# KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

## ELECTRICAL ENGINEERING DEPARTMENT

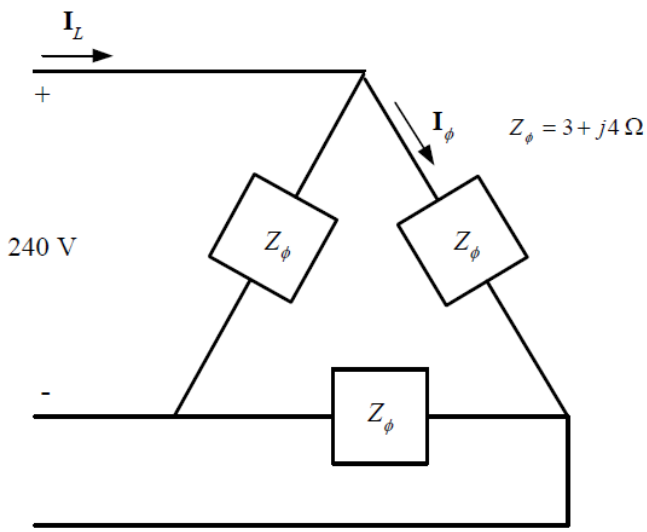
## EE 306 – Term 171

**HW # 1: Three-Phase Circuits**
**Due Date: October 2nd, 2017**

**Problem # 1:**

Three impedances of $4 + j3\ \Omega$ are $\Delta$-connected and tied to a three-phase 208-V power line. Find $I_\phi$, $I_L$, $P$, $Q$, $S$, and the power factor of this load.

**Solution of Problem # 1:**




$V_L = V_\phi = 208\text{ V}$, and $Z_\phi = 4 + j3\ \Omega = 5\angle 36.87°\ \Omega$, so

$$I_\phi = \frac{V_\phi}{Z_\phi} = \frac{208\text{ V}}{5\ \Omega} = 41.6\text{ A}$$

$$I_L = \sqrt{3} I_\phi = \sqrt{3}(41.6\text{ A}) = 72.05\text{ A}$$

$$P = 3\frac{V_\phi^2}{Z}\cos\theta = 3\frac{(208\text{ V})^2}{5\ \Omega}\cos\ 36.87° = 20.77\text{ kW}$$

$$Q = 3\frac{V_\phi^2}{Z}\sin\theta = 3\frac{(208\text{ V})^2}{5\ \Omega}\sin\ 36.87° = 15.58\text{ kvar}$$

$$S = \sqrt{P^2 + Q^2} = 25.96\text{ kVA}$$

$$\text{PF} = \cos\ \theta = 0.8\text{ lagging}$$

**Problem # 2:**

Prove that the line voltage of a Y-connected generator with an *acb* phase sequence lags the corresponding phase voltage by 30°. Draw a phasor diagram showing the phase and line voltages for this generator.

**Solution of Problem # 2:**

If the generator has an *acb* phase sequence, then the three phase voltages will be

$$\mathbf{V}_{an} = V_\phi \angle 0°$$

$$\mathbf{V}_{bn} = V_\phi \angle -240°$$

$$\mathbf{V}_{cn} = V_\phi \angle -120°$$

The relationship between line voltage and phase voltage is derived below. By Kirchhoff's voltage law, the line-to-line voltage $\mathbf{V}_{ab}$ is given by

$$\mathbf{V}_{ab} = \mathbf{V}_a - \mathbf{V}_b$$

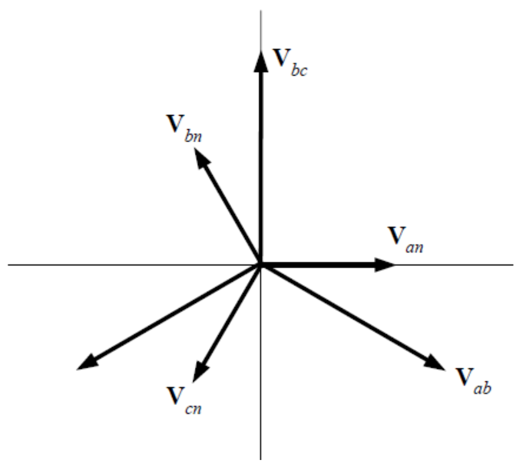
$$\mathbf{V}_{ab} = V_\phi \angle 0° - V_\phi \angle -240°$$

$$\mathbf{V}_{ab} = V_\phi - \left( -\frac{1}{2}V_\phi + j\frac{\sqrt{3}}{2}V_\phi \right) = \frac{3}{2}V_\phi - j\frac{\sqrt{3}}{2}V_\phi$$

$$\mathbf{V}_{ab} = \sqrt{3}V_\phi \left( \frac{\sqrt{3}}{2} - j\frac{1}{2} \right)$$

$$\mathbf{V}_{ab} = \sqrt{3}V_\phi \angle -30°$$

Thus the line voltage *lags* the corresponding phase voltage by 30°. The phasor diagram for this connection is shown below.



**Problem # 3:**
A balanced 3-phase, 173-V, 60-Hz source supplies the two following loads:

- A Δ-connected load with a phase impedance of (18+*j*24) Ω,
- A Y-connected load with a phase impedance of 10∠53.13° Ω.

Find:

a. The power factor of the entire load.
b. The total line current supplied.
c. The total real, reactive, and apparent powers.

**Solution of Problem # 3:**

Convert Δ to Y $$Z_y = \frac{18 + j\,24}{3} = 6 + j\,8$$

Parallel combination of the 2 loads (per phase)

$$Z_T = \frac{(6 = j\,8)(10\angle 53.1^0)}{6 + j\,8 + 10\angle 53.1^0} = 5\angle 53.1^0$$

a. Power factor= cos(53.1$^0$) = 0.6 lag

b. $$I_L = I_{ph} = \frac{173/\sqrt{3}\angle 0^0}{5\angle 53.1^0} = 20\angle -53.1^0 A$$

c.

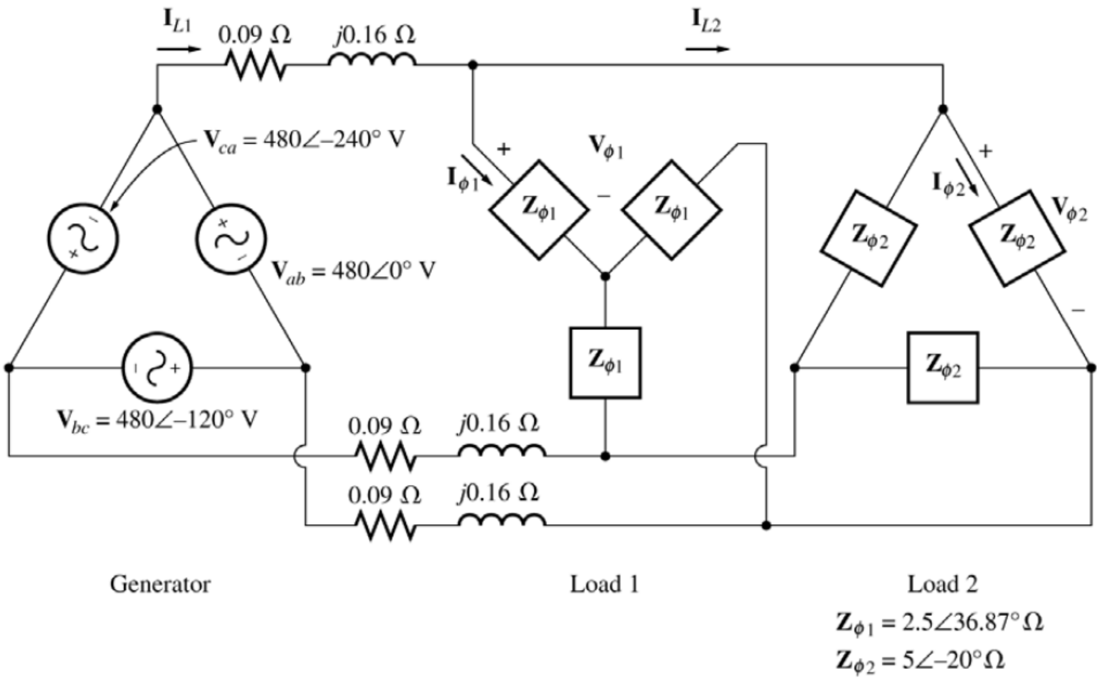
$$P_T = \sqrt{3}x\,173x\,20x\,0.6 = 3.596kW$$

$$Q_T = \sqrt{3}x\,173x\,20x\,0.8 = 4.794kVAR$$

$$|S_T| = \sqrt{3}x\,173x\,20 = 5.993kVA$$

**Problem # 4:**

Consider the three-phase circuit below




*(a)* What is the line voltage of the two loads?

*(b)* What is the voltage drop on the transmission lines?

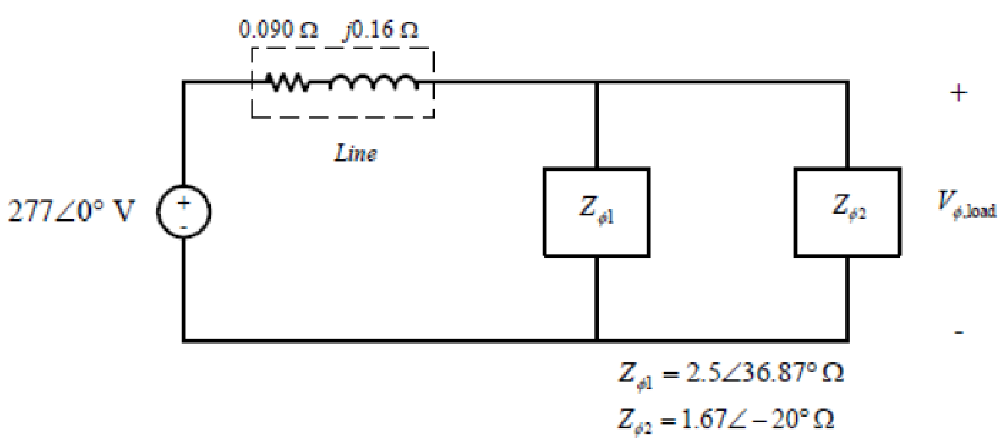
*(c)* Find the real and reactive powers supplied to each load.

*(d)* Find the real and reactive power losses in the transmission line.

*(e)* Find the real power, reactive power, and power factor supplied by the generator.

**Solution of Problem # 4:**

To solve this problem, first convert the delta-connected load 2 to an equivalent wye (by dividing the impedance by 3), and get the per-phase equivalent circuit.



*(a)* The phase voltage of the equivalent Y-loads can be found by nodal analysis.

$$\frac{\mathbf{V}_{\phi,\text{load}} - 277\angle 0^\circ \text{ V}}{0.09 + j0.16\ \Omega} + \frac{\mathbf{V}_{\phi,\text{load}}}{2.5\angle 36.87^\circ\ \Omega} + \frac{\mathbf{V}_{\phi,\text{load}}}{1.67\angle -20^\circ\ \Omega} = 0$$

$$(5.443\angle -60.6^\circ)\ \left(\mathbf{V}_{\phi,\text{load}} - 277\angle 0^\circ \text{ V}\right) + (0.4\angle -36.87^\circ)\mathbf{V}_{\phi,\text{load}} + (0.6\angle 20^\circ)\mathbf{V}_{\phi,\text{load}} = 0$$

$$(5.955\angle -53.34^\circ)\ \mathbf{V}_{\phi,\text{load}} = 1508\angle -60.6^\circ$$

$$\mathbf{V}_{\phi,\text{load}} = 253.2\angle -7.3^\circ \text{ V}$$

Therefore, the line voltage at the loads is $V_L\sqrt{3}\ V_\phi = 439\text{ V}$.

*(b)* The voltage drop in the transmission lines is

$$\Delta\mathbf{V}_{\text{line}} = \mathbf{V}_{\phi,\text{gen}} - \mathbf{V}_{\phi,\text{load}} = 277\angle 0^\circ \text{ V} - 253.2\angle -7.3^\circ = 41.3\angle 52^\circ \text{ V}$$

*(c)* The real and reactive power of each load is

$$P_1 = 3\frac{V_\phi^2}{Z}\cos\theta = 3\frac{(253.2 \text{ V})^2}{2.5\ \Omega}\cos\ 36.87^\circ = 61.6 \text{ kW}$$

$$Q_1 = 3\frac{V_\phi^2}{Z}\sin\theta = 3\frac{(253.2 \text{ V})^2}{2.5\ \Omega}\sin\ 36.87^\circ = 46.2 \text{ kvar}$$

$$P_2 = 3\frac{V_\phi^2}{Z}\cos\theta = 3\frac{(253.2 \text{ V})^2}{1.67\ \Omega}\cos\ (-20^\circ) = 108.4 \text{ kW}$$

$$Q_2 = 3\frac{V_\phi^2}{Z}\sin\theta = 3\frac{(253.2 \text{ V})^2}{1.67\ \Omega}\sin\ (-20^\circ) = -39.5 \text{ kvar}$$

*(d)* The line current is

$$\mathbf{I}_{\text{line}} = \frac{\Delta\mathbf{V}_{\text{line}}}{Z_{\text{line}}} = \frac{41.3\angle 52^\circ \text{ V}}{0.09 + j0.16\ \Omega} = 225\angle -8.6^\circ \text{A}$$

Therefore, the loses in the transmission line are

$$P_{\text{line}} = 3I_{\text{line}}^2 R_{\text{line}} = 3\ (225 \text{ A})^2 (0.09\ \Omega) = 13.7 \text{ kW}$$

$$Q_{\text{line}} = 3I_{\text{line}}^2 X_{\text{line}} = 3\ (225 \text{ A})^2 (0.16\ \Omega) = 24.3 \text{ kvar}$$

*(e)* The real and reactive power supplied by the generator is

$$P_{\text{gen}} = P_{\text{line}} + P_1 + P_2 = 13.7 \text{ kW} + 61.6 \text{ kW} + 108.4 \text{ kW} = 183.7 \text{ kW}$$

$$Q_{\text{gen}} = Q_{\text{line}} + Q_1 + Q_2 = 24.3 \text{ kvar} + 46.2 \text{ kvar} - 39.5 \text{ kvar} = 31 \text{ kvar}$$

The power factor of the generator is

$$\text{PF} = \cos\left[\tan^{-1}\frac{Q_{\text{gen}}}{P_{\text{gen}}}\right] = \cos\left[\tan^{-1}\frac{31 \text{ kvar}}{183.7 \text{ kW}}\right] = 0.986 \text{ lagging}$$

**Problem # 5:**

A single phase electrical load draws 10 MW at 0.6 power factor lagging.

a. Find the real and reactive power absorbed by the load
b. Draw the power triangle.
c. Determine the kVAR of a capacitor to be connected across the load to raise the power factor to 0.95.

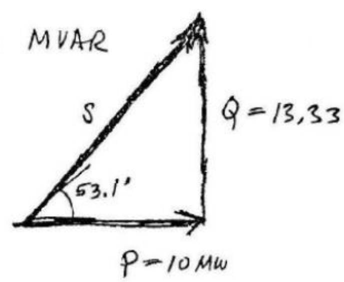
**Solution of Problem # 5:**

$P = 10\ MW$ , $PF = 0{,}6$ lagging

$\theta = \cos^{-1} 0.6 = 53.1°$

ⓐ $P = 10\ MW$

$Q = P \tan\theta = 10 \tan 53{,}1° = 13{,}33\ MVAR$




ⓒ $\theta_{new} = \cos^{-1} 0.95 = 18{,}2°$

$Q_{new} = P \tan\theta_{new} = 10 \tan 18.2°$

$= 3.29\ MVAR$

$= Q_{old} + Q_{cap}$

$Q_{cap} = 3{,}29 - 13{,}33 = -10\ MVAR = -10{,}000\ KVAR$