2-1. A 100-kVA 8000/277-V distribution transformer has the following resistances and reactances:

$R_P = 5\ \Omega$ $\quad$ $R_S = 0.005\ \Omega$

$X_P = 6\ \Omega$ $\quad$ $X_S = 0.006\ \Omega$

$R_C = 50\ \text{k}\Omega$ $\quad$ $X_M = 10\ \text{k}\Omega$

The excitation branch impedances are given referred to the high-voltage side of the transformer.

(a) Find the equivalent circuit of this transformer referred to the low-voltage side.

(c) Assume that this transformer is supplying rated load at 277 V and 0.85 PF lagging. What is this transformer's input voltage? What is its voltage regulation?

(d) What are the copper losses and core losses in this transformer under the conditions of part *(c)*?

(e) What is the transformer's efficiency under the conditions of part *(c)*?

SOLUTION

(a) The turns ratio of this transformer is a = 8000/277 = 28.88. Therefore, the primary impedances referred to the low voltage (secondary) side are

$$R_P' = \frac{R_P}{a^2} = \frac{5\ \Omega}{(28.88)^2} = 0.006\ \Omega$$

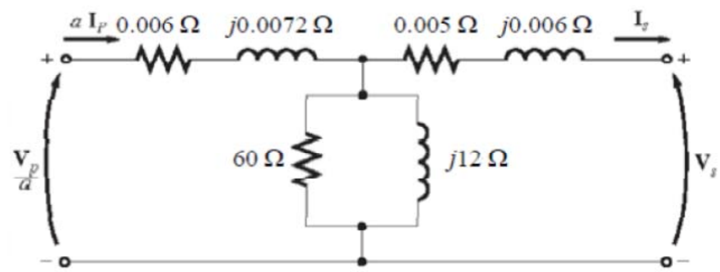
$$X_P' = \frac{R_P}{a^2} = \frac{6\ \Omega}{(28.88)^2} = 0.0072\ \Omega$$

and the excitation branch elements referred to the secondary side are

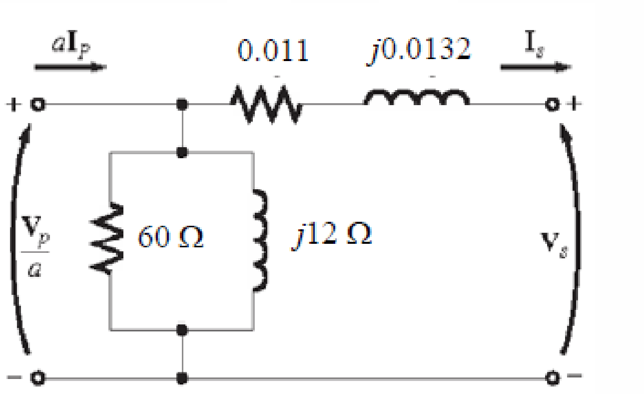
$$R_C' = \frac{R_C}{a^2} = \frac{50\ \text{k}\Omega}{(28.88)^2} = 60\ \Omega$$

$$X_M' = \frac{X_M}{a^2} = \frac{10\ \text{k}\Omega}{(28.88)^2} = 12\ \Omega$$

The resulting equivalent circuit is



(c) To simplify the calculations, use the simplified equivalent circuit referred to the secondary side of the transformer:



The secondary current in this transformer is

$$\mathbf{I}_S = \frac{100\text{ kVA}}{277\text{ V}}\angle -31.8^\circ\text{ A} = 361\angle -31.8^\circ\text{ A}$$

Therefore, the primary voltage on this transformer (referred to the secondary side) is

$$\mathbf{V}_P' = \mathbf{V}_S + \left(R_{EQ} + jX_{EQ}\right)\mathbf{I}_S$$

$$\mathbf{V}_P' = 277\angle 0^\circ\text{ V} + (0.011 + j0.0132)(361\angle -31.8^\circ\text{ A}) = 283\angle 0.4^\circ\text{ V}$$

The voltage regulation of the transformer under these conditions is

$$\text{VR} = \frac{283 - 277}{277}\times 100\% = 2.2\%$$

(d) Under the conditions of part *(c)*, the transformer's output power copper losses and core losses are:

$$P_{OUT} = S\cos\theta = (100\text{ kVA})(0.85) = 85\text{ kW}$$

$$P_{CU} = (I_S)^2 R_{EQ} = (361)^2(0.11) = 1430\text{ W}$$

$$P_{core} = \frac{V_P'^2}{R_C} = \frac{283^2}{50} = 1602\text{ W}$$

(e) The efficiency of this transformer is

$$\eta = \frac{P_{OUT}}{P_{OUT} + P_{CU} + P_{core}}\times 100\% = \frac{85{,}000}{85{,}000 + 1430 + 1602}\times 100\% = 96.6\%$$

2-2. A single-phase power system is shown in Figure P2-1. The power source feeds a 100-kVA 14/2.4-kV transformer through a feeder impedance of 38.2 + *j*140 Ω. The transformer's equivalent series impedance referred to its low-voltage side is 0.10 + *j*0.4 Ω. The load on the transformer is 90 kW at 0.8 PF lagging and 2300 V.

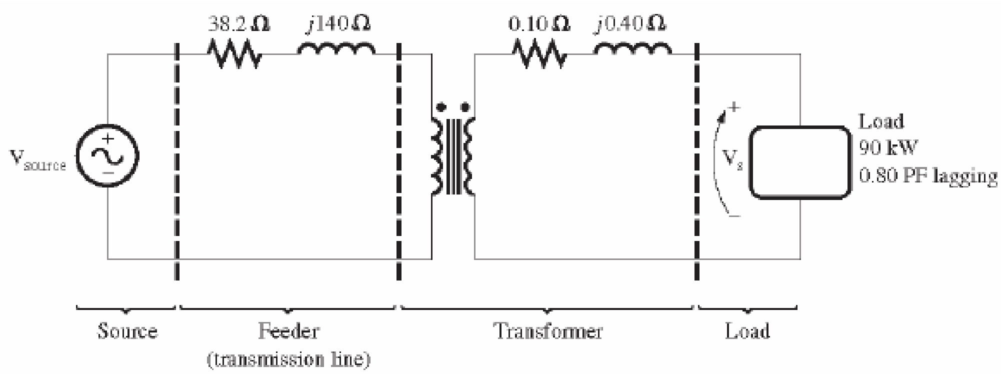


FIGURE P2–1
The circuit of Problem 2–2.

(a) What is the voltage at the power source of the system?

(b) What is the voltage regulation of the transformer?

(c) How efficient is the overall power system?

SOLUTION

To solve this problem, we will refer the circuit to the secondary (low-voltage) side. The feeder's impedance referred to the secondary side is

$$Z_{\text{line}}' = \left(\frac{2.4\text{ kV}}{14\text{ kV}}\right)^2 (38.2\ \Omega + j140\ \Omega) = 1.12 + j4.11\ \Omega$$

The secondary current $\mathbf{I}_S$ is given by

$$I_S = \frac{90\text{ kW}}{(2400\text{ V})(0.8)} = 46.88\text{ A}$$

The power factor is 0.80 lagging, so the impedance angle $\theta = \cos^{-1}(0.8) = 36.87°$, and the phasor current is

$$\mathbf{I}_S = 46.88\angle -36.87°\text{ A}$$

(a) The voltage at the power source of this system (referred to the secondary side) is

$$\mathbf{V}_{\text{source}}' = \mathbf{V}_S + \mathbf{I}_S Z_{\text{line}}' + \mathbf{I}_S Z_{\text{EQ}}$$

$$\mathbf{V}_{\text{source}}' = 2400\angle 0°\text{ V} + (46.88\angle -36.87°\text{ A})(1.12 + j4.11\ \Omega) + (46.88\angle -36.87°\text{ A})(0.10 + j0.40\ \Omega)$$

$$\mathbf{V}_{\text{source}}' = 2576\angle 3.0°\text{ V}$$

Therefore, the voltage at the power source is

$$\mathbf{V}_{\text{source}} = (2576\angle 3.0°\text{ V})\frac{14\text{ kV}}{2.4\text{ kV}} = 15.5\angle 3.0°\text{ kV}$$

(b) To find the voltage regulation of the transformer, we must find the voltage at the primary side of the transformer (referred to the secondary side) under these conditions:

$$\mathbf{V}_P' = \mathbf{V}_S + \mathbf{I}_S Z_{EQ}$$

$$\mathbf{V}_P' = 2400\angle 0° \text{ V} + (46.88\angle -36.87° \text{ A})(0.10 + j0.40\ \Omega) = 2415\angle 0.3° \text{ V}$$

There is a voltage drop of 15 V under these load conditions. Therefore the voltage regulation of the transformer is

$$\text{VR} = \frac{2415 - 2400}{2400} \times 100\% = 0.63\%$$

(c) The overall efficiency of the power system will be the ratio of the output power to the input power. The output power supplied to the load is $P_{\text{OUT}} = 90$ kW. The input power supplied by the source is

$$P_{\text{IN}} = P_{\text{OUT}} + P_{\text{LOSS}} = P_{\text{OUT}} + I^2R = (90 \text{ kW}) + (46.88 \text{ A})^2 (1.22\ \Omega) = 92.68 \text{ kW}$$

$$P_{\text{IN}} = V_{\text{source}}' I_S \cos\theta = (2415 \text{ V})(46.88 \text{ A}) \cos 36.57° = 90.93 \text{ kW}$$

Therefore, the efficiency of the power system is

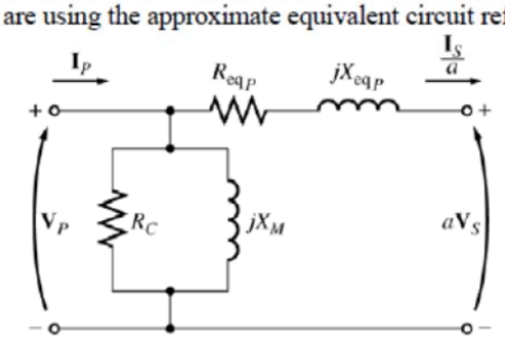
$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{90 \text{ kW}}{92.68 \text{ kW}} \times 100\% = 97.1\%$$

2-4. The secondary winding of a real transformer has a terminal voltage of $v_s(t) = 282.8 \sin 377t$ V. The turns ratio of the transformer is 100:200 (a = 0.50). If the secondary current of the transformer is $i_s(t) = 7.07 \sin(377t - 36.87°)$ A, what is the primary current of this transformer? What are its voltage regulation and efficiency? The impedances of this transformer referred to the primary side are

$$R_{\text{eq}} = 0.20\ \Omega \qquad R_C = 300\ \Omega$$

$$X_{\text{eq}} = 0.80\ \Omega \qquad X_M = 100\ \Omega$$

SOLUTION The equivalent circuit of this transformer is shown below. (Since no particular equivalent circuit was specified, we are using the approximate equivalent circuit referred to the primary side.)



(a) $R_{\text{eq}P} = R_P + a^2R_S$

$X_{\text{eq}P} = X_P + a^2X_S$

The secondary voltage and current are

$$\mathbf{V}_S = \frac{282.8}{\sqrt{2}}\angle 0° \text{ V} = 200\angle 0° \text{ V}$$

$$\mathbf{I}_S = \frac{7.07}{\sqrt{2}}\angle -36.87° \text{ A} = 5\angle -36.87° \text{ A}$$

The secondary voltage referred to the primary side is

$$\mathbf{V}_S' = a\mathbf{V}_S = 100\angle 0° \text{ V}$$

The secondary current referred to the primary side is

$$\mathbf{I}_S' = \frac{\mathbf{I}_S}{a} = 10\angle -36.87° \text{ A}$$

The primary circuit voltage is given by

$$\mathbf{V}_P = \mathbf{V}_S' + \mathbf{I}_S'\left(R_{\text{eq}} + jX_{\text{eq}}\right)$$

$$\mathbf{V}_P = 100\angle 0° \text{ V} + (10\angle -36.87° \text{ A})(0.20\ \Omega + j0.80\ \Omega) = 106.5\angle 2.8° \text{ V}$$

The excitation current of this transformer is

$$\mathbf{I}_{\text{EX}} = \mathbf{I}_C + \mathbf{I}_M = \frac{106.5\angle 2.8° \text{ V}}{300\ \Omega} + \frac{106.5\angle 2.8° \text{ V}}{j100\ \Omega} = 0.355\angle 2.8° + 1.065\angle -87.2°$$

$$\mathbf{I}_{\text{EX}} = 1.12\angle -68.8° \text{ A}$$

Therefore, the total primary current of this transformer is

$$\mathbf{I}_P = \mathbf{I}_S' + \mathbf{I}_{\text{EX}} = 10\angle -36.87° + 1.12\angle -68.8° = 11.0\angle -40.0° \text{ A}$$

The voltage regulation of the transformer at this load is

$$\text{VR} = \frac{V_P - aV_S}{aV_S} \times 100\% = \frac{106.5 - 100}{100} \times 100\% = 6.5\%$$

The input power to this transformer is

$$P_{\text{IN}} = V_P I_P \cos\theta = (106.5 \text{ V})(11.0 \text{ A})\cos\left[2.8° - (-40.0°)\right]$$

$$P_{\text{IN}} = (106.5 \text{ V})(11.0 \text{ A})\cos 42.8° = 860 \text{ W}$$

The output power from this transformer is

$$P_{\text{OUT}} = V_S I_S \cos\theta = (200 \text{ V})(5 \text{ A})\cos(36.87°) = 800 \text{ W}$$

Therefore, the transformer's efficiency is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{800 \text{ W}}{860 \text{ W}} \times 100\% = 93.0\%$$

2-6. A 1000-VA 230/115-V transformer has been tested to determine its equivalent circuit. The results of the tests are shown below.

Open-circuit test (on secondary side)	Short-circuit test (on primary side)
$V_{OC} = 115$ V	$V_{SC} = 17.1$ V
$I_{OC} = 0.11$ A	$I_{SC} = 8.7$ A
$P_{OC} = 3.9$ W	$P_{SC} = 38.1$ W

(a) Find the equivalent circuit of this transformer referred to the low-voltage side of the transformer.

(b) Find the transformer's voltage regulation at rated conditions and (1) 0.8 PF lagging, (2) 1.0 PF, (3) 0.8 PF leading.

(c) Determine the transformer's efficiency at rated conditions and 0.8 PF lagging.

SOLUTION

(a) OPEN CIRCUIT TEST (referred to the low-voltage or secondary side):

$$|Y_{EX}| = |G_C - jB_M| = \frac{0.11\text{ A}}{115\text{ V}} = 0.0009565\text{ S}$$

$$\theta = \cos^{-1}\frac{P_{OC}}{V_{OC}I_{OC}} = \cos^{-1}\frac{3.9\text{ W}}{(115\text{ V})(0.11\text{ A})} = 72.0°$$

$$Y_{EX} = G_C - jB_M = 0.0009565\angle -72°\text{ S} = 0.0002956 - j0.0009096\text{ S}$$

$$R_C = \frac{1}{G_C} = 3383\ \Omega$$

$$X_M = \frac{1}{B_M} = 1099\ \Omega$$

SHORT CIRCUIT TEST (referred to the high-voltage or primary side):

$$|Z_{EQ}| = |R_{EQ} + jX_{EQ}| = \frac{17.1\text{ V}}{8.7\text{ A}} = 1.97\ \Omega$$

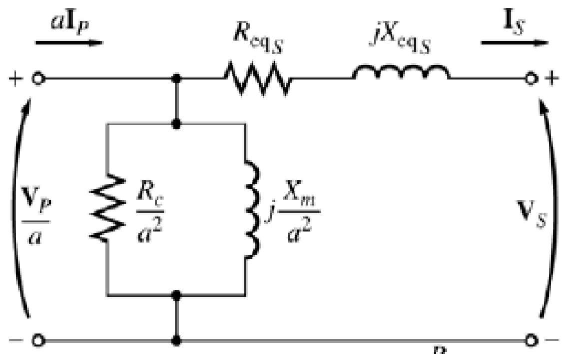
$$\theta = \cos^{-1}\frac{P_{SC}}{V_{SC}I_{SC}} = \cos^{-1}\frac{38.1\text{ W}}{(17.1\text{ V})(8.7\text{ A})} = 75.2°$$

$$Z_{EQ} = R_{EQ} + jX_{EQ} = 1.97\angle 75.2°\ \Omega = 0.503 + j1.905\ \Omega$$

$$R_{EQ} = 0.503\ \Omega$$

$$X_{EQ} = j1.905\ \Omega$$

To convert the equivalent circuit to the secondary side, divide each series impedance by the square of the turns ratio ($a = 230/115 = 2$). Note that the excitation branch elements are already on the secondary side. The resulting equivalent circuit is shown below:



(b)

$$R_{\text{eq}_S} = \frac{R_p}{a^2} + R_S$$

$$X_{\text{eq}_S} = \frac{X_p}{a^2} + X_S$$

$R_{\text{EQ,S}} = 0.126\ \Omega$ $\qquad X_{\text{EQ,S}} = j0.476\ \Omega$

$R_{C,S} = 3383\ \Omega$ $\qquad X_{M,S} = 1099\ \Omega$

(b) To find the required voltage regulation, we will use the equivalent circuit of the transformer referred to the secondary side. The rated secondary current is

$$I_S = \frac{1000\text{ VA}}{115\text{ V}} = 8.70\text{ A}$$

We will now calculate the primary voltage referred to the secondary side and use the voltage regulation equation for each power factor.

(1) <u>0.8 PF Lagging:</u>

$$\mathbf{V}_P' = \mathbf{V}_S + Z_{\text{EQ}}\mathbf{I}_S = 115\angle 0°\text{ V} + (0.126 + j0.476\ \Omega)(8.7\angle -36.87°\text{ A})$$

$$\mathbf{V}_P' = 118.4\angle 1.3°\text{ V}$$

$$\text{VR} = \frac{118.4\text{-}115}{115} \times 100\% = 2.96\%$$

(2) <u>1.0 PF:</u>

$$\mathbf{V}_P' = \mathbf{V}_S + Z_{\text{EQ}}\mathbf{I}_S = 115\angle 0°\text{ V} + (0.126 + j0.476\ \Omega)(8.7\angle 0.0°\text{ A})$$

$$\mathbf{V}_P' = 116.2\angle 2.04°\text{ V}$$

$$\text{VR} = \frac{116.2\text{-}115}{115} \times 100\% = 1.04\%$$

(3) <u>0.8 PF Leading:</u>

$$\mathbf{V}_P' = \mathbf{V}_S + Z_{\text{EQ}}\mathbf{I}_S = 115\angle 0°\text{ V} + (0.126 + j0.476\ \Omega)(8.7\angle 36.87°\text{ A})$$

$$\mathbf{V}_P' = 113.5\angle 2.0°\text{ V}$$

$$\text{VR} = \frac{113.5\text{-}115}{115} \times 100\% = -1.3\%$$

(c) At rated conditions and 0.8 PF lagging, the output power of this transformer is

$$P_{\text{OUT}} = V_S I_S \cos\theta = (115\text{ V})(8.7\text{ A})(0.8) = 800\text{ W}$$

The copper and core losses of this transformer are

$$P_{\text{CU}} = I_S^{\,2} R_{\text{EQ},S} = (8.7\text{ A})^2 (0.126\ \Omega) = 9.5\text{ W}$$

$$P_{\text{core}} = \frac{\left(V_P'\right)^2}{R_C} = \frac{(118.4\text{ V})^2}{3383\ \Omega} = 4.1\text{ W}$$

Therefore the efficiency of this transformer at these conditions is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{OUT}} + P_{\text{CU}} + P_{\text{core}}} \times 100\% = \frac{800\text{ W}}{800\text{ W} + 9.5\text{ W} + 4.1\text{ W}} = 98.3\%$$

2-7. A 30-kVA 8000/230-V distribution transformer has an impedance referred to the primary of $20 + j100\ \Omega$. The components of the excitation branch referred to the primary side are $R_C = 100\text{ k}\Omega$ and $X_M = 20\text{ k}\Omega$.

(a) If the primary voltage is 7967 V and the load impedance is $Z_L = 2.0 + j0.7\ \Omega$, what is the secondary voltage of the transformer? What is the voltage regulation of the transformer?

(b) If the load is disconnected and a capacitor of $-j3.0\ \Omega$ is connected in its place, what is the secondary voltage of the transformer? What is its voltage regulation under these conditions?

SOLUTION

(a) The easiest way to solve this problem is to refer all components to the *primary* side of the transformer. The turns ratio is a = 8000/230 = 34.78. Thus the load impedance referred to the primary side is

$$Z_L' = (34.78)^2 (2.0 + j0.7\ \Omega) = 2419 + j847\ \Omega$$

The referred secondary current is

$$\mathbf{I}_S' = \frac{7967\angle 0°\text{ V}}{(20 + j100\ \Omega) + (2419 + j847\ \Omega)} = \frac{7967\angle 0°\text{ V}}{2616\angle 21.2°\ \Omega} = 3.045\angle -21.2°\text{ A}$$

and the referred secondary voltage is

$$\mathbf{V}_S' = \mathbf{I}_S' Z_L' = (3.045\angle -21.2°\text{ A})(2419 + j847\ \Omega) = 7804\angle -1.9°\text{ V}$$

The actual secondary voltage is thus

$$\mathbf{V}_S = \frac{\mathbf{V}_S'}{a} = \frac{7804\angle -1.9°\text{ V}}{34.78} = 224.4\angle -1.9°\text{ V}$$

The voltage regulation is

$$\text{VR} = \frac{7967\text{-}7804}{7804} \times 100\% = 2.09\%$$

(b) The easiest way to solve this problem is to refer all components to the *primary* side of the transformer. The turns ratio is again $a = 34.78$. Thus the load impedance referred to the primary side is

$$Z_L' = (34.78)^2 (-j3.0\ \Omega) = -j3629\ \Omega$$

The referred secondary current is

$$\mathbf{I}_S' = \frac{7967\angle 0^\circ \text{ V}}{(20 + j100\ \Omega) + (-j3629\ \Omega)} = \frac{7967\angle 0^\circ \text{ V}}{3529\angle -89.7^\circ\ \Omega} = 2.258\angle 89.7^\circ \text{ A}$$

and the referred secondary voltage is

$$\mathbf{V}_S' = \mathbf{I}_S' Z_L' = (2.258\angle 89.7^\circ \text{ A})(-j3629\ \Omega) = 8194\angle -0.3^\circ \text{ V}$$

The actual secondary voltage is thus

$$\mathbf{V}_S = \frac{\mathbf{V}_S'}{a} = \frac{8194\angle -0.3^\circ \text{ V}}{34.78} = 235.6\angle -0.3^\circ \text{ V}$$

The voltage regulation is

$$\text{VR} = \frac{7967 - 8914}{8194} \times 100\% = -10.6\%$$