KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
Electrical Engineering Department
EE 306 – Term 162
HW # 3: Single Phase Transformers
(Solution)

Solution P1:

(a) $V_2 = \frac{N_2}{N_1} V_1 = \frac{500}{1000} \times 220 = 110\,V.$

(b) $I_2 = \frac{5000}{110} = 45.4545\,A$

$Z_2 = \frac{110}{45.4545} = 2.42\,\Omega$

(c) $Z_2' = \left(\frac{1000}{500}\right)^2 \times 2.42 = 9.68\,\Omega$

Solution P2:

(a)

$V_{H(rated)} = 1000\,V, \quad I_{H(rated)} = \frac{100\times10^3}{1000} = 100\,A.$

$V_{L(rated)} = 100\,V, \quad I_{L(rated)} = \frac{100\times10^3}{100} = 1000\,A$

From open circuit test,

$R_{cL} = \frac{100^2}{400} = 25\,\Omega.$

$I_{cL} = \frac{100}{25} = 4\,A.$

$I_{mL} = \sqrt{6^2 - 4^2} = 4.47\,A$

$X_{mL} = \frac{100}{4.47} = 22.37\,\Omega$

Turns ratio $a = \frac{1000}{100} = 10$

Refer to high voltage side,

$R_{cH} = 25 \times 10^2 = 2500\,\Omega, \quad X_{mH} = 2237\,\Omega$

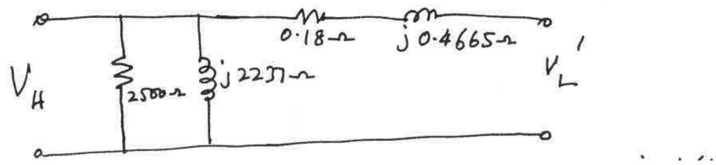
From short circuit test,

$R_{eq\,H} = \frac{1800}{100^2} = 0.18\,\Omega.$

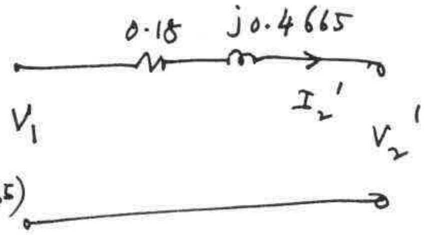
$Z_{eq\,H} = \frac{50}{100} = 0.5\,\Omega$

$X_{eq\,H} = \sqrt{0.5^2 - 0.18^2} = 0.4665\,\Omega$

Equivalent circuit referred to H.V. side



(b)



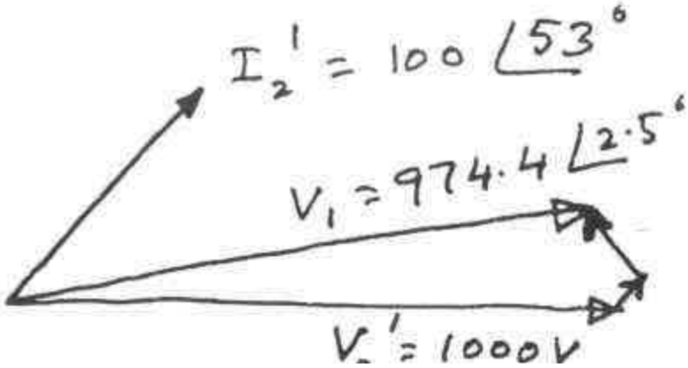
$$V_1 = V_2' + I_2' Z_{eq\,H}$$

$$= 1000\angle 0^\circ + 100\angle 53^\circ (0.18 + j0.4665)$$

$$= 974.4 \angle 2.5^\circ$$

$$V.R = \frac{974.4 - 1000}{1000} \times 100\,\% = -2.56\,\%$$

Phaser Diagram



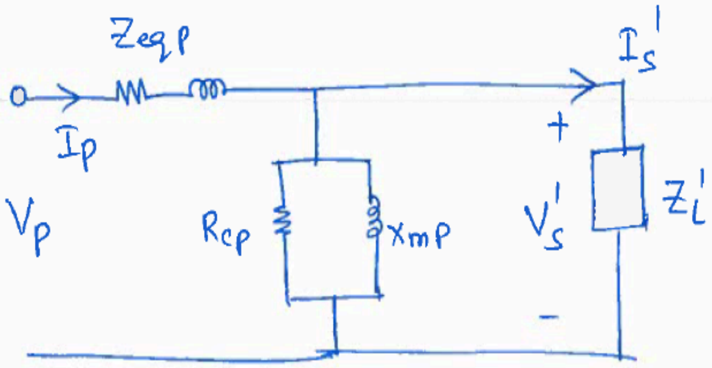
Solution P3:

Turns ratio $a = \dfrac{8000}{230} = 34.78$

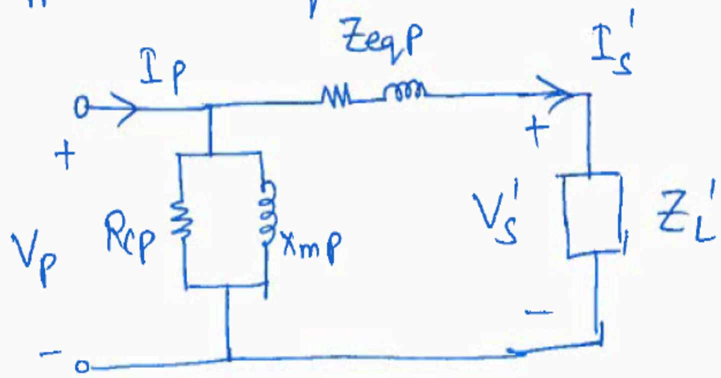
$Z_{eqp} = (20 + j100)\ \Omega$

$R_{cp} = 100\ k\Omega \qquad X_{mp} = 20\ k\Omega$

(a) Equivalent circuit referred to the primary side



Approximate equivalent circuit,



The load impedance referred to the primary side,

$$Z_L' = a^2 Z_L$$

$$Z_L' = (34.78)^2 (2.0 + j0.7) = 2419 + j847$$

The referred secondary current,

$$I_s' = \frac{V_P}{Z_{eqp} + Z_L'}$$

$$= \frac{7967\angle 0^\circ}{(20 + j100) + (2419 + j847)}$$

$$I_s' = 3.045\angle -21.2^\circ \text{ A}$$

The refered secondary voltage is,

$$V_s' = I_s' Z_L' = (3.045\angle -21.2^\circ)(2419 + j847)$$

$$V_s' = 7804\angle -1.9^\circ \text{ V}$$

The actual secondary voltage is

$$V_s = \frac{V_s'}{a} = \frac{7804\angle -1.9^\circ}{34.78} = 224.4\angle -1.$$

The voltage regulation is

$$VR = \frac{|V_P| - |V_s'|}{|V_s'|} \times 100\ \%$$

$$= \frac{7967 - 7804}{7804} \times 100\ \%$$

$$VR = 2.09\%$$

(b) Now the load is disconnected. Connect a capa
with $Z_L = -j3.0\,\Omega$ as load.

The load impedance referred to the primary side is

$$Z_L' = a^2 Z_L = (34.78)^2(-j3.0) = -j3629\,\Omega$$

The refferred Secondary current is

$$I_s' = \frac{V_P}{Z_L' + Z_{eqP}}$$

$$= \frac{7967\angle 0^\circ}{(-j3629) + (20 + j100)}$$

$$I_s' = 2.258\angle 89.7^\circ\ A$$

The referred Secondary voltage is

$$V_s' = I_s' Z_L' = (2.258\angle 89.7^\circ)(-j3629)$$

$$V_s' = 8194\angle -0.3^\circ \text{ V}$$

The actual secondary voltage is

$$V_s = \frac{V_s'}{a} = \frac{8194\angle -0.3^\circ}{34.78} = 235.6\angle -0.3^\circ \text{ V}$$

The voltage regulation is

$$VR = \frac{|V_P| - |V_s'|}{|V_s'|} \times 100\%$$

$$= \frac{7967 - 8194}{8194} \times 100\%$$

$$VR = -10.6\%$$

(c)

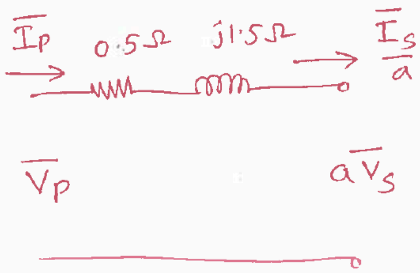
It has been observed from (a) and (b) that leading load or pure capacitor leads to negative voltage regulation and we can clearly see that output secondary voltage has increased in part (b) as compared with Part (a).

Solution P4:

Turn's Ratio $a = \frac{2400}{240} = 10$

Take the secondary voltage as reference phasor.

$\bar{V}_S = 240\angle 0^\circ$ V



The Secondary current is

$$I_S = \frac{150{,}000}{240}\angle -\cos^{-1}0.85$$

$$= 625\angle -31.8^\circ \text{ A}$$

$$P_{output} = \frac{150{,}000 \times 0.85}{\cancel{0.85}} = 127{,}500 \text{ W}$$

$$P_{cu} = I_P^2 R_{eq\,P} = \left(\frac{625}{10}\right)^2 \cdot (0.5) = 1953 \text{ W}$$

$$P_{core} = 600 \text{ W}$$

The efficiency is,

$$\eta = \frac{P_{output}}{P_{output} + P_{core} + P_{cu}} \times 100$$

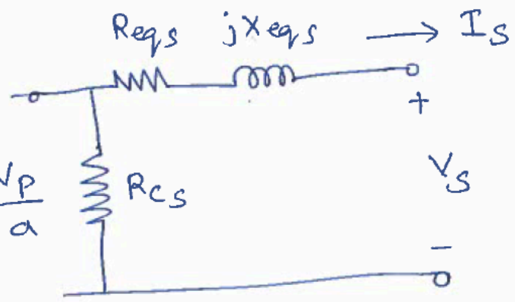
$$= \frac{127{,}500}{127{,}500 + 600 + 1953} \times 100$$

$$\eta = 98\%$$

Solution P5:

Secondary rated current $I_S = \frac{1000}{115} = 8.695\,A$

given, $VR = -1.5\%$



$$\frac{\frac{V_P}{a} - V_S}{V_S} = \frac{-1.5}{100}$$

$$\frac{V_P}{a} = \left(1 - \frac{1.5}{100}\right) V_S$$

$$\left|\frac{V_P}{a}\right| = 113.3\ V$$

Now by KVL

$$\frac{V_P}{a} = V_S + I_S\,(R_{eqs} + jX_{eqs})$$

From copper losses $P_{cu} = 10.6\,W$

$$I_S^2 R_{eqs} = 10.6\,W$$

$$\Rightarrow \boxed{R_{eqs} = 0.14\,\Omega}$$

$$|113.3| = 115 + (8.69\angle{+36.87})\,(R_{eqs} + jX_{eqs})$$

$$113.3 = 115 + (6.95 + j5.21)(0.14 + jX_{eqs})$$

Taking magnitudes,

$$\left[(115 + 0.973 - 5.21X_{eqs})^2 + (j16.95X_{eqs} + 0.72)\right]^2 = 1$$

$$75.4466\,X_{eqs}^2 - 1198.392\,X_{eqs} + 613.3284 = C$$

$$\boxed{X_{eqs} = 0.5294\,\Omega}$$

Now, from efficiency $\eta = \dfrac{P_{out}}{P_{out} + P_{cu} + P_{core}}$

$$0.949 = \frac{115 \times 8.695 \times 0.8}{115 \times 8.695 \times 0.8 + 0.6 + P}$$

$$\Rightarrow \boxed{P_{core} = 32.3893\ W}$$

By, $P_{core} = \dfrac{(V_P/a)^2}{R_{Cs}} \Rightarrow R_{Cs} = \dfrac{(113.3)^2}{32.3893}$

$$\boxed{R_{Cs} = 396.32\,\Omega}$$

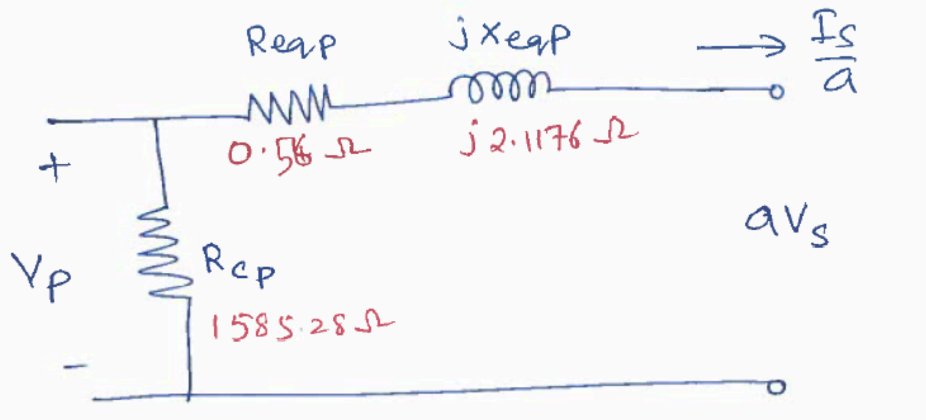
Now, refered to H.V. Side

$$R_{eqP} = a^2 R_{eqs} = 2^2 \times 0.14 = 0.56\,\Omega$$

$$X_{eqP} = a^2 X_{eqs} = 2^2 \times 0.5294 = 2.1176\,\Omega$$

$$R_{CP} = a^2 R_{Cs} = 2^2 \times 396.32 = 1585.28\,\Omega$$

Equivalent circuit Refered to H.V. Side



Solution P6:

(a)

$$P_{out} = V_S I_S \cos\theta_S = 0.7 \times 30{,}000 \times 0.7 = 14{,}700\ W$$

$$P_{core} = 400\ W$$

$$P_{cu} = I_S^2 R_{eq,s} \Rightarrow R_{eq,s} = \frac{P_{cu}}{I_S^2} = \frac{1200}{\left(\frac{30{,}000}{240}\right)^2} = 0.0768\ \Omega$$

$$P_{cu} @\ 0.7 \text{ load} \Rightarrow P_{cu} = 0.7^2 * 1200 = 588\ W$$

$$\eta = \frac{P_{out}}{P_{out} + P_{core} + P_{cu}} \times 100$$

$$= \frac{14{,}700}{14{,}700 + 400 + 588} \times 100 = 93.7\ \%$$

(b)

I_S at max. efficiency

$$I_S = \left(\frac{P_{cu}}{R_{eq,s}}\right)^{1/2} = 7\left(\frac{400}{0.0768}\right)^{1/2}$$

$$I_s = 72.168\ A$$

$$P_{out}\Big|_{\eta max} = V_S I_S \cos\theta_S$$

$$= 240 \times 72.168 \times 1$$

$$P_{out}\Big|_{\eta max} = 17320.5\ W$$

(c)

$$\eta_{max} = \frac{P_{out}}{P_{out} + P_{core} + P_{cu}} \times 100$$

$$= \frac{17320.5}{17320.5 + 400 + 400} \times 100$$

$$\eta_{max} = 95.58\%.$$

(d)

Output KVA at $\eta_{max} = 17.320$

Rated KVA $= 30$

η_{max} occurs at $\frac{17.320}{30} = 57.7\%$ of full load.