Problem 1

A 9 kVA, 208 V, 1200 rpm, three phase, 60 Hz, Y-connected synchronous generator has a field winding resistance of 4.5 Ω and armature winding impedance of 0.3+j5Ω per phase. When the generator operates at its full load and 0.8 PF lagging, the field winding current is 5 A. The rotational loss is 500 W. Determine:

(a) The voltage regulation

(b) Efficiency

(c) The torque applied by the prime mover

Solution

$$V_\phi = \frac{208\ V}{\sqrt{3}} = 120\ V$$

$$I_a = \frac{S}{3V_\phi} = \frac{9\ kVA}{3 * 120} = 25\ A$$

→ For 0.8 PF lagging $I_a = 25\angle -36.87^\circ\ A$

→ The Generated voltage E_A:

$$E_A = V_\phi + I_a(R_a + jX_s)$$

$$= 120\angle 0^\circ + 25\angle -36.87\ [0.3 + j5]$$

$$= 222.534\ \angle 25.414^\circ\ V$$

(a) $$VR = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\%$$

$$= \frac{222.534 - 120}{120} \times 100\% = 85.45\%$$

(b) $\eta = \dfrac{P_{out}}{P_{in}}$

$P_{out} = 3 V_\phi I_\phi \cos\theta$

$= 3 * 120 * 25 * 0.8$

$= 7.2 \ kW$

Copper loss $= 3 * I_a^2 R_a$

$= 3 * 25^2 * 0.3$

$= 562.5 \ W$

Power developed: $P_{dev} = 7.2 kW + 0.562 kW = 7.76 kW$

The constant loss = Rotational + $P_{field\ wdg}$

$= 500 + 5^2 * 4.5 = 0.6125 \ kW$

Total $P_{input} = P_{input} = P_{out}$ + losses

$= 7.76 kW + 0.6125 kW = 8.375 kW$

$\eta = \dfrac{7.2 kW}{8.375 kW} = 0.86 = 86\%$

(c). $P_{input(mechanical)} = P_{dev} + P_{rotational}$

$= 7.76 kW + 0.5 kW = 8.263 kW$

$T_{prim} * \omega = P_{in}$

$T_{prime} = \dfrac{P_{in}}{\omega} = \dfrac{8.263 kW}{\frac{2\pi}{60} * 1200} = 65.75 \ N\text{-}m$

Problem 2

A three phase, 14 kV, 10 MVA, 60 Hz, two pole, 0.85 PF lagging, star connected, synchronous generator has X_s= 20 Ω per phase and R_a= 2 Ω per phase. The generator is connected to an infinite bus.

(a) Determine the excitation voltage at the rated condition. Draw the phasor diagram for this condition.

(b) Determine the torque angle at the rated condition.

(c) If the field current is kept constant, determine the maximum power the generator can supply. Neglect R_a.

(d) For the condition in part (c), determine the generator current and the power factor. Draw the phasor diagram for this condition.

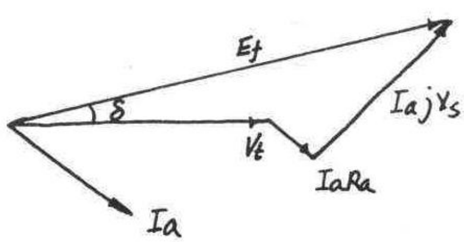
Problem 2 Solution

(a) $V_t|_{rated} = \frac{14\times10^3}{\sqrt{3}} = 8083\ V/phase \quad I_a|_{rated} = \frac{10\times10^6}{\sqrt{3}\times14\times10^3} = 412.4A$

$\theta = \cos^{-1} 0.85 = 31.8°$

$E_f = 8083\angle0° + (412.4\angle{-31.8°})\times2 + (412.4\angle{-31.8°})\times20\angle90°$

$= 8083 + 700 - j434.6 + 4346.3 + j7009.9 = 13129.3 + j6575.3 = 14{,}683.8\angle26°V$



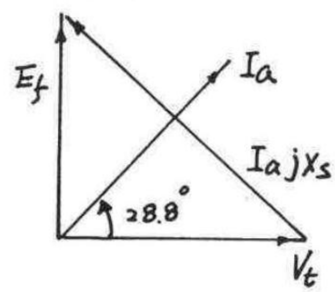
(b) The torque angle $\delta = 26^0$

(c) $P_{max} = \dfrac{3 \times 8083 \times 14683}{20} = 817.8\,MW$

(d) $I_a = \dfrac{14683\angle 90° - 8083\angle 0°}{20\angle 90°} = \dfrac{j14683 - 8083}{j20} = \dfrac{16760.83\angle 118.8°}{20\angle 90°}$

$= 838.04\angle 28.8°\,A$

$PF = \cos 28.8° = 0.876$ (leading)



Problem 3

A three phase, 120 MVA, 12 kV, 60 Hz, two pole, 0.85 PF lagging, Y-connected steam turbine driven alternator(synchronous generator) has a stator resistance of R_a= 0.018 Ω and a synchronous reactance of X_s= 1.02 Ω. At full load (rated) condition, the efficiency is 92% (the field winding loss is neglected). At this condition, determine

(a) The synchronous speed
(b) The power loss in the armature resistance
(c) The rotational loss (Neglect
(d) The torque applied to the shaft by the steam turbine prime mover

(a) $$\omega_s = \frac{120*60}{2} = 3600 rpm$$

$$V\phi = \frac{12kV}{\sqrt{3}} = 6928.4V$$

$$I_a = \frac{120\,MVA}{6928.4V} = 5773.7\,A$$

(b) $$P_{Ra} = 3*I_a^{\,2}R_a = 3*5773.7^2*0.018 = 1.8\,MW$$

$$P_{out} = 120*0.85 = 102\ MW$$

$$Turbine\ power\ P_{in} = \frac{102}{0.92} = 110.8696\ MW$$

$$Total\ loss = 110.8696 - 102 = 8.896\ MW$$

(c) $$Rotational\ loss = 8.896 - 1.8 = 7.0696\,MW$$

(d) $$Torque = \frac{110.8696\,MW}{(3600*2\pi)/60} = 294.09\,kN-M$$

Problem 4

A 208-V Y-connected synchronous motor is drawing 40 A at unity power factor from a 208-V power system. The field current flowing under these conditions is 2.7 A. Its synchronous reactance is 0.8 Ω. Assume a linear open-circuit characteristic.

(a) Find the torque angle δ.

(b) How much field current would be required to make the motor operate at 0.8 PF leading?

(c) What is the new torque angle in part *(b)*?

SOLUTION

(a) The phase voltage of this motor is V_ϕ = 120 V, and the armature current is $\mathbf{I}_A = 40\angle 0^\circ$ A. Therefore, the internal generated voltage is

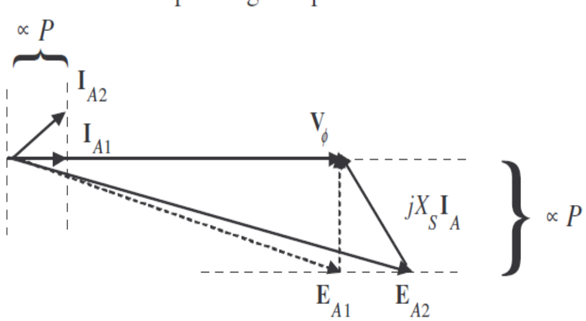
$$\mathbf{E}_A = \mathbf{V}_\phi - R_A\mathbf{I}_A - jX_S\mathbf{I}_A$$

$$\mathbf{E}_A = 120\angle 0^\circ \text{ V} - j(0.8\ \Omega)(40\angle 0^\circ \text{ A})$$

$$\mathbf{E}_A = 124\angle -14.9^\circ \text{ V}$$

The torque angle δ of this machine is –14.9°.

(b) A phasor diagram of the motor operating at a power factor of 0.78 leading is shown below.



Since the power supplied by the motor is constant, the quantity $I_A \cos\theta$, which is directly proportional to power, must be constant. Therefore,

$$I_{A2}(0.8) = (40 \text{ A})(1.00)$$

$$\mathbf{I}_{A2} = 50\angle 36.87° \text{ A}$$

The internal generated voltage required to produce this current would be

$$\mathbf{E}_{A2} = \mathbf{V}_\phi - R_A\mathbf{I}_{A2} - jX_S\mathbf{I}_{A2}$$
$$\mathbf{E}_{A2} = 120\angle 0° \text{ V} - j(0.8\ \Omega)(50\angle 36.87° \text{ A})$$
$$\mathbf{E}_{A2} = 147.5\angle -12.5° \text{ V}$$

The internal generated voltage E_A is directly proportional to the field flux, and we have assumed in this problem that the flux is directly proportional to the field current. Therefore, the required field current is

$$I_{F2} = \frac{E_{A2}}{E_{A1}} I_{F1} = \frac{147 \text{ V}}{124 \text{ V}}(2.7 \text{ A}) = 3.20 \text{ A}$$

(c) The new torque angle δ of this machine is $-12.5°$.

Problem 5

A 480-V, 100-kW, 50-Hz, four-pole, Y-connected synchronous motor has a rated power factor of 0.85 leading. At full load, the efficiency is 91%. The armature resistance is 0.08 Ω, and the synchronous reactance is 1.0 Ω. Find the following quantities for this machine when it is operating at full load:

(a) Output torque

(b) Input power

(c) n_m (Mechanical speed of the machine)

(d) E_A

(e) $|I_A|$

(f) P_{conv}

(g) Rotational loss= $P_{mech} + P_{core} + P_{stray}$

SOLUTION

(a) Since this machine has 8 poles, it rotates at a speed of

$$n_m = \frac{120 f_e}{P} = \frac{120(50\text{ Hz})}{4} = 1500\text{ r/min}$$

If the output power is 100 kW, the output torque is

$$\tau_{\text{load}} = \frac{P_{\text{out}}}{\omega_{\text{m}}} = \frac{(100{,}000\text{ W})}{(1500\text{ r/min})\left(\dfrac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right)} = 637\text{ N}\cdot\text{m}$$

(b) The input power is

$$P_{\text{IN}} = \frac{P_{\text{OUT}}}{\eta} = \frac{100\text{ kW}}{0.91} = 110\text{ kW}$$

(c) The mechanical speed is

$$n_m = 1500\text{ r/min}$$

(d) The armature current is

$$I_A = I_L = \frac{P}{\sqrt{3}\,V_T\,\text{PF}} = \frac{110\text{ kW}}{\sqrt{3}(480\text{ V})(0.85)} = 156\text{ A}$$

$$\mathbf{I}_A = 156\angle 31.8^\circ\text{ A}$$

Therefore, $\mathbf{E}_A$ is

$$\mathbf{E}_A = \mathbf{V}_\phi - R_A\mathbf{I}_A - jX_S\mathbf{I}_A$$
$$\mathbf{E}_A = (277\angle 0^\circ\text{ V}) - (0.08\ \Omega)(156\angle 31.8^\circ\text{ A}) - j(1.0\ \Omega)(156\angle 31.8^\circ\text{ A})$$
$$\mathbf{E}_A = 375\angle -21.8^\circ\text{ V}$$

(e) The magnitude of the armature current is 375 A.

(f) The power converted from electrical to mechanical form is given by the equation $P_{\text{conv}} = P_{\text{IN}} - P_{\text{CU}}$

$$P_{\text{CU}} = 3I_A^{\,2}R_A = 3(156\text{ A})^2(0.08\ \Omega) = 5.8\text{ kW}$$

$$P_{\text{conv}} = P_{\text{IN}} - P_{\text{CU}} = 110\text{ kW} - 5.8\text{ kW} = 104.2\text{ kW}$$

(g) The mechanical, core, and stray losses are given by the equation

$$P_{\text{mech}} + P_{\text{core}} + P_{\text{stray}} = P_{\text{conv}} - P_{\text{OUT}} = 104.2\text{ kW} - 100\text{ kW} = 4.2\text{ kW}$$