Chapter 5: *Synchronous Motors*

5-1. A 480-V, 60 Hz, 400-hp 0.8-PF-leading eight-pole Δ-connected synchronous motor has a synchronous reactance of 0.6 Ω and negligible armature resistance. Ignore its friction, windage, and core losses for the purposes of this problem. Assume that $\left|\mathbf{E}_A\right|$ is directly proportional to the field current I_F (in other words, assume that the motor operates in the linear part of the magnetization curve), and that $\left|\mathbf{E}_A\right|$ = 480 V when $I_F = 4$ A.

(a) What is the speed of this motor?

(b) If this motor is initially supplying 400 hp at 0.8 PF lagging, what are the magnitudes and angles of $\mathbf{E}_A$ and $\mathbf{I}_A$?

(c) How much torque is this motor producing? What is the torque angle δ? How near is this value to the maximum possible induced torque of the motor for this field current setting?

(d) If $\left|\mathbf{E}_A\right|$ is increased by 30 percent, what is the new magnitude of the armature current? What is the motor's new power factor?

(e) Calculate and plot the motor's V-curve for this load condition.

SOLUTION

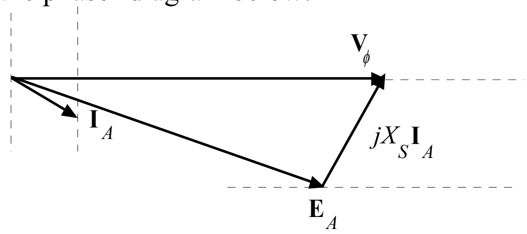
(a) The speed of this motor is given by

$$n_m = \frac{120 f_{se}}{P} = \frac{120\left(60\text{ Hz}\right)}{8} = 900\text{ r/min}$$

(b) If losses are being ignored, the output power is equal to the input power, so the input power will be

$$P_{\text{IN}} = \left(400\text{ hp}\right)\left(746\text{ W/hp}\right) = 298.4\text{ kW}$$

This situation is shown in the phasor diagram below:



The line current flow under these circumstances is

$$I_L = \frac{P}{\sqrt{3}\ V_T\ \text{PF}} = \frac{298.4\text{ kW}}{\sqrt{3}\left(480\text{ V}\right)\left(0.8\right)} = 449\text{ A}$$

Because the motor is Δ-connected, the corresponding phase current is $I_A = 449/\sqrt{3} = 259$ A. The angle of the current is $-\cos^{-1}\left(0.80\right) = -36.87°$, so $\mathbf{I}_A = 259\angle -36.87°$ A. The internal generated voltage $\mathbf{E}_A$ is

$$\mathbf{E}_A = \mathbf{V}_\phi - jX_S\mathbf{I}_A$$

$$\mathbf{E}_A = \left(480\angle 0°\text{ V}\right) - j\left(0.6\ \Omega\right)\left(259\angle -36.87°\text{ A}\right) = 406\angle -17.8°\text{ V}$$

(c) This motor has 6 poles and an electrical frequency of 60 Hz, so its rotation speed is n_m = 1200 r/min. The induced torque is

$$\tau_{\text{ind}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{298.4 \text{ kW}}{(900 \text{ r/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)} = 3166 \text{ N}\cdot\text{m}$$

The maximum possible induced torque for the motor at this field setting is the maximum possible power divided by ω_m

$$\tau_{\text{ind,max}} = \frac{3V_\phi E_A}{\omega_m X_S} = \frac{3(480 \text{ V})(406 \text{ V})}{(900 \text{ r/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)(0.6\ \Omega)} = 10{,}340 \text{ N}\cdot\text{m}$$

The current operating torque is about 1/3 of the maximum possible torque.

(d) If the magnitude of the internal generated voltage E_A is increased by 30%, the new torque angle can be found from the fact that $E_A \sin\delta \propto P = \text{constant}$.

$$E_{A2} = 1.30\ E_{A1} = 1.30(406 \text{ V}) = 487.2 \text{ V}$$

$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_1\right) = \sin^{-1}\left(\frac{406 \text{ V}}{487.2 \text{ V}}\sin(-17.8°)\right) = -14.8°$$

The new armature current is

$$\mathbf{I}_{A2} = \frac{\mathbf{V}_\phi - \mathbf{E}_{A2}}{jX_S} = \frac{480\angle 0° \text{ V} - 487.2\angle -14.8° \text{ V}}{j0.6\ \Omega} = 208\angle -4.1° \text{ A}$$

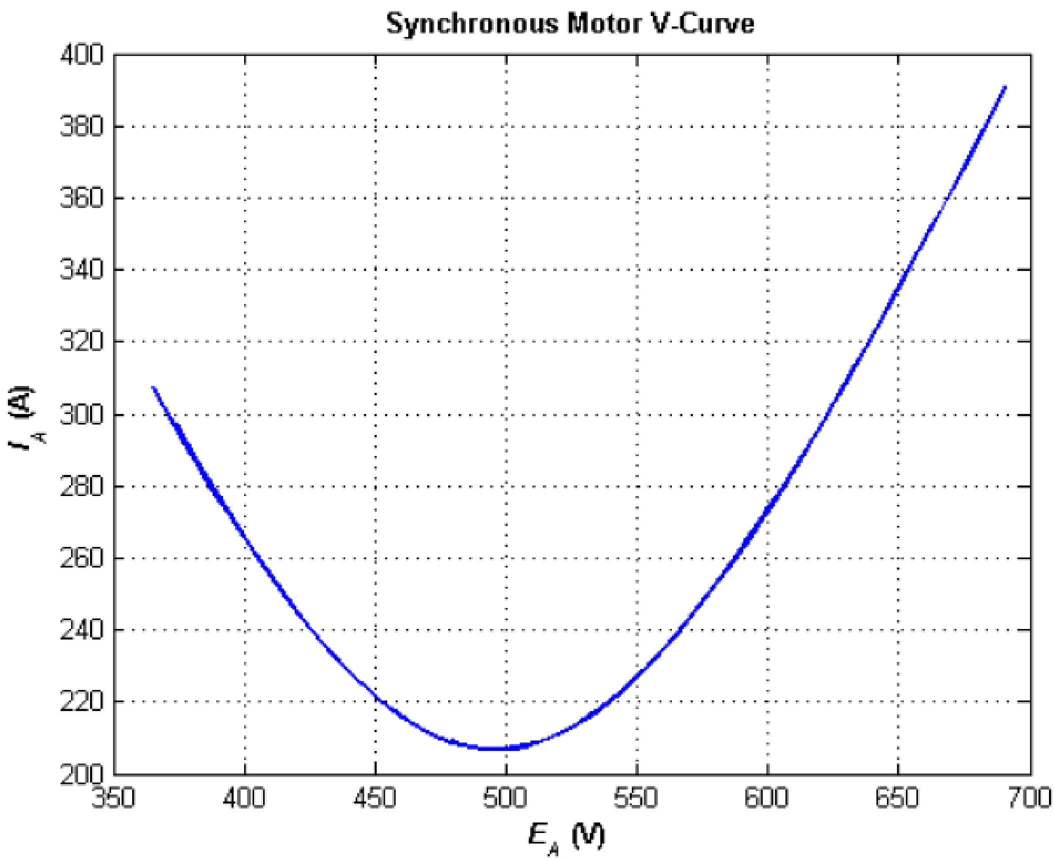
The magnitude of the armature current is 208 A, and the power factor is cos (-24.1°) = 0.913 lagging.

(e) A MATLAB program to calculate and plot the motor's V-curve is shown below:

```
% M-file: prob5_1e.m
% M-file create a plot of armature current versus Ea
%   for the synchronous motor of Problem 5-1.

% Initialize values
Ea = (0.90:0.01:1.70)*406;        % Magnitude of Ea volts
Ear = 406;                        % Reference Ea
deltar = -17.8 * pi/180;          % Reference torque angle
Xs = 0.6;                         % Synchronous reactance (ohms)
Vp = 480;                         % Phase voltage at 0 degrees
Ear = Ear * (cos(deltar) + j * sin(deltar));

% Calculate delta2
delta2 = asin ( abs(Ear) ./ abs(Ea) .* sin(deltar) );

% Calculate the phasor Ea
Ea = Ea .* (cos(delta2) + j .* sin(delta2));

% Calculate Ia
Ia = ( Vp - Ea ) / ( j * Xs);

% Plot the v-curve
```

```
figure(1);
plot(abs(Ea),abs(Ia),'b','Linewidth',2.0);
xlabel('\bf\itE_{A}\rm\bf (V)');
ylabel('\bf\itI_{A}\rm\bf (A)');
title ('\bfSynchronous Motor V-Curve');
grid on;
```

The resulting plot is shown below



5-3. A 230-V, 50 Hz, two-pole synchronous motor draws 40 A from the line at unity power factor and full load. Assuming that the motor is lossless, answer the following questions:

(a) What is the output torque of this motor? Express the answer both in newton-meters and in pound-feet.

(b) What must be done to change the power factor to 0.85 leading? Explain your answer, using phasor diagrams.

(c) What will the magnitude of the line current be if the power factor is adjusted to 0.85 leading?

SOLUTION

(a) If this motor is assumed lossless, then the input power is equal to the output power. The input power to this motor is

$$P_{\text{IN}} = \sqrt{3} V_T I_L \cos\theta = \sqrt{3}(230 \text{ V})(40 \text{ A})(1.0) = 15.93 \text{ kW}$$

The rotational speed of the motor is

$$n_m = \frac{120 f_{se}}{P} = \frac{120(50 \text{ Hz})}{4} = 1500 \text{ r/min}$$

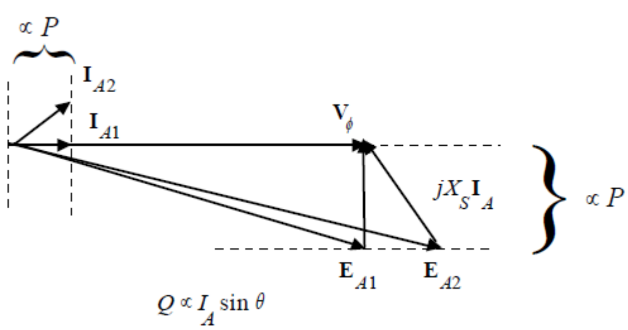
The output torque would be

$$\tau_{\text{LOAD}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{15.93 \text{ kW}}{(1500 \text{ r/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)} = 101.4 \text{ N} \cdot \text{m}$$

In English units,

$$\tau_{\text{LOAD}} = \frac{7.04\, P_{\text{OUT}}}{n_m} = \frac{(7.04)(15.93 \text{ kW})}{(1500 \text{ r/min})} = 74.8 \text{ lb} \cdot \text{ft}$$

(b) To change the motor's power factor to 0.8 leading, its field current must be increased. Since the power supplied to the load is independent of the field current level, an increase in field current increases $|\mathbf{E}_A|$ while keeping the distance $E_A \sin\delta$ constant. This increase in E_A changes the angle of the current $\mathbf{I}_A$, eventually causing it to reach a power factor of 0.8 leading.



(c) The magnitude of the line current will be

$$I_L = \frac{P}{\sqrt{3}\, V_T \text{ PF}} = \frac{15.93 \text{ kW}}{\sqrt{3}(230 \text{ V})(0.8)} = 50.0 \text{ A}$$

5-7. A 208-V Y-connected synchronous motor is drawing 50 A at unity power factor from a 208-V power system. The field current flowing under these conditions is 2.7 A. Its synchronous reactance is 1.6 Ω. Assume a linear open-circuit characteristic.

(a) Find $\mathbf{V}_\phi$ and $\mathbf{E}_A$ for these conditions.

(b) Find the torque angle δ.

(c) What is the static stability power limit under these conditions?

(d) How much field current would be required to make the motor operate at 0.80 PF leading?

(e) What is the new torque angle in part *(d)*?

SOLUTION

(a) The phase voltage of this motor is V_ϕ = 120 V, and the armature current is $\mathbf{I}_A = 50\angle 0°\ \text{A}$. Therefore, the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi - R_A\mathbf{I}_A - jX_S\mathbf{I}_A$$

$$\mathbf{E}_A = 120\angle 0°\ \text{V} - j(1.6\ \Omega)(50\angle 0°\ \text{A})$$

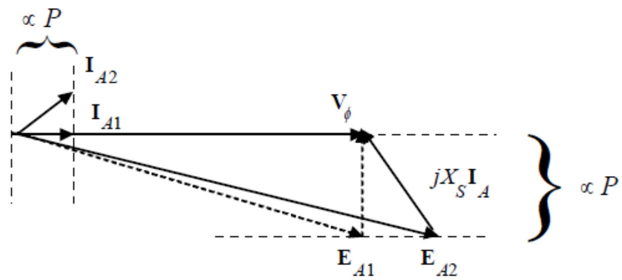
$$\mathbf{E}_A = 144\angle -33.7°\ \text{V}$$

(b) The torque angle δ of this machine is –33.7°.

(c) The static stability power limit is given by

$$P_{\max} = \frac{3V_\phi E_A}{X_S} = \frac{3(120 \text{ V})(144 \text{ V})}{(1.6\ \Omega)} = 32.4 \text{ kW}$$

(d) A phasor diagram of the motor operating at a power factor of 0.78 leading is shown below.



Since the power supplied by the motor is constant, the quantity $I_A \cos\theta$, which is directly proportional to power, must be constant. Therefore,

$$I_{A2}(0.8) = (50 \text{ A})(1.00)$$
$$\mathbf{I}_{A2} = 62.5\angle 36.87° \text{ A}$$

The internal generated voltage required to produce this current would be

$$\mathbf{E}_{A2} = \mathbf{V}_\phi - R_A\mathbf{I}_{A2} - jX_S\mathbf{I}_{A2}$$
$$\mathbf{E}_{A2} = 120\angle 0° \text{ V} - j(1.6\ \Omega)(62.50\angle 36.87° \text{ A})$$
$$\mathbf{E}_{A2} = 197\angle -23.9° \text{ V}$$

The internal generated voltage E_A is directly proportional to the field flux, and we have assumed in this problem that the flux is directly proportional to the field current. Therefore, the required field current is

$$I_{F2} = \frac{E_{A2}}{E_{A1}} I_{F1} = \frac{197 \text{ V}}{144 \text{ V}}(2.7 \text{ A}) = 3.70 \text{ A}$$

(e) The new torque angle δ of this machine is –23.9°.

5-10. A synchronous machine has a synchronous reactance of 1.0 Ω per phase and an armature resistance of 0.1 Ω per phase. If $\mathbf{E}_A$ = 460∠-10° V and $\mathbf{V}_\phi$ = 480∠0° V, is this machine a motor or a generator? How much power P is this machine consuming from or supplying to the electrical system? How much reactive power Q is this machine consuming from or supplying to the electrical system?

SOLUTION This machine is a motor, *consuming* power from the power system, because $\mathbf{E}_A$ is lagging $\mathbf{V}_\phi$. It is also *consuming* reactive power, because $E_A \cos\delta < V_\phi$. The current flowing in this machine is

$$\mathbf{I}_A = \frac{\mathbf{V}_\phi - \mathbf{E}_A}{R_A + jX_S} = \frac{480\angle 0° \text{ V} - 460\angle -10° \text{ V}}{0.1 + j1.0 \ \Omega} = 83.9\angle -13° \text{ A}$$

Therefore the real power consumed by this motor is

$$P = 3V_\phi I_A \cos\theta = 3(480 \text{ V})(89.3 \text{ A})\cos(13°) = 125.3 \text{ kW}$$

and the reactive power consumed by this motor is

$$Q = 3V_\phi I_A \sin\theta = 3(480 \text{ V})(89.3 \text{ A})\sin(13°) = 28.9 \text{ kVAR}$$