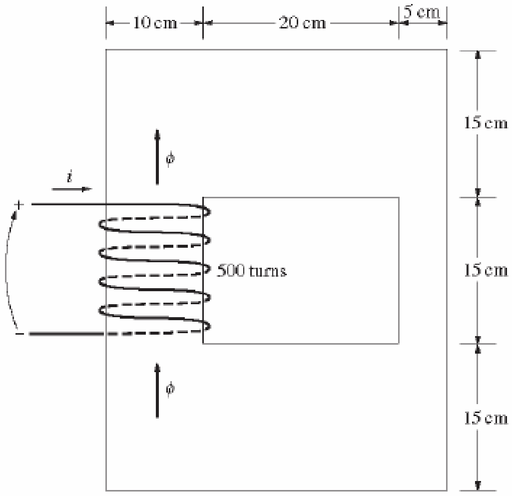
1-5. A ferromagnetic core is shown in Figure P1-2. The depth of the core is 5 cm. The other dimensions of the core are as shown in the figure. Find the value of the current that will produce a flux of 0.005 Wb. With this current, what is the flux density at the top of the core? What is the flux density at the right side of the core? Assume that the relative permeability of the core is 800.



SOLUTION There are three regions in this core. The top and bottom form one region, the left side forms a second region, and the right side forms a third region. If we assume that the mean path length of the flux is in the center of each leg of the core, and if we ignore spreading at the corners of the core, then the path lengths are $l_1 = 2(27.5 \text{ cm}) = 55$ cm, $l_2 = 30$ cm, and $l_3 = 30$ cm. The reluctances of these regions are:

$$\mathcal{R}_1 = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_o A} = \frac{0.55 \text{ m}}{(800)(4\pi \times 10^{-7} \text{ H/m})(0.05 \text{ m})(0.15 \text{ m})} = 72.9 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_2 = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_o A} = \frac{0.30 \text{ m}}{(800)(4\pi \times 10^{-7} \text{ H/m})(0.05 \text{ m})(0.10 \text{ m})} = 59.7 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_3 = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_o A} = \frac{0.30 \text{ m}}{(800)(4\pi \times 10^{-7} \text{ H/m})(0.05 \text{ m})(0.05 \text{ m})} = 119.4 \text{ kA} \cdot \text{t/Wb}$$

The total reluctance is thus

$$\mathcal{R}_{TOT} = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 = 72.9 + 59.7 + 119.4 = 252 \text{ kA} \cdot \text{t/Wb}$$

and the magnetomotive force required to produce a flux of 0.005 Wb is

$$\mathcal{F} = \phi \mathcal{R} = (0.005 \text{ Wb})(252 \text{ kA} \cdot \text{t/Wb}) = 1260 \text{ A} \cdot \text{t}$$

and the required current is

$$i = \frac{\mathcal{F}}{N} = \frac{1260 \text{ A} \cdot \text{t}}{500 \text{ t}} = 2.5 \text{ A}$$

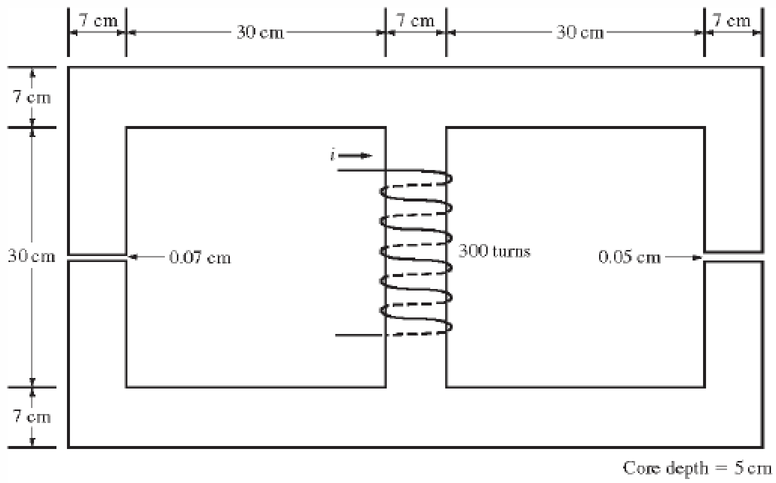
The flux density on the top of the core is

$$B = \frac{\phi}{A} = \frac{0.005 \text{ Wb}}{(0.15 \text{ m})(0.05 \text{ m})} = 0.67 \text{ T}$$

The flux density on the right side of the core is

$$B = \frac{\phi}{A} = \frac{0.005 \text{ Wb}}{(0.05 \text{ m})(0.05 \text{ m})} = 2.0 \text{ T}$$

1-6. A ferromagnetic core with a relative permeability of 1500 is shown in Figure P1-3. The dimensions are as shown in the diagram, and the depth of the core is 5 cm. The air gaps on the left and right sides of the core are 0.050 and 0.070 cm, respectively. Because of fringing effects, the effective area of the air gaps is 5 percent larger than their physical size. If there are 300 turns in the coil wrapped around the center leg of the core and if the current in the coil is 1.0 A, what is the flux in each of the left, center, and right legs of the core? What is the flux density in each air gap?



SOLUTION This core can be divided up into five regions. Let $\mathcal{R}_1$ be the reluctance of the left-hand portion of the core, $\mathcal{R}_2$ be the reluctance of the left-hand air gap, $\mathcal{R}_3$ be the reluctance of the right-hand portion of the core, $\mathcal{R}_4$ be the reluctance of the right-hand air gap, and $\mathcal{R}_5$ be the reluctance of the center leg of the core. Then the total reluctance of the core is

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_5 + \frac{(\mathcal{R}_1 + \mathcal{R}_2)(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}$$

$$\mathcal{R}_1 = \frac{l_1}{\mu_r \mu_0 A_1} = \frac{1.11\text{ m}}{(1500)(4\pi \times 10^{-7}\text{ H/m})(0.07\text{ m})(0.05\text{ m})} = 168\text{ kA}\cdot\text{t/Wb}$$

$$\mathcal{R}_2 = \frac{l_2}{\mu_0 A_2} = \frac{0.0007\text{ m}}{(4\pi \times 10^{-7}\text{ H/m})(0.07\text{ m})(0.05\text{ m})(1.05)} = 152\text{ kA}\cdot\text{t/Wb}$$

$$\mathcal{R}_3 = \frac{l_3}{\mu_r \mu_0 A_3} = \frac{1.11\text{ m}}{(1500)(4\pi \times 10^{-7}\text{ H/m})(0.07\text{ m})(0.05\text{ m})} = 168\text{ kA}\cdot\text{t/Wb}$$

$$\mathcal{R}_4 = \frac{l_4}{\mu_0 A_4} = \frac{0.0005\text{ m}}{(4\pi \times 10^{-7}\text{ H/m})(0.07\text{ m})(0.05\text{ m})(1.05)} = 108\text{ kA}\cdot\text{t/Wb}$$

$$\mathcal{R}_5 = \frac{l_5}{\mu_r \mu_0 A_5} = \frac{0.37\text{ m}}{(1500)(4\pi \times 10^{-7}\text{ H/m})(0.07\text{ m})(0.05\text{ m})} = 56.1\text{ kA}\cdot\text{t/Wb}$$

The total reluctance is

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_5 + \frac{(\mathcal{R}_1 + \mathcal{R}_2)(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} = 56.1 + \frac{(168+152)(168+108)}{168+152+168+108} = 204\text{ kA}\cdot\text{t/Wb}$$

The total flux in the core is equal to the flux in the center leg:

$$\phi_{\text{center}} = \phi_{\text{TOT}} = \frac{\mathcal{F}}{\mathcal{R}_{\text{TOT}}} = \frac{(300\text{ t})(1.0\text{ A})}{204\text{ kA}\cdot\text{t/Wb}} = 0.00147\text{ Wb}$$

The fluxes in the left and right legs can be found by the "flux divider rule", which is analogous to the current divider rule.

$$\phi_{\text{left}} = \frac{(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}\phi_{\text{TOT}} = \frac{(168+108)}{168+152+168+108}(0.00147 \text{ Wb}) = 0.00068 \text{ Wb}$$

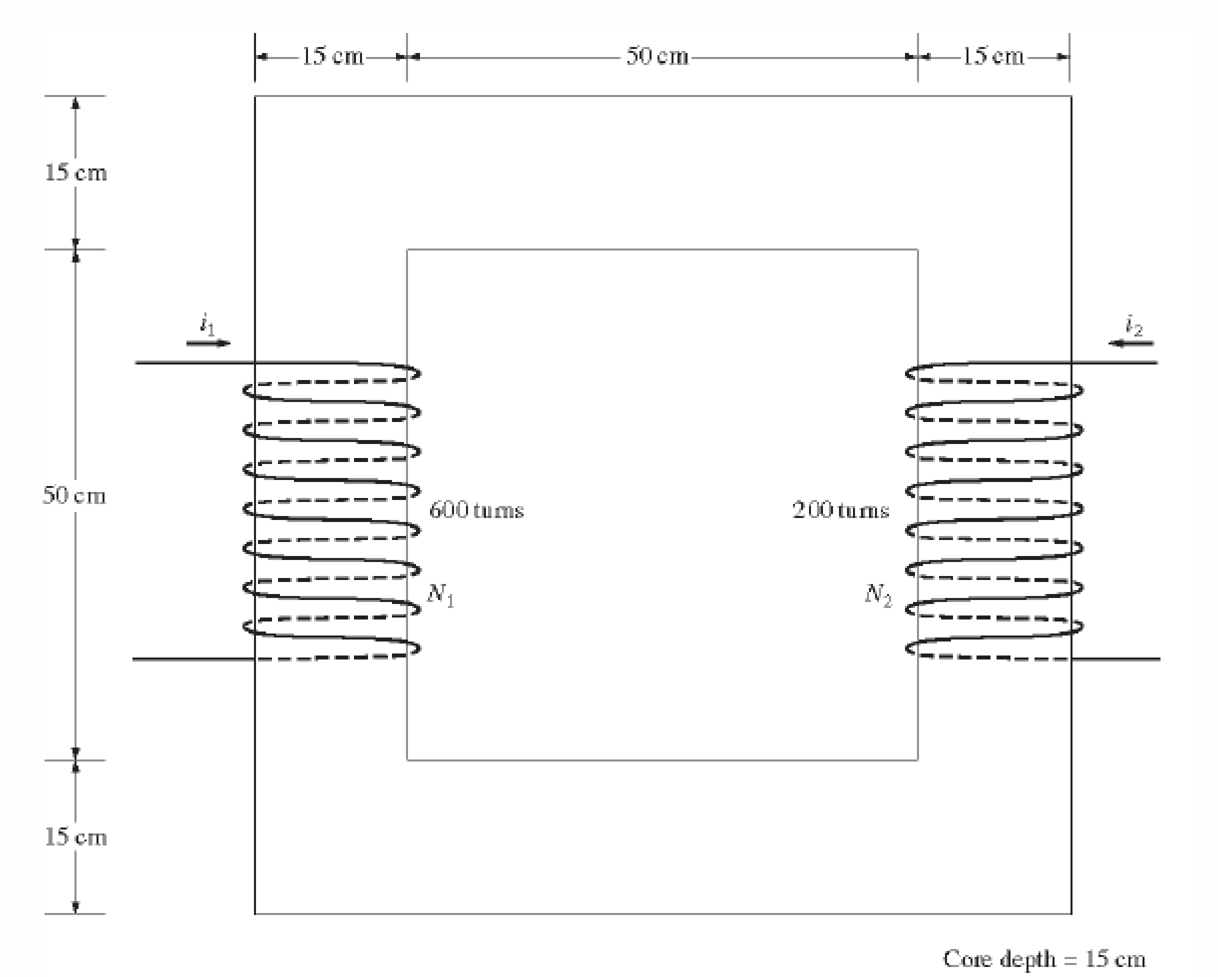
$$\phi_{\text{right}} = \frac{(\mathcal{R}_1 + \mathcal{R}_2)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}\phi_{\text{TOT}} = \frac{(168+152)}{168+152+168+108}(0.00147 \text{ Wb}) = 0.00079 \text{ Wb}$$

The flux density in the air gaps can be determined from the equation $\phi = BA$:

$$B_{\text{left}} = \frac{\phi_{\text{left}}}{A_{\text{eff}}} = \frac{0.00068 \text{ Wb}}{(0.07 \text{ cm})(0.05 \text{ cm})(1.05)} = 0.185 \text{ T}$$

$$B_{\text{right}} = \frac{\phi_{\text{right}}}{A_{\text{eff}}} = \frac{0.00079 \text{ Wb}}{(0.07 \text{ cm})(0.05 \text{ cm})(1.05)} = 0.215 \text{ T}$$

1-7. A two-legged core is shown in Figure P1-4. The winding on the left leg of the core (N_1) has 600 turns, and the winding on the right (N_2) has 200 turns. The coils are wound in the directions shown in the figure. If the dimensions are as shown, then what flux would be produced by currents $i_1 = 0.5$ A and $i_2 = 1.0$ A? Assume $\mu_r = 1200$ and constant.



SOLUTION The two coils on this core are wound so that their magnetomotive forces are additive, so the total magnetomotive force on this core is

$$\mathcal{F}_{\text{TOT}} = N_1 i_1 + N_2 i_2 = (600\text{ t})(0.5\text{ A}) + (200\text{ t})(1.00\text{ A}) = 500\text{ A}\cdot\text{t}$$

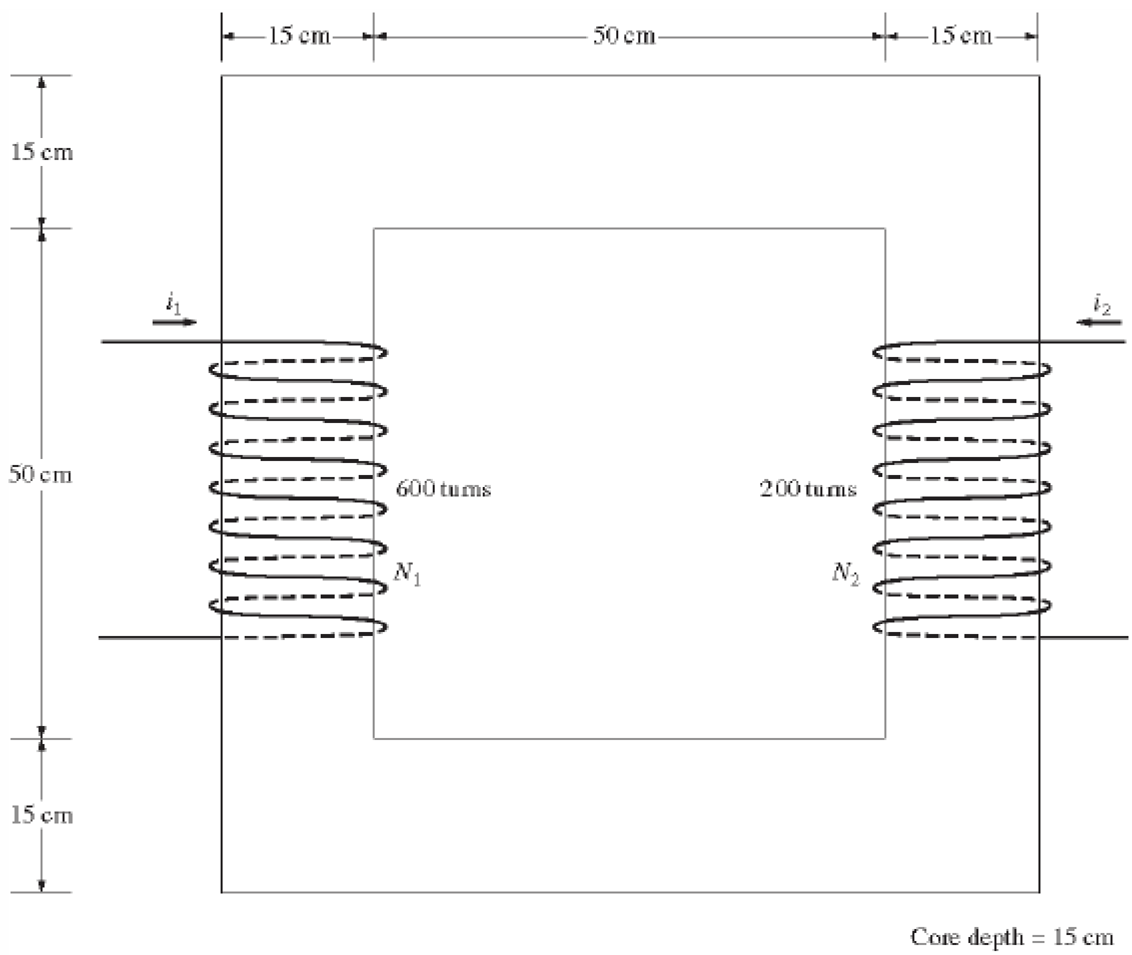
The total reluctance in the core is

$$\mathcal{R}_{\text{TOT}} = \frac{l}{\mu_r \mu_0 A} = \frac{2.60\text{ m}}{(1200)(4\pi\times10^{-7}\text{ H/m})(0.15\text{ m})(0.15\text{ m})} = 76.6\text{ kA}\cdot\text{t/Wb}$$

and the flux in the core is:

$$\phi = \frac{\mathcal{F}_{\text{TOT}}}{\mathcal{R}_{\text{TOT}}} = \frac{500\text{ A}\cdot\text{t}}{76.6\text{ kA}\cdot\text{t/Wb}} = 0.00653\text{ Wb}$$

1-12. The core shown in Figure P1-4 is made of a steel whose magnetization curve is shown in Figure P1-9. Repeat Problem 1-7, but this time do *not* assume a constant value of μ_r. How much flux is produced in the core by the currents specified? What is the relative permeability of this core under these conditions? Was the assumption in Problem 1-7 that the relative permeability was equal to 1200 a good assumption for these conditions? Is it a good assumption in general?



SOLUTION The magnetization curve for this core is shown below:

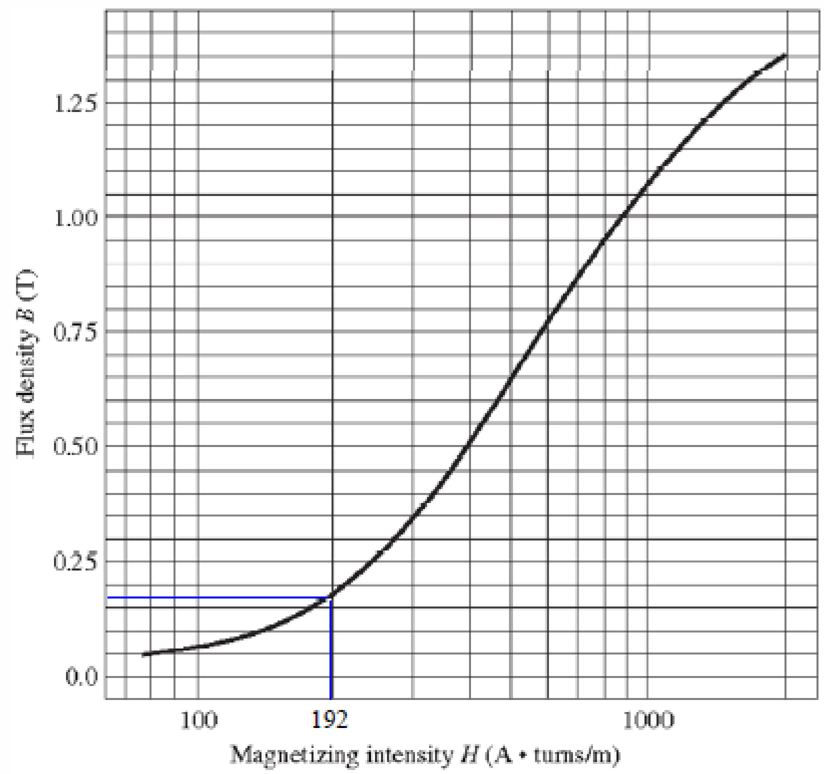


FIGURE P1–9
The magnetization curve for the core material of Problems 1–12 and 1–14.

The two coils on this core are wound so that their magnetomotive forces are additive, so the total magnetomotive force on this core is

$$\mathcal{F}_{\text{TOT}} = N_1 i_1 + N_2 i_2 = (600\text{ t})(0.5\text{ A}) + (200\text{ t})(1.00\text{ A}) = 500\text{ A}\cdot\text{t}$$

Therefore, the magnetizing intensity H is

$$H = \frac{\mathcal{F}}{l_c} = \frac{500\text{ A}\cdot\text{t}}{2.60\text{ m}} = 192\text{ A}\cdot\text{t/m}$$

From the magnetization curve,

$$B = 0.17\text{ T}$$

and the total flux in the core is

$$\phi_{\text{TOT}} = BA = (0.17\text{ T})(0.15\text{ m})(0.15\text{ m}) = 0.00383\text{ Wb}$$

The relative permeability of the core can be found from the reluctance as follows:

$$\mathcal{R} = \frac{\mathcal{F}_{\text{TOT}}}{\phi_{\text{TOT}}} = \frac{l}{\mu_r \mu_0 A}$$

Solving for μ_r yields

$$\mu_r = \frac{\phi_{\text{TOT}}\ l}{\mathcal{F}_{\text{TOT}}\ \mu_0\ A} = \frac{(0.00383\text{ Wb})(2.6\text{ m})}{(500\text{ A}\cdot\text{t})(4\pi\times 10^{-7}\text{ H/m})(0.15\text{ m})(0.15\text{ m})} = 704$$