KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

ELECTRICAL ENGINEERING DEPARTMENT

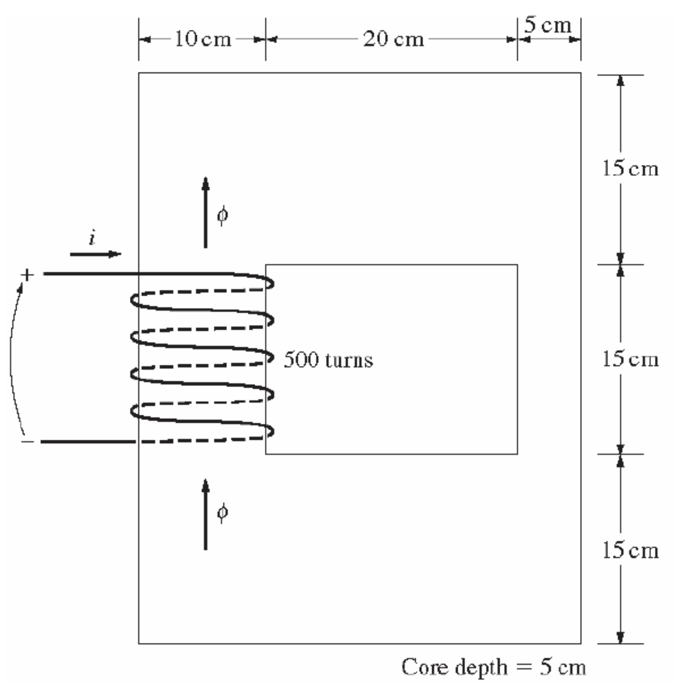
EE 306 – Term 171

HW # 2: Magnetic Circuits

Due Date: October 11th, 2017

Problem # 1:

A ferromagnetic core is shown below. The depth of the core is 5 cm. The other dimensions of the core are as shown in the figure. Find the value of the current that will produce a flux of 0.005 Wb. With this current, what is the flux density at the top of the core? What is the flux density at the right side of the core? Assume that the relative permeability of the core is 800.



SOLUTION There are three regions in this core. The top and bottom form one region, the left side forms a second region, and the right side forms a third region. If we assume that the mean path length of the flux is in the center of each leg of the core, and if we ignore spreading at the corners of the core, then the path lengths are $l_1 = 2(27.5\text{ cm}) = 55\text{ cm}$, $l_2 = 30\text{ cm}$, and $l_3 = 30\text{ cm}$. The reluctances of these regions are:

$$\mathcal{R}_1 = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_o A} = \frac{0.55 \text{ m}}{(800)(4\pi \times 10^{-7} \text{ H/m})(0.05 \text{ m})(0.15 \text{ m})} = 72.9 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_2 = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_o A} = \frac{0.30 \text{ m}}{(800)(4\pi \times 10^{-7} \text{ H/m})(0.05 \text{ m})(0.10 \text{ m})} = 59.7 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_3 = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_o A} = \frac{0.30 \text{ m}}{(800)(4\pi \times 10^{-7} \text{ H/m})(0.05 \text{ m})(0.05 \text{ m})} = 119.4 \text{ kA} \cdot \text{t/Wb}$$

The total reluctance is thus

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 = 72.9 + 59.7 + 119.4 = 252 \text{ kA} \cdot \text{t/Wb}$$

and the magnetomotive force required to produce a flux of 0.005 Wb is

$$\mathcal{F} = \phi \mathcal{R} = (0.005 \text{ Wb})(252 \text{ kA} \cdot \text{t/Wb}) = 1260 \text{ A} \cdot \text{t}$$

and the required current is

$$i = \frac{\mathcal{F}}{N} = \frac{1260 \text{ A} \cdot \text{t}}{500 \text{ t}} = 2.5 \text{ A}$$

The flux density on the top of the core is

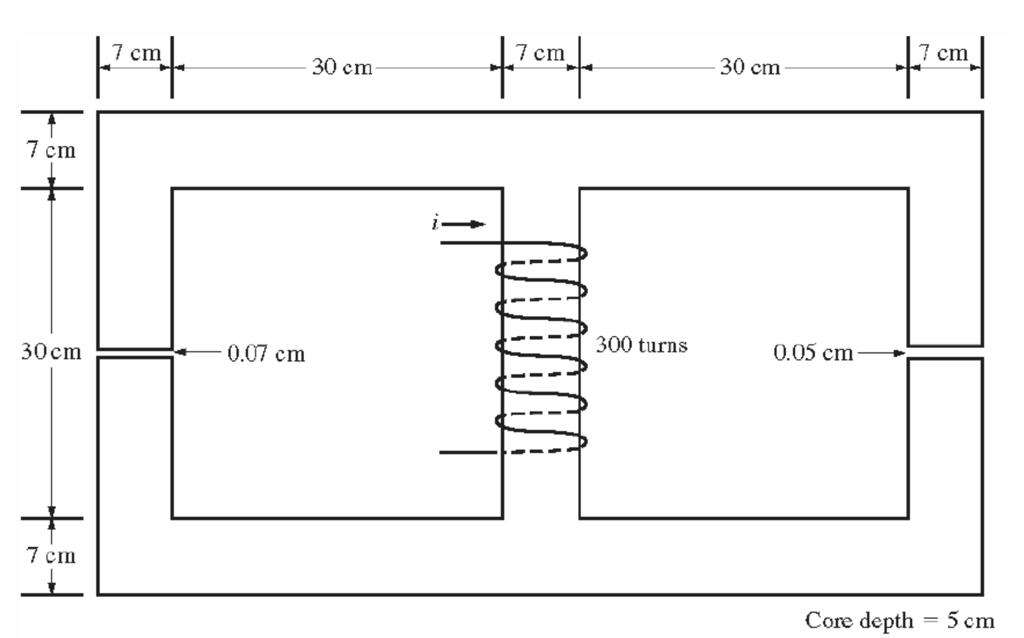
$$B = \frac{\phi}{A} = \frac{0.005 \text{ Wb}}{(0.15 \text{ m})(0.05 \text{ m})} = 0.67 \text{ T}$$

The flux density on the right side of the core is

$$B = \frac{\phi}{A} = \frac{0.005 \text{ Wb}}{(0.05 \text{ m})(0.05 \text{ m})} = 2.0 \text{ T}$$

Problem # 2:

A ferromagnetic core with a relative permeability of 1500 is shown below. The dimensions are as shown in the diagram, and the depth of the core is 5 cm. The air gaps on the left and right sides of the core are 0.050 and 0.070 cm, respectively. Because of fringing effects, the effective area of the air gaps is 5 percent larger than their physical size. If there are 300 turns in the coil wrapped around the center leg of the core and if the current in the coil is 1.0 A, what is the flux in each of the left, center, and right legs of the core? What is the flux density in each air gap?



SOLUTION This core can be divided up into five regions. Let $\mathcal{R}_1$ be the reluctance of the left-hand portion of the core, $\mathcal{R}_2$ be the reluctance of the left-hand air gap, $\mathcal{R}_3$ be the reluctance of the right-hand portion of the core, $\mathcal{R}_4$ be the reluctance of the right-hand air gap, and $\mathcal{R}_5$ be the reluctance of the center leg of the core. Then the total reluctance of the core is

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_5 + \frac{(\mathcal{R}_1 + \mathcal{R}_2)(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}$$

$$\mathcal{R}_1 = \frac{l_1}{\mu_r \mu_0 A_1} = \frac{1.11 \text{ m}}{(1500)(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.05 \text{ m})} = 168 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_2 = \frac{l_2}{\mu_0 A_2} = \frac{0.0007 \text{ m}}{(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.05 \text{ m})(1.05)} = 152 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_3 = \frac{l_3}{\mu_r \mu_0 A_3} = \frac{1.11 \text{ m}}{(1500)(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.05 \text{ m})} = 168 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_4 = \frac{l_4}{\mu_0 A_4} = \frac{0.0005 \text{ m}}{(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.05 \text{ m})(1.05)} = 108 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_5 = \frac{l_5}{\mu_r \mu_0 A_5} = \frac{0.37 \text{ m}}{(1500)\left(4\pi \times 10^{-7} \text{ H/m}\right)(0.07 \text{ m})(0.05 \text{ m})} = 56.1 \text{ kA} \cdot \text{t/Wb}$$

The total reluctance is

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_5 + \frac{(\mathcal{R}_1 + \mathcal{R}_2)(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} = 56.1 + \frac{(168+152)(168+108)}{168+152+168+108} = 204 \text{ kA} \cdot \text{t/Wb}$$

The total flux in the core is equal to the flux in the center leg:

$$\phi_{\text{center}} = \phi_{\text{TOT}} = \frac{\mathcal{F}}{\mathcal{R}_{\text{TOT}}} = \frac{(300 \text{ t})(1.0 \text{ A})}{204 \text{ kA} \cdot \text{t/Wb}} = 0.00147 \text{ Wb}$$

The fluxes in the left and right legs can be found by the "flux divider rule", which is analogous to the current divider rule.

$$\phi_{\text{left}} = \frac{(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} \phi_{\text{TOT}} = \frac{(168+108)}{168+152+168+108}(0.00147 \text{ Wb}) = 0.00068 \text{ Wb}$$

$$\phi_{\text{right}} = \frac{(\mathcal{R}_1 + \mathcal{R}_2)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} \phi_{\text{TOT}} = \frac{(168+152)}{168+152+168+108}(0.00147 \text{ Wb}) = 0.00079 \text{ Wb}$$

The flux density in the air gaps can be determined from the equation $\phi = BA$:

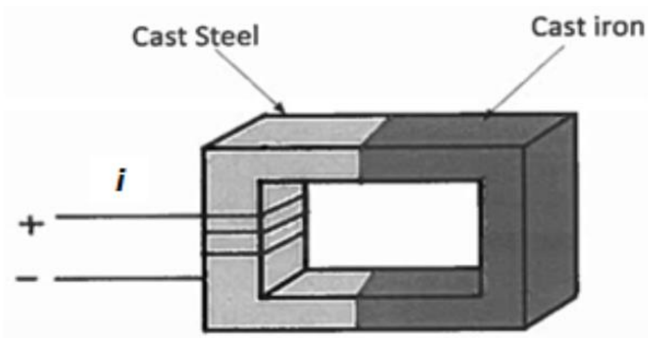
$$B_{\text{left}} = \frac{\phi_{\text{left}}}{A_{\text{eff}}} = \frac{0.00068 \text{ Wb}}{(0.07 \text{ cm})(0.05 \text{ cm})(1.05)} = 0.185 \text{ T}$$

$$B_{\text{right}} = \frac{\phi_{\text{right}}}{A_{\text{eff}}} = \frac{0.00079 \text{ Wb}}{(0.07 \text{ cm})(0.05 \text{ cm})(1.05)} = 0.215 \text{ T}$$

Problem # 3:

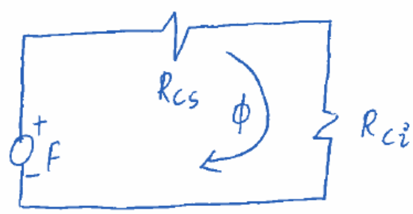
Consider a magnetic circuit as shown below. The core of the circuit is composed of cast steel and cast iron. Each material has a mean length of 20 cm. The cross section area of the core is 16 cm^2. The coil has 350 turns and it carries a current of 1.2 A. The relative permeability of the cast steel is 800 and that of cast iron is 250. Determine the following:

1) The flux in the core
2) The total flux linkage
3) The magnetic flux density B in the core



Solution

Magnetic equivalent circuit



$$F = Ni = 350 \times 1.2 = 420$$

$$\boxed{F = 420 \text{ A.t}}$$

$$R_{cs} = \frac{l_{cs}}{\mu_{cs} A_{cs}} = \frac{20 \times 10^{-2}}{800 \times 4\pi \times 10^{-7} \times 16 \times 10^{-4}}$$

$$\boxed{R_{cs} = 124339.7 \text{ A.t/Wb}}$$

$$R_{ci} = \frac{l_{ci}}{\mu_{ci} A_{ci}} = \frac{20 \times 10^{-2}}{250 \times 4\pi \times 10^{-7} \times 16 \times 10^{-4}}$$

$$\boxed{R_{ci} = 397887.3 \ A.t/Wb}$$

$$\Rightarrow R_{eq} = R_{cs} + R_{ci}$$

$$R_{eq} = 522227.05 \ At/Wb$$

$$F = Ni = \phi R_{eq}$$

$$\Rightarrow \phi = \frac{F}{R_{eq}} = \frac{420}{5222270.05}$$

1)

$$\boxed{\phi = 8.042 \times 10^{-4} \ Wb}$$

2) $\lambda = N\phi = 350 \times 8.042 \times 10^{-4}$

$$\boxed{\lambda = 0.2814 \ Wb.turn}$$

3) $B = \phi / A = \frac{8.042 \times 10^{-4}}{16 \times 10^{-4}}$

$$\boxed{B = 0.502 T}$$

Problem # 4:

A square magnetic core has a mean path length of 55 cm and a cross-sectional area of 150 cm^2. A 200-turn coil of wire is wrapped around one leg of the core. The magnetization curve of the core material is shown below.

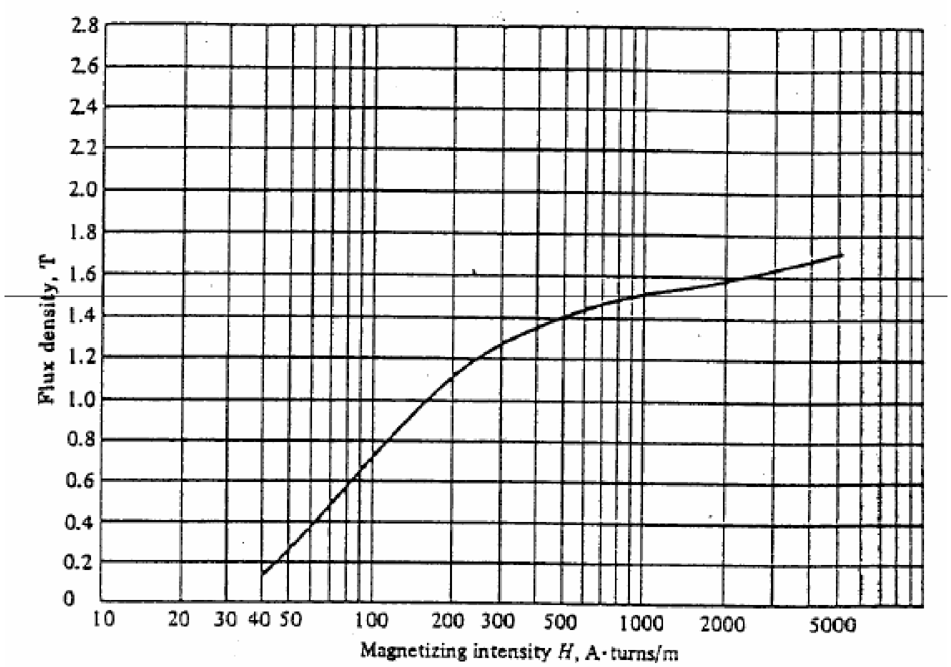
(a) How much current is required to produce 12 mWb of flux in the core?

(b) What is the relative permeability of the core at that level of current?

(c) What is its reluctance?

(d) Repeat part (a) if an air-gap of length 1 mm is cut across the core.

Assume a 5% increase in the effective air-gap area to account for fringing.



Solution:

$$B = \frac{\Phi}{A} = \frac{0.012}{0.015} = 0.8\ T$$

From the magnetization curve, the corresponding H

$H \cong 115\ AT/m$

Hence, $F = Hl = 115 * 0.55 = 63.25\ AT$

So,

a) $I = \frac{F}{N} = \frac{63.25}{200} = 0.316\ A$

b) $\mu = \frac{B}{H} = \frac{0.8}{115} = 0.00696\ H/m$

$\mu_r = \frac{\mu}{\mu_0} = \frac{0.00696}{4\pi * 10^{-7}} = 5540$

c) $\mathcal{R} = \frac{F}{\Phi} = \frac{63.25}{0.012} = 5270\ AT/Wb$

or, $\mathcal{R} = \frac{l}{\mu A} = \frac{0.55}{5540 * 4\pi * 10^{-7} * 150 * 10^{-4}} = 5270\ AT/Wb$

d) With the same flux $\Phi = 12\ mWb$, $B_c = 0.8\ T$

$H_c = 115\ AT/m$, $l_c = 0.55\ m$ (Neglect l_g; $l_g \ll l_c$)

$B_g = \frac{\Phi}{A_g} = \frac{0.012}{1.05 * 150 * 10^{-4}} = 0.762\ T$

$H_g = \frac{B_g}{\mu_g} = \frac{0.762}{4\pi * 10^{-7}} = 0.061 * 10^{7}\ AT/m$, $F_g = H_g l_g$

$F_{total} = F_c + F_g = 63.25 + 0.061 * 10^{7} * 1 * 10^{-3} = 673.25$

$I = \frac{F_{tot}}{N} = \frac{673.25}{200} = 3.366\ A$

Problem # 5:

The total core loss for a specimen of magnetic sheet steel is found to be 1800 W at 60 Hz. If the flux density is kept constant and the frequency of the supply increases 50%, the total core loss is found to be 3000 W. Compute the separate hysteresis and eddy-current losses at both frequencies.

Solution:

For Constant B

$P_h \propto f$ i.e. $P_h = Af$ where A is constant

$P_e \propto f^2$ i.e $P_e = Bf^2$ — B — —

at 60 Hz

$$1800 = A * 60 + B * (60)^2 \longrightarrow ①$$

at 90 Hz

$$3000 = A * 90 + B * (90)^2 \longrightarrow ②$$

① by 3 and ② by 2 and Subtract

$$3 * 1800 - 2 * 3000 = 3B * (60)^2 - 2B * (90)^2$$

$$-600 = -5400\,B$$

So, $B = 0.111$ sub. in ①

$A = 23.333$

Hence; At 60 Hz: $P_h = 23.333 * 60 = 1400$ W

$P_e = 0.111 * (60)^2 = 400$ W

At 90 Hz: $P_h = 23.333 * 90 = 2100$ W

$P_e = 0.111 * (90)^2 = 900$ W