectively, are unexcited. Determine the flux densities in each of the air gaps $g_{1}, g_{2}$, and $g_{3}$.

## Solution:

$$
\begin{aligned}
F_{1} & =80 \times 12=960 \mathrm{At} \\
R_{g_{2}}=R_{g_{1}} & =\frac{5 \times 10^{-3}}{r_{0} \times 5 \times 10^{-4}}=7.950^{\circ} \times 10^{6} \mathrm{At} / \mathrm{Wh} \\
R_{g_{3}} & =\frac{10 \times 10^{-3}}{\mu_{0} \times 10 \times 10^{-4}}=7.958 \times 10^{6} \mathrm{At} / \mathrm{wb} \\
R_{t} & =R_{g_{1}}+\frac{R_{g_{2}} R_{93}}{R_{g_{2}}+R_{g_{3}}}=11.937 \times 10^{6} \mathrm{At} / \mathrm{Wb} \\
\phi_{1} & =\frac{F_{1}}{R_{t}}=\frac{960}{11.937 \times 10^{6}}=80.4 \times 10^{-6} R_{g_{2}} R_{2} \quad \mathrm{~Wb}_{1}=\frac{\phi_{1}}{A_{1}}=.161 \mathrm{~T} \\
\phi_{2} & =\Phi_{3}=\frac{1}{2} \Phi_{1}=40.2 \times 10^{-6} \mathrm{~Wb} \quad B_{2}=0.08 \mathrm{~T} \\
B_{3} & =0.04 \mathrm{~T}
\end{aligned}
$$

Problem \# 6:
The core loss of a magnetic core is 2000 W at 50 Hz . Keeping the flux density constant, the frequency of the supply is raised to 75 Hz resulting in core loss of 3200 W. Compute separately hysteresis and eddy current losses at both the frequencies.

Solution:

For constant Bax

$$
w_{n}=A F \text { and } w_{e}=B f^{2}
$$

$$
w_{t}=w_{n}+w_{e}=A F+B f^{2}
$$

$$
\text { at so Hz: } \quad \begin{align*}
2000 & =A \times 50+B \times 50^{2}  \tag{1}\\
3200 & =A \times 75+B \times 75^{2} \tag{3}
\end{align*}
$$

solving, $x 2$ for $A$ and $B$

$$
\begin{array}{ll}
A=34.667 & \text { anal } B=0.10667 \\
\text { at } 50 \mathrm{~Hz} & a+75 \mathrm{~Hz} \\
W_{h}=1733.35 \mathrm{w} & W_{h}=2600 \mathrm{w} \\
W_{c}=266.65 \mathrm{w} & W_{c}=600 \mathrm{w}
\end{array}
$$

