KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

ELECTRICAL ENGINEERING DEPARTMENT

EE 463 – Term 161

HW # 4: Unsymmetrical Faults

Key Solutions

From Text: 10.1; 10.9; 10.13; 10.16

Extra Problems:

Problem # 1)

A 60-Hz turbogenerator is rated 500 MVA, 22 kV. It is Y-connected and solidly grounded and is operating at rated voltage at no load. It is disconnected from the rest of the system. Its reactances are $X_d'' = X_1 = X_2 = 0.15$ and $X_0 = 0.05$ per unit. Find the ratio of the subtransient line current for a single line-to-ground fault to the subtransient line current for a symmetrical three-phase fault.

Problem # 2)

Find the ratio of the subtransient line current for a line-to-line fault to the subtransient current for a symmetrical three-phase fault on the generator of

Problem #1.

10.1. Obtain the symmetrical components for the set of unbalanced voltages $V_a = 300\angle{-120°}$, $V_b = 200\angle 90°$, and $V_c = 100\angle{-30°}$.
The commands

```
Vabc = [300  -120
        200    90
        100   -30];
V012 = abc2sc(Vabc);  % Symmetrical components of phase a
V012p= rec2pol(V012)          % Rectangular to polar form
```

result in

```
V012p =
         42.2650 -120.0000
        193.1852 -135.0000
          86.9473  -84.8961
```

10.9. A generator having a solidly grounded neutral and rated 50-MVA, 30-kV has positive-, negative-, and zero-sequence reactances of 25, 15, and 5 percent, respectively. What reactance must be placed in the generator neutral to limit the fault current for a bolted line-to-ground fault to that for a bolted three-phase fault?

The generator base impedance is

$$Z_B = \frac{(30)^2}{50} = 18 \;\; \Omega$$

The three-phase fault current is

$$I_{f\,3\phi} = \frac{1}{0.25} = 4.0 \;\; \text{pu}$$

The line-to-ground fault current is

$$I_{f\,LG} = \frac{3}{0.25 + 0.15 + 0.05 + 3X_n} = 4.0 \;\; \text{pu}$$

Solving for X_n, results in

$$\begin{aligned} X_n &= 0.1 \quad \text{pu} \\ &= (0.1)(18) = 1.8 \;\; \Omega \end{aligned}$$

10.13. Repeat Problem 10.11 for a bolted double line-to-ground fault on phases b and c.

The positive- and zero-sequence fault currents in phase a are

$$\begin{aligned} I_a^1 &= \frac{1}{j0.105 + j\left(\frac{(0.085)(0.06)}{0.085+0.06}\right)} = -j7.13407 \text{ pu} \\ I_a^0 &= -\frac{1-(j0.105)(-j7.13407)}{j0.06} = j4.182 \text{ pu} \end{aligned}$$

The fault current is

$$I_f = 3I_a^0 = 12.546\angle 90^\circ$$

10.16. For Problem 10.15, obtain the bus impedance matrices for the sequence networks. A bolted single line-to-ground fault occurs at bus 1. Find the fault current, the three-phase bus voltages during fault, and the line currents in each phase. Check your results using the **zbuild** and **lgfault** programs.

First, we obtain the positive-sequence bus impedance matrix. Add branch 1, $z_{30} = j0.1$ between node $q = 3$ and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{33} = z_{30} = j0.1$$

Next, add branch 2, $z_{40} = j0.1$ between node $q = 4$ and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{33} & 0 \\ 0 & Z_{44} \end{bmatrix} = \begin{bmatrix} j0.1 & 0 \\ 0 & j0.1 \end{bmatrix}$$

Add branch 3, $z_{24} = j0.25$ between the new node $q = 2$ and the existing node $p = 4$. According to rule 2, we get

$$\mathbf{Z}_{bus}^{(3)} = \begin{bmatrix} j0.35 & 0 & j0.1 \\ 0 & j0.1 & 0 \\ j0.1 & 0 & j0.1 \end{bmatrix}$$

Add branch 4, $z_{13} = j0.25$ between the new node $q = 1$ and the existing node $p = 3$. According to rule 2, we get

$$\mathbf{Z}_{bus}^{(4)} = \begin{bmatrix} j0.35 & 0 & j0.1 & 0 \\ 0 & j0.35 & 0 & j0.1 \\ j0.1 & 0 & j0.1 & 0 \\ 0 & j0.1 & 0 & j0.1 \end{bmatrix}$$

Add link 5, $z_{12} = j0.3$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\mathbf{Z}_{bus}^{(5)} = \left[\begin{array}{cccc|c} j0.35 & 0 & j0.1 & 0 & -j0.35 \\ 0 & j0.35 & 0 & j0.1 & j0.35 \\ j0.1 & 0 & j0.1 & 0 & -j0.1 \\ 0 & j0.1 & 0 & j0.1 & j0.1 \\ \hline -j0.35 & j0.35 & -j0.1 & j0.1 & j1 \end{array}\right]$$

From (9.58)

$$\frac{\Delta \mathbf{Z}\, \Delta \mathbf{Z}^T}{Z_{44}} = \frac{1}{j1}\begin{bmatrix} -j0.35 \\ j0.35 \\ -j0.1 \\ j0.1 \end{bmatrix}\begin{bmatrix} -j0.35 & j0.35 & -j0.1 & j0.1 \end{bmatrix}$$

$$= \begin{bmatrix} j0.1225 & -j0.1225 & j0.0350 & -j0.0350 \\ -j0.1225 & j0.1225 & -j0.0350 & j0.0350 \\ j0.0350 & -j0.0350 & j0.0100 & -j0.0100 \\ -j0.0350 & j0.0350 & -j0.0100 & j0.0100 \end{bmatrix}$$

From (9.59), the new positive-sequence bus impedance matrix is

$$\mathbf{Z}_{bus}^1 = \begin{bmatrix} j0.35 & 0 & j0.1 & 0 \\ 0 & j0.35 & 0 & j0.1 \\ j0.1 & 0 & j0.1 & 0 \\ 0 & j0.1 & 0 & j0.1 \end{bmatrix} - \begin{bmatrix} j0.1225 & -j0.1225 & j0.0350 & -j0.0350 \\ -j0.1225 & j0.1225 & -j0.0350 & j0.0350 \\ j0.0350 & -j0.0350 & j0.0100 & -j0.0100 \\ -j0.0350 & j0.0350 & -j0.0100 & j0.0100 \end{bmatrix}$$

$$= \begin{bmatrix} j0.2275 & j0.1225 & j0.0650 & j0.0350 \\ j0.1225 & j0.2275 & j0.0350 & j0.0650 \\ j0.0650 & j0.0350 & j0.0900 & j0.0100 \\ j0.0350 & j0.0650 & j0.0100 & j0.0900 \end{bmatrix}$$

Next, we obtain the zero-sequence bus impedance matrix. Add branch 1, $z_{10} = j0.25$ between node $q = 1$ and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{11} = z_{10} = j0.25$$

Next, add branch 2, $z_{20} = j0.25$ between node $q = 2$ and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{11} & 0 \\ 0 & Z_{22} \end{bmatrix} = \begin{bmatrix} j0.25 & 0 \\ 0 & j0.25 \end{bmatrix}$$

Add link 3, $z_{12} = j0.5$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\mathbf{Z}_{bus}^{(3)} = \left[\begin{array}{cc|c} j0.25 & 0 & -j0.25 \\ 0 & j0.25 & j0.25 \\ \hline -j0.25 & j0.25 & j1 \end{array}\right]$$

From (9.58)

$$\frac{\Delta \mathbf{Z}\, \Delta \mathbf{Z}^T}{Z_{44}} = \frac{1}{j1}\begin{bmatrix} -j0.25 \\ j0.25 \end{bmatrix}\begin{bmatrix} -j0.25 & j0.25 \end{bmatrix}$$

$$= \begin{bmatrix} j0.0625 & -j0.0625 \\ -j0.0625 & j0.0625 \end{bmatrix}$$

From (9.59), the new positive-sequence bus impedance matrix is

$$\begin{aligned}\mathbf{Z}_{bus}^{0} &= \begin{bmatrix} j0.25 & 0 \\ 0 & j0.25 \end{bmatrix} - \begin{bmatrix} j0.0625 & -j0.0625 \\ -j0.0625 & j0.0625 \end{bmatrix} \\ &= \begin{bmatrix} j0.1875 & j0.0625 \\ j0.0625 & j0.1875 \end{bmatrix}\end{aligned}$$

For a bolted single line-to-ground fault at bus 1, from (10.90), the symmetrical components of fault current is given by

$$\begin{aligned}I_1^0(F) = I_1^1(F) = I_1^2(F) &= \frac{1.0}{Z_{11}^1 + Z_{11}^2 + Z_{11}^0} \\ &= \frac{1.0}{j0.2275 + j0.2275 + j0.1875} = -j1.5564\end{aligned}$$

The fault current is

$$I_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j1.5564 \\ -j1.5564 \\ -j1.5564 \end{bmatrix} = \begin{bmatrix} 4.6693\angle{-90^\circ} \\ 0\angle 0^\circ \\ 0\angle 0^\circ \end{bmatrix}$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 - Z_{11}^0 I_1^0 \\ V_1^1(0) - Z_{11}^1 I_1^1 \\ 0 - Z_{11}^2 I_1^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.1875(-j1.5564) \\ 1 - j0.2275(-j1.5564) \\ 0 - j0.2275(-j1.5564) \end{bmatrix} = \begin{bmatrix} -0.2918 \\ 0.6459 \\ -0.3541 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 - Z_{21}^0 I_1^0 \\ V_2^1(0) - Z_{21}^1 I_1^1 \\ 0 - Z_{21}^2 I_1^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.0625(-j1.5564) \\ 1 - j0.1225(-j1.5564) \\ 0 - j0.1225(-j1.5564) \end{bmatrix} = \begin{bmatrix} -0.0973 \\ 0.8093 \\ -0.1907 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 - Z_{31}^0 I_1^0 \\ V_3^1(0) - Z_{31}^1 I_1^1 \\ 0 - Z_{31}^2 I_1^2 \end{bmatrix} = \begin{bmatrix} 0 - j0(-j1.5564) \\ 1 - j0.0650(-j1.5564) \\ 0 - j0.0650(-j1.5564) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.8988 \\ -0.1012 \end{bmatrix}$$

$$V_4^{012}(F) = \begin{bmatrix} 0 - Z_{41}^0 I_1^0 \\ V_4^1(0) - Z_{41}^1 I_1^1 \\ 0 - Z_{41}^2 I_1^2 \end{bmatrix} = \begin{bmatrix} 0 - j0(-j1.5564) \\ 1 - j0.0350(-j1.5564) \\ 0 - j0.0350(-j1.5564) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.9455 \\ -0.0545 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.2918 \\ 0.6459 \\ -0.3541 \end{bmatrix} = \begin{bmatrix} 0\angle{-180^\circ} \\ 0.9704\angle{-116.815^\circ} \\ 0.9704\angle{+116.815^\circ} \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.0973 \\ 0.8093 \\ -0.1907 \end{bmatrix} = \begin{bmatrix} 0.5214\angle 0^\circ \\ 0.9567\angle -115.151^\circ \\ 0.9567\angle +115.151^\circ \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.8988 \\ -0.1012 \end{bmatrix} = \begin{bmatrix} 0.7977\angle 0^\circ \\ 0.9535\angle -114.727^\circ \\ 0.9535\angle +114.727^\circ \end{bmatrix}$$

$$V_4^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.9455 \\ -0.0545 \end{bmatrix} = \begin{bmatrix} 0.8911\angle 0^\circ \\ 0.9739\angle -117.223^\circ \\ 0.9739\angle +117.7223^\circ \end{bmatrix}$$

The symmetrical components of fault currents in lines for phase a are

$$I_{21}^{012} = \begin{bmatrix} \frac{V_2^0(F)-V_1^0(F)}{z_{12}^0} \\ \frac{V_2^1(F)-V_1^1(F)}{z_{12}^1} \\ \frac{V_2^2(F)-V_1^2(F)}{z_{12}^2} \end{bmatrix} = \begin{bmatrix} \frac{-0.0973-(-0.2918)}{j0.5} \\ \frac{0.8093-0.6459)}{j0.3} \\ \frac{-0.1907-(-0.3541)}{j0.3} \end{bmatrix} = \begin{bmatrix} 0.3891\angle -90^\circ \\ 0.5447\angle -90^\circ \\ 0.5447\angle -90^\circ \end{bmatrix}$$

$$I_{31}^{012} = \begin{bmatrix} \frac{V_3^0(F)-V_1^0(F)}{z_{13}^0} \\ \frac{V_3^1(F)-V_1^1(F)}{z_{13}^1} \\ \frac{V_3^2(F)-V_1^2(F)}{z_{13}^2} \end{bmatrix} = \begin{bmatrix} \frac{0-(-0.2918)}{\infty} \\ \frac{0.8988-0.6459)}{j0.25} \\ \frac{-0.1012-(-0.3541)}{j0.25} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0117\angle -90^\circ \\ 1.0117\angle -90^\circ \end{bmatrix}$$

$$I_{42}^{012} = \begin{bmatrix} \frac{V_4^0(F)-V_2^0(F)}{z_{24}^0} \\ \frac{V_4^1(F)-V_2^1(F)}{z_{24}^1} \\ \frac{V_4^2(F)-V_2^2(F)}{z_{24}^2} \end{bmatrix} = \begin{bmatrix} \frac{0-(-0.0973)}{\infty} \\ \frac{0.9455-0.8093)}{j0.25} \\ \frac{-0.0545-(-0.1907)}{j0.25} \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5447\angle -90^\circ \\ 0.5447\angle -90^\circ \end{bmatrix}$$

The line fault currents are

$$I_{21}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.3891\angle -90^\circ \\ 0.5447\angle -90^\circ \\ 0.5447\angle -90^\circ \end{bmatrix} = \begin{bmatrix} 1.4784\angle -90^\circ \\ 0.1556\angle 90^\circ \\ 0.1556\angle 90^\circ \end{bmatrix}$$

$$I_{31}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 1.0117\angle 90^\circ \\ 1.0117\angle 90^\circ \end{bmatrix} = \begin{bmatrix} 2.0233\angle -90^\circ \\ 1.0117\angle 90^\circ \\ 1.0117\angle 90^\circ \end{bmatrix}$$

$$I_{42}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.5447\angle -90^\circ \\ 0.5447\angle -90^\circ \end{bmatrix} = \begin{bmatrix} 1.0895\angle -90^\circ \\ 0.5447\angle 90^\circ \\ 0.5447\angle 90^\circ \end{bmatrix}$$

Problem # 1)

Solution:

Single line-to-ground fault:

$$\begin{aligned} I_a^{(1)} &= \frac{1}{j0.15 + j0.15 + j0.15} = -j2.857 \text{ per unit} \\ I_a &= 3I_a^{(1)} = -j8.571 \text{ per unit} \end{aligned}$$

Three-phase fault:

$$I_a = \frac{1}{j0.15} = -j6.667 \text{ per unit}$$

The ratio is $8.571/6.667 = 1.286/1$.

Problem # 2)

Solution:

Line-to-line fault:

$$\begin{aligned} I_a^{(1)} &= \frac{1}{j0.15 + j0.15} = -j3.333 \text{ per unit} \\ I_a^{(2)} &= -I_a^{(1)} = j3.333 \text{ per unit} \\ I_b^{(1)} &= a^2 I_a^{(1)} = 3.333\underline{/150^\circ} \text{ per unit} \\ I_{b2} &= aI_a^{(2)} = 3.333\underline{/210^\circ} \text{ per unit} \\ I_b &= I_b^{(1)} + I_{b2} = -5.773 \text{ per unit} \end{aligned}$$

the ratio is now

$5.773/6.667 = 0.866/1$