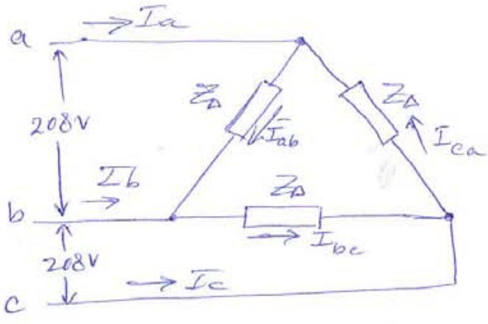
Three-phase circuits

Problem 1 :



$\overline{V}_{an} = 120\angle 0^\circ \Rightarrow V_{ab} = \sqrt{3}\,120\angle 30^\circ$

$\left.\begin{aligned} \overline{V}_{ab} &= 208\angle 30^\circ \text{ V} \\ \overline{V}_{bc} &= 208\angle -90^\circ \text{ V} \\ \overline{V}_{ca} &= 208\angle +150 \text{ V} \end{aligned}\right\}$ Phase voltages

(a)

$\left.\begin{aligned} \overline{I}_{ab} &= \frac{\overline{V}_{ab}}{Z_\Delta} = \frac{208\angle 30^\circ}{20\angle -30} = 10.4\angle 60^\circ \text{ A} \\ \overline{I}_{bc} &= \frac{\overline{V}_{bc}}{Z_\Delta} = \frac{208\angle -90^\circ}{20\angle -30} = 10.4\angle -60^\circ \text{ A} \\ \overline{I}_{ca} &= \frac{\overline{V}_{ca}}{Z_\Delta} = \frac{208\angle 150^\circ}{20\angle -30} = 10.4\angle 180^\circ \text{ A} \end{aligned}\right\}$ phase currents

(b)

$\left.\begin{aligned} \overline{I}_a &= \sqrt{3}\,\overline{I}_{ab}\angle -30^\circ = \sqrt{3}\,10.4\angle 60^\circ - 30^\circ = 18\angle 30^\circ \text{ A} \\ \overline{I}_b &= 18\angle -90^\circ \text{ A} \\ \overline{I}_c &= 18\angle 150^\circ \text{ A} \end{aligned}\right\}$ line currents

(c) $PF = \cos(-30^\circ) = 0.866$ (leading)

(d) $S_{3ph} = 3\,V_{ph}\cdot I_{ph} = \sqrt{3}\,V_L I_L = \sqrt{3}\;208 \times 18 = 6485 \text{ VA}$

$P_{3\phi} = 3\,V_{ph}\cdot I_{ph}\cdot\cos\phi = \sqrt{3}\,V_L I_L \cos\phi = 3 \times 208 \times 10.4 \times 0.866$

$P_{3\phi} = 5.616 \text{ KW}$

$Q_{3\phi} = \sqrt{S_{3\phi}^2 - P_{3\phi}^2} = 3.243 \text{ kVAr}$

Problem 2

Load#1

$Z_\Delta = 10\angle 30^\circ \ \Omega/ph = 8.66 + j5 \ \Omega/ph$

$\Delta - Y$ trans. $\quad Z_{Y1} = 2.887 + j1.667 = 3.333\angle 30^\circ \ \Omega/ph$

Load #2 $\quad Z_{Y2} = 5\angle -36.87^\circ = 4 - j3 \ \Omega/ph$

$Z_t = Z_{Y1} // Z_{Y2} = \dfrac{16.665\angle -6.87}{6.887 - j1.333} = \dfrac{16.665\angle -6.87}{7.015\angle -10.954}$

$= 2.376\angle 4.084 \ \Omega/ph$

a) the overall power factor $= \cos(4.084) = 0.997$ lagging

b) the total current supplied $= \dfrac{V_{ph}}{Z_t} = \dfrac{480/\sqrt{3}}{2.376} = 116.64 \text{ A}$

c) $P_t = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} * 480 * 116.64 * 0.997$

$= 9.668 \text{ KW}$

$Q_t = \sqrt{3} V_L I_L \sin\phi = \sqrt{3} * 480 * 116.64 * \sin(4.084) -$

$= 6.906 \text{ KVAR}$

$S_t = \sqrt{3} V_L I_L = \sqrt{3} * 480 * 116.64$

$= 9.697 \text{ KVA}$

Problem 3

$$Z_{ph} = 20 + j15\ \Omega = 25\angle 36.87$$

$$V_{ph} = \frac{400}{\sqrt{3}}\angle 0 = 231\angle 0$$

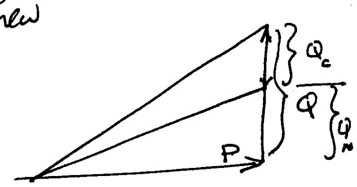
a) $$I_{ph} = I_L = \frac{V_{ph}}{Z_{ph}} = \frac{231\angle 0}{25\angle 36.87} = 9.24\angle -36.87^\circ$$

b) $$P = \sqrt{3} * V_L I_L \cos 36.87 = \sqrt{3} * 400 * 9.24 * 0.8$$
$$= 5120\ W$$

$$Q = \sqrt{3} V_L I_L \sin 36.87 = \sqrt{3} * 400 * 9.24 * 0.6$$
$$= 3840\ VAR$$

When the capacitor bank is connected

$$pf_{new} = 0.95 \ ; \ \phi_{new} = 18.19^\circ \ ; \ \tan\phi_{new} = 0.3286$$



real Power remains Constant

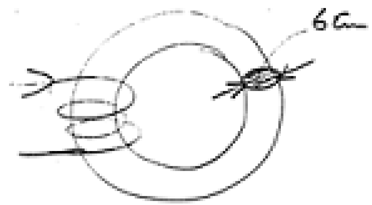
$$Q_{new} = P\tan\phi_{new} = 1682\ VAR$$
$$Q_c = Q - Q_{new}$$
$$Q_c = 3840 - 1682 = 2158\ VAR$$

$$Q_{cph} = \frac{2158}{3} = 719.3\ VAR$$

$$Q_{cph} = \frac{V_L^2}{X_c} \longrightarrow X_c = \frac{V_L^2}{Q_{cph}} = \frac{(400)^2}{719.3} = 222.44\ \Omega = \frac{1}{\omega C}$$

$$C = \frac{1}{2\pi * 50 * 222.44} = 14.31\ \mu F$$

Problem 4



$l = \pi d_m = \pi * 25 = 78.64$ cm

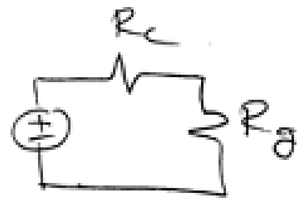
$A = \pi * \frac{(6)^2}{4} = 28.27$ cm^2

a) $B = \frac{1.7 * 10^{-3}}{28.27 * 10^{-4}} = 0.6$ T

From table $H = 600$ AT/m

so $NI = Hl$ $\quad I = \frac{600 * 78.64 * 10^{-2}}{600}$

$I = \underline{0.7864}$ A



(b) Same Φ and B

$H_c = 600$ AT/m

$l_c = 78.64$ cm

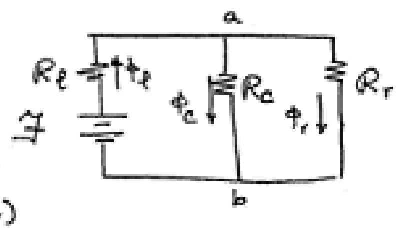
$H_g = \frac{B}{\mu_0} = \frac{0.6}{4\pi * 10^{-7}} = 477.5 * 10^3$ AT/m

$l_g = 2 * 10^{-3}$ m

$NI = H_g l_g + H_c l_c = 1427$ AT

$I = \underline{2.378}$ A

Problem 5



$\phi_c = 0.36 \text{ mWb}$

$B_c = \frac{\phi_c}{A} = \frac{0.36 * 10^{-3}}{4 * 10^{-4}} = \underline{0.9} \text{ Wb/m}^2$

$H_c = 150 \text{ AT/m}$ (From B-H Curve)

$\therefore AT_{ab} = H_c \ell_c = 150 * 0.06 = 9 \text{ AT}$ $\quad (\ell_c = 4+1+1 = 6 \text{ Cm})$

$= H_r \ell_r = H_r * (6+6+6) * 10^{-2}$

$\therefore H_r = \frac{9}{18 * 10^{-2}} = 50 \text{ AT/m}$

$\therefore B_r$ (Corresponding Value from B-H Curve) $= 0.35 \text{ Wb/m}^2$

$\therefore \phi_r = B_r A_r = 0.35 * 4 * 10^{-4} = \underline{0.14} \text{ mWb}$

$\therefore \phi_\ell = \phi_c + \phi_r = 0.36 + 0.14 = \underline{0.5} \text{ mWb}$

$\therefore B_\ell = \frac{\phi_\ell}{A} = \frac{0.5 * 10^{-3}}{4 * 10^{-4}} = 1.25 \text{ Wb/m}^2$

$\therefore H_\ell = 500 \text{ AT/m}$ (from B-H Curve).

$\therefore H_\ell \ell_\ell = 500 * (6+6+6) * 10^{-2} = \underline{90} \text{ ATs}$

$\therefore \mathcal{F} = H_\ell \ell_\ell + H_c \ell_c = 90 + 9 = 99 \text{ ATs}$

$= N I$

$\therefore I = \frac{99}{300} = \underline{\underline{0.33}} \text{ Amps}$

Problem 6

$g_c = 1\,mm$; $g_A = 2\,mm$, $A = (5)(5) = 25\,cm^2 = 25\times10^{-4}\,m^2$

$N = 200$ turns , $R = 2.5\,\Omega$

(a) $\mathcal{R}_c = \dfrac{g_c}{\mu_0 A} = \dfrac{1\times10^{-3}}{(4\pi\times10^{-7})(25\times10^{-4})} = 318{,}310\ At/wb$

$\phi_c = BA = (0.75)(25\times10^{-4}) = 1.875\times10^{-3}\ Wb$

$NI = \mathcal{R}_c\,\phi_c = (318{,}310)(1.875\times10^{-3}) = 596.83\ At$

$I = \dfrac{596.83}{200} = 2.984\ A$

$V = RI = (2.5)(2.984) = 7.46\ V$

(b) $\mathcal{R}_A = \dfrac{g_A}{\mu_0 A} = \dfrac{2\times10^{-3}}{(4\pi\times10^{-7})(25\times10^{-4})} = 636{,}620\ At/wb$

$\phi_A = \dfrac{NI}{\mathcal{R}_A} = \dfrac{596.83}{636{,}620} = 9.375\times10^{-4}\ Wb$

$\phi_B = \phi_A + \phi_c = 9.375\times10^{-4} + 18.75\times10^{-4} = 28.125\times10^{-4}$
$= 2.8125\times10^{-3}\ Wb$