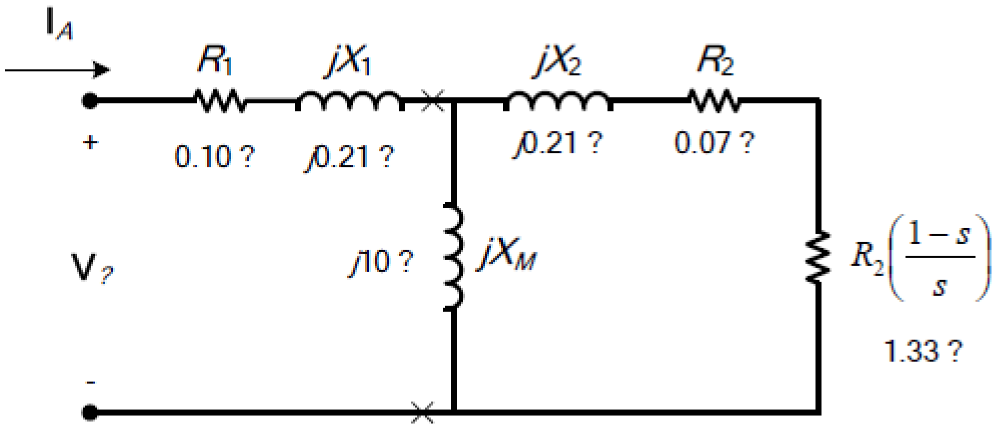
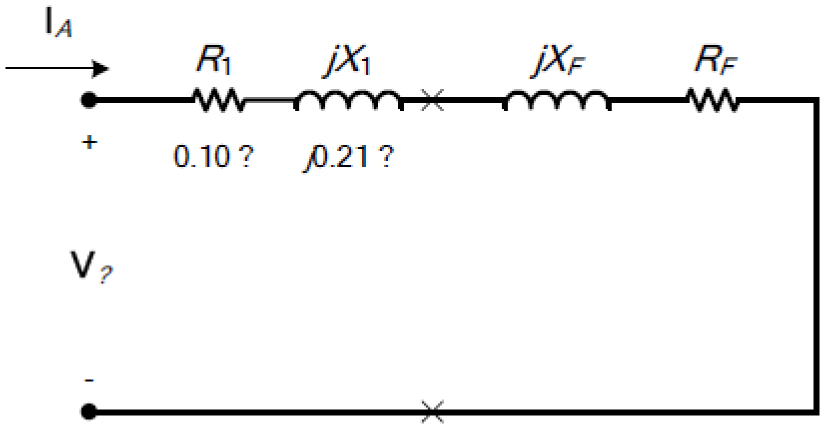
Q1)

The equivalent circuit of this induction motor is shown below:



(a) The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\frac{1}{jX_M} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{j10\ \Omega} + \frac{1}{1.40 + j0.21}} = 1.318 + j0.386 = 1.374\angle 16.3^\circ\ \Omega$$

The phase voltage is $208/\sqrt{3}$ = 120 V, so line current I_L is

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{120\angle 0^\circ\ \text{V}}{0.10\ \Omega\ + j0.21\ \Omega + \ 1.318\ \Omega + j0.386\ \Omega}$$

$$I_L = I_A = 78.0\angle -22.8^\circ\ \text{A}$$

(b) The stator copper losses are

$$P_{SCL} = 3I_A^{\,2}R_1 = 3(78.0\ \text{A})^2(0.10\ \Omega) = 1825\ \text{W}$$

(c) The air gap power is $P_{AG} = 3I_2^{\,2}\dfrac{R_2}{s} = 3I_A^{\,2}R_F$

(Note that $3I_A^{\,2}R_F$ is equal to $3I_2^{\,2}\dfrac{R_2}{s}$, since the only resistance in the original rotor circuit was R_2/s, and the resistance in the Thevenin equivalent circuit is R_F. The power consumed by the Thevenin equivalent circuit must be the same as the power consumed by the original circuit.)

$$P_{AG} = 3I_2^{\,2}\frac{R_2}{s} = 3I_A^{\,2}R_F = 3(78.0\ \text{A})^2(1.318\ \Omega) = 24.0\ \text{kW}$$

(d) The power converted from electrical to mechanical form is

$$P_{conv} = (1-s)P_{AG} = (1-0.05)(24.0\ \text{kW}) = 22.8\ \text{kW}$$

(e) The induced torque in the motor is

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{24.0\ \text{kW}}{(1800\ \text{r/min})\left(\dfrac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right)} = 127.4\ \text{N}\cdot\text{m}$$

(f) The output power of this motor is

$$P_{OUT} = P_{conv} - P_{mech} - P_{core} - P_{misc} = 22.8\ \text{kW} - 500\ \text{W} - 400\ \text{W} - 0\ \text{W} = 21.9\ \text{kW}$$

The output speed is

$$n_m = (1-s)\ n_{sync} = (1-0.05)(1800\ \text{r/min}) = 1710\ \text{r/min}$$

Therefore the load torque is

$$\tau_{load} = \frac{P_{OUT}}{\omega_m} = \frac{21.9\ \text{kW}}{(1710\ \text{r/min})\left(\dfrac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right)} = 122.3\ \text{N}\cdot\text{m}$$

(g) The overall efficiency is

$$\eta = \frac{P_{OUT}}{P_{IN}} \times 100\% = \frac{P_{OUT}}{3V_\phi I_A\cos\theta} \times 100\%$$

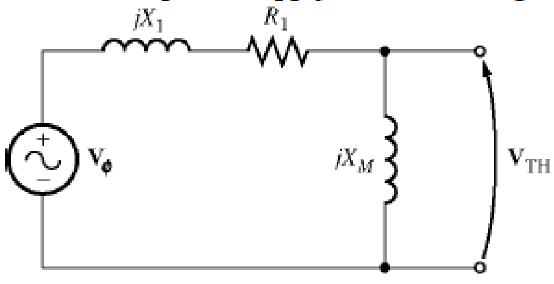
$$\eta = \frac{21.9\ \text{kW}}{3(120\ \text{V})(78.0\ \text{A})\cos 22.8^\circ} \times 100\% = 84.6\%$$

(h) The motor speed in revolutions per minute is 1710 r/min. The motor speed in radians per second is

$$\omega_m = (1710\ \text{r/min})\left(\frac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 179\ \text{rad/s}$$

Q2)

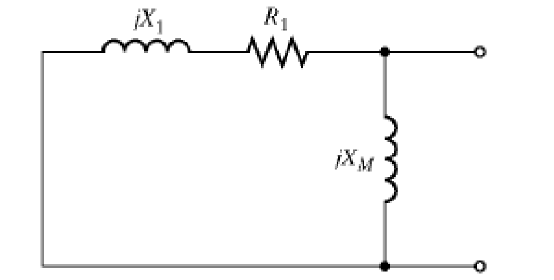
SOLUTION The slip at pullout torque is found by calculating the Thevenin equivalent of the input circuit from the rotor back to the power supply, and then using that with the rotor circuit model.



$$\mathbf{V}_{TH} = \frac{jX_M}{R_1 + jX_1 + jX_M}\mathbf{V}_\phi$$

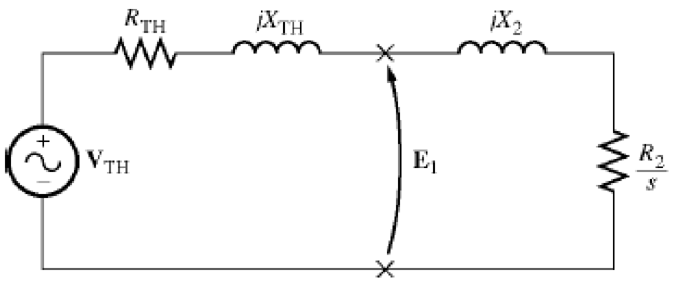
$$V_{TH} = \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}\mathbf{V}_\phi$$

(*a*)



$$Z_{TH} = \frac{jX_M(R_1 + jX_1)}{R_1 + j(X_1 + X_M)}$$

(*b*)



(*c*)

$$Z_{TH} = \frac{jX_M(R_1 + jX_1)}{R_1 + j(X_1 + X_M)} = \frac{(j10\ \Omega)(0.10\ \Omega\ + j0.21\ \Omega)}{0.10\ \Omega\ + j(0.21\ \Omega + 10\ \Omega)} = 0.0959 + j0.2066\ \Omega = 0.2278\angle 65.1^\circ\ \Omega$$

$$\mathbf{V}_{TH} = \frac{jX_M}{R_1 + j(X_1 + X_M)}\mathbf{V}_\phi = \frac{(j10\ \Omega)}{0.1\ \Omega\ +\ j(0.23\ \Omega + 10\ \Omega)}\ (120\angle 0°\ \text{V}) = 117.5\angle 0.6°\ \text{V}$$

The slip at pullout torque is

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}{}^2 + (X_{TH} + X_2)^2}}$$

$$s_{max} = \frac{0.070\ \Omega}{\sqrt{(0.0959\ \Omega)^2 + (0.2066\ \Omega\ + 0.210\ \Omega)^2}} = 0.164$$

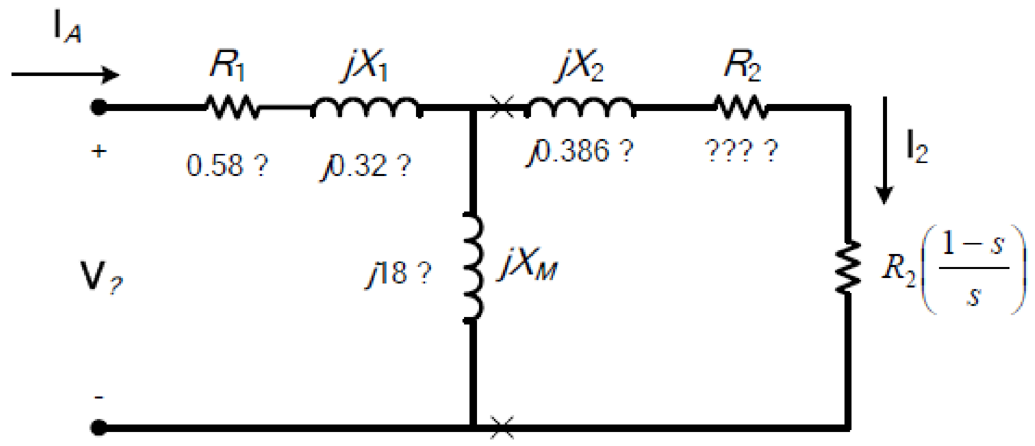
The pullout torque of the motor is

$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}\right]}$$

$$\tau_{max} = \frac{3(117.5\ \text{V})^2}{2(188.5\ \text{rad/s})\left[0.0959\ \Omega + \sqrt{(0.0959\ \Omega)^2 + (0.2066\ \Omega\ + 0.210\ \Omega)^2}\right]}$$

$$\tau_{max} = 210\ \text{N}\cdot\text{m}$$

Q3)



The Thevenin equivalent of the input circuit is:

$$Z_{TH} = \frac{jX_M(R_1 + jX_1)}{R_1 + j(X_1 + X_M)} = \frac{(j18\ \Omega)(0.58\ \Omega\ +\ j0.32\ \Omega)}{0.58\ \Omega\ +\ j(0.32\ \Omega + 18\ \Omega)} = 0.559 + j0.332\ \Omega = 0.651\angle 30.7°\ \Omega$$

$$\mathbf{V}_{TH} = \frac{jX_M}{R_1 + j(X_1 + X_M)}\mathbf{V}_\phi = \frac{(j18\ \Omega)}{0.58\ \Omega\ +\ j(0.32\ \Omega + 18\ \Omega)}(266\angle 0°\ \text{V}) = 261\angle 1.8°\ \text{V}$$

(a) If losses are neglected, the induced torque in a motor is equal to its load torque. At full load, the output power of this motor is 75 hp and its slip is 1.2%, so the induced torque is

$$n_m = (1-0.035)(1800\ \text{r/min}) = 1737\ \text{r/min}$$

$$\tau_{\text{ind}} = \tau_{\text{load}} = \frac{(75\ \text{hp})(746\ \text{W/hp})}{(1737\ \text{r/min})\left(\frac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\frac{1\,\text{min}}{60\ \text{s}}\right)} = 307.6\ \text{N}\cdot\text{m}$$

The induced torque is given by the equation

$$\tau_{\text{ind}} = \frac{3V_{\text{TH}}^2 R_2/s}{\omega_{\text{sync}}\left[\left(R_{\text{TH}} + R_2/s\right)^2 + \left(X_{\text{TH}} + X_2\right)^2\right]}$$

Substituting known values and solving for R_2/s yields

$$\tau_{\text{ind}} = \frac{3V_{\text{TH}}{}^2 (R_2/s)}{\omega_{\text{sync}}\left[\left(R_{\text{TH}} + R_2/s\right)^2 + \left(X_{\text{TH}} + X_2\right)^2\right]}$$

$$307.6\ \text{N}\cdot\text{m} = \frac{3(261\ \text{V})^2 R_2/s}{(188.5\ \text{rad/s})\left[\left(0.559 + R_2/s\right)^2 + (0.332+0.386)^2\right]}$$

$$57{,}983 = \frac{205{,}932\ \ R_2/s}{\left[\left(0.559 + R_2/s\right)^2 + 0.516\right]}$$

$$\left[\left(0.559 + R_2/s\right)^2 + 0.516\right] = 3.552\ \ R_2/s$$

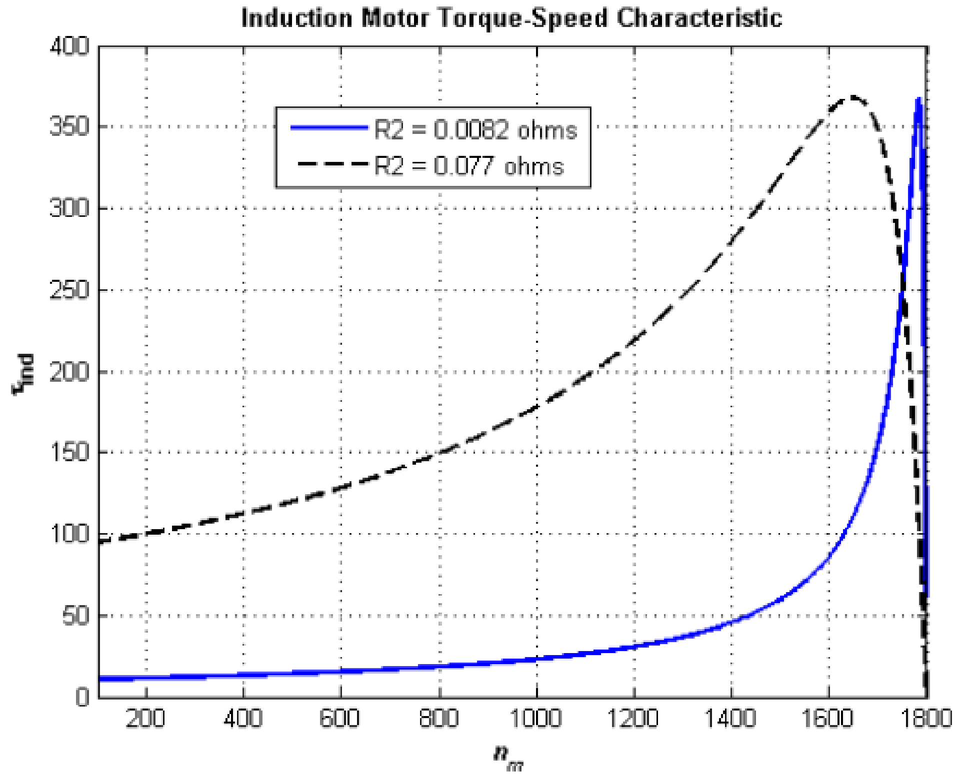
$$\left[0.3125 + 1.118 R_2/s + \left(R_2/s\right)^2 + 0.516\right] = 3.552\ \ R_2/s$$

$$\left(\frac{R_2}{s}\right)^2 - 2.434\left(\frac{R_2}{s}\right) + 0.516 = 0$$

$$\left(\frac{R_2}{s}\right) = 0.2346,\quad 2.199$$

$$R_2 = 0.0082\ \Omega,\ \ 0.077\ \Omega$$

These two solutions represent two situations in which the torque-speed curve would go through this specific torque-speed point. The two curves are plotted below. As you can see, only the 0.077 Ω solution is realistic, since the 0.0082 Ω solution passes through this torque-speed point at an unstable location on the back side of the torque-speed curve.



(b) The slip at pullout torque can be found by calculating the Thevenin equivalent of the input circuit from the rotor back to the power supply, and then using that with the rotor circuit model. The Thevenin equivalent of the input circuit was calculated in part *(a)*. The slip at pullout torque is

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}{}^2 + \left(X_{TH} + X_2\right)^2}}$$

$$s_{max} = \frac{0.077\ \Omega}{\sqrt{(0.559\ \Omega)^2 + (0.332\ \Omega\ + 0.386\ \Omega)^2}} = 0.0846$$

The rotor speed a maximum torque is

$$n_{pullout} = (1-s)\ n_{sync} = (1-0.0846)(1800\ \text{r/min}) = 1648\ \text{r/min}$$

and the pullout torque of the motor is

$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + \left(X_{TH} + X_2\right)^2}\right]}$$

$$\tau_{\max} = \frac{3(261\text{ V})^2}{2(188.5\text{ rad/s})\left[0.559\ \Omega + \sqrt{(0.559\ \Omega)^2 + (0.332\ \Omega\ + 0.368\ \Omega)^2}\right]}$$

$$\tau_{\max} = 373\text{ N}\cdot\text{m}$$

(c) The starting torque of this motor is the torque at slip $s = 1$. It is

$$\tau_{\text{ind}} = \frac{3V_{\text{TH}}^2 R_2 / s}{\omega_{\text{sync}}\left[(R_{\text{TH}} + R_2/s)^2 + (X_{\text{TH}} + X_2)^2\right]}$$

$$\tau_{\text{ind}} = \frac{3(261\text{ V})^2(0.077\ \Omega)}{(188.5\text{ rad/s})\left[(0.559 + 0.077\ \Omega)^2 + (0.332 + 0.368)^2\right]} = 93.3\text{ N}\cdot\text{m}$$

Q4)

5.14 (a) $P_{rot} = 500 - 3 \times 6.5^2 \times \frac{0.54}{2} = 465.78\text{ W}$

(b) From no-load test $R_1 = \frac{0.54}{2} = 0.27\Omega$

$V_1 = \frac{208}{\sqrt{3}} = 120.1\text{ V}$

$Z_{NL} = \frac{120.1}{6.5} = 18.48\ \Omega$

$R_{NL} = \frac{500}{3 \times 6.5^2} = 3.94\ \Omega$

$X_{NL} = \sqrt{18.48^2 - 3.94^2} = 18.05\ \Omega$

$X_1 + X_m = X_{NL} = 18.05\ \Omega$

From blocked - rotor test

$R_{BL} = \frac{1250}{3 \times 25^2} = 0.6667\,\Omega$

$R_2' = 0.6667 - 0.27 = 0.3967\,\Omega$

$Z_{BL} = \frac{44/\sqrt{3}}{25} = 1.0162\,\Omega$

$X_{BL} = \sqrt{1.0162^2 - 0.6667^2} = 0.7669\,\Omega$

$X_1 = X_2' = 0.3834\,\Omega$

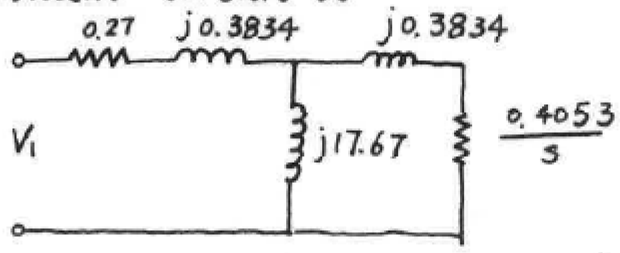
$X_m = 18.05 - 0.3834 = 17.6666\,\Omega$

Precise value for R_2'

$R = R_{BL} - R_1 = 0.6667 - 0.27 = 0.3967\,\Omega$

$R_2' = \frac{0.3834 + 17.6667}{17.6667} \times 0.3967 = 0.4053\,\Omega$

Equivalent Circuit is



(c) Neglecting X_m, $S_b \simeq \frac{R_2'}{\sqrt{R_1^2 + (X_1 + X_2')^2}} = \frac{0.4053}{\sqrt{0.27^2 + 0.7668^2}}$

$= 0.4985$

Maximum torque occurs at 49.85% of synchrouns speed.

It is class - D type motor

(d) $\frac{R_2'}{s} = \frac{0.4053}{0.1} = 4.053\,\Omega$

$Z_1 = 0.27 + j0.3834 + \frac{j17.67(4.053 + j0.3834)}{4.053 + j18.05} = 4.273\angle 21.83^\circ$

$$I_1 = \frac{208/\sqrt{3}}{4.273\angle 21.83^\circ} = 28.104\angle -21.83^\circ$$

$$P_s = 3 \times 120.1 \times 28.104 \times \cos(-21.83^\circ) = 9396.809\,W$$

$$P_{ag} = 9396.809 - 3 \times 28.104^2 \times 0.27 = 8757.045\,W$$

$$P_{mech} = (1-0.1) \times 8757.045 = 7881.341\,W$$

$$P_{out} = P_{shaft} = P_{mech} - P_{rot} = 7881.341 - 465.78 = 7415.561\,W$$

$$HP = \frac{7415.561}{746} = 9.94\,(HP)$$

Q5)

a. $f = \frac{np}{120}$ gives $60 = \frac{n \cdot 10}{120}$ gives $n = 12 \cdot 60 = 720rpm$

b. Plug and chug…

$$T_{\max} = \frac{V_t^2}{2\omega_s\left[R_1 + \sqrt{R_1^2 + X_{eq}^2}\right]} = \frac{4160^2}{2 \cdot \frac{377}{5}\left[0.8 + \sqrt{0.8^2 + 2^2}\right]} = 38.8knt - m$$

c. More plug and chug

$$s_{T_{\max}} = \frac{R_2}{\sqrt{R_1^2 + X_{eq}^2}} = \frac{0.2}{\sqrt{0.8^2 + 2^2}} = 0.093 = 9.3\%$$

d. $f_r = sf_s = 0.093 \cdot 60 = 5.58Hz$

e. At start, s = 1, (1-s)/s = 0

$\bar{I}_{st} = \frac{V_t}{R_{eq} + jX_{eq}}$ so $|I_{st}| = \frac{V_t}{\sqrt{R_{eq}^2 + X_{eq}^2}}$ note, use line to neutral voltage to compute current

$$|I_{st}| = \frac{4160/\sqrt{3}}{\sqrt{1^2 + 2^2}} = \frac{4160}{\sqrt{5} \cdot \sqrt{3}} = 1074A$$

f. $T_{st} = \frac{V_t^2 R_2}{\omega_s\left(R_{eq}^2 + X_{eq}^2\right)}$ gives $4000 = \frac{V_t^2 \cdot 0.2}{\frac{377}{5}\left(1^2 + 2^2\right)}$ gives $V_t = 2750V$

(Reducing voltage is a good way to limit both starting torque and starting current for induction motors.)

g. The small slip assumption is

$$T \cong \frac{V_t^2 s}{\omega_s R_2} \text{ gives} \qquad s = \frac{T\omega_s R_2}{V_t^2} = \frac{4000 \cdot \frac{377}{5} \cdot 0.2}{4160^2} = 0.0035$$

$$n = (1-s)n_s = (1-0.0035)\cdot 720 = 717.5rpm$$