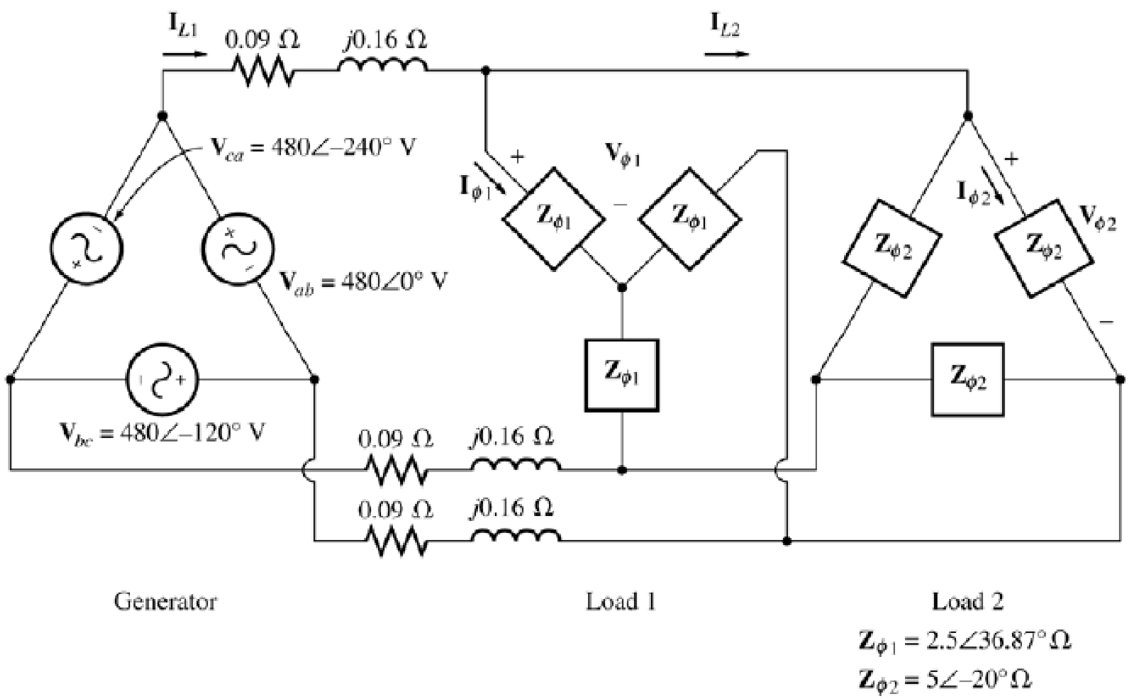
A-2. Figure PA-1 shows a three-phase power system with two loads. The Δ-connected generator is producing a line voltage of 480 V, and the line impedance is $0.09 + j0.16\ \Omega$. Load 1 is Y-connected, with a phase impedance of $2.5\angle 36.87°\ \Omega$ and load 2 is Δ-connected, with a phase impedance of $5\angle -20°\ \Omega$.



(a) What is the line voltage of the two loads?

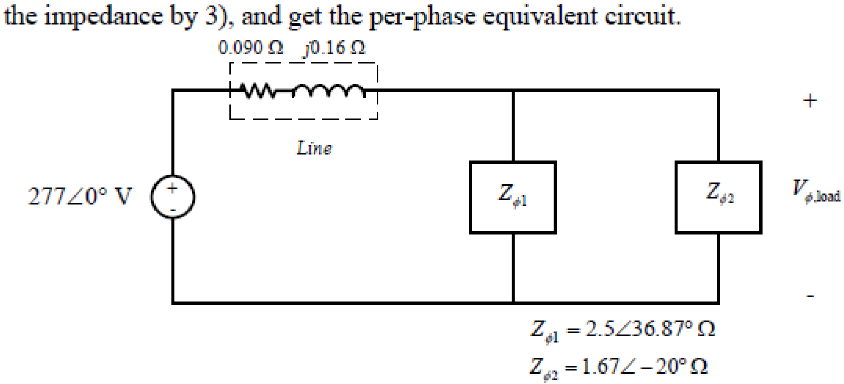
(b) What is the voltage drop on the transmission lines?

(c) Find the real and reactive powers supplied to each load.

(d) Find the real and reactive power losses in the transmission line.

(e) Find the real power, reactive power, and power factor supplied by the generator.

SOLUTION To solve this problem, first convert the delta-connected load 2 to an equivalent wye (by dividing the impedance by 3), and get the per-phase equivalent circuit.



(a) The phase voltage of the equivalent Y-loads can be found by nodal analysis.

$$\frac{\mathbf{V}_{\phi,\text{load}} - 277\angle 0° \text{ V}}{0.09 + j0.16\ \Omega} + \frac{\mathbf{V}_{\phi,\text{load}}}{2.5\angle 36.87°\ \Omega} + \frac{\mathbf{V}_{\phi,\text{load}}}{1.67\angle -20°\ \Omega} = 0$$

$$(5.443\angle -60.6°)\ \left(\mathbf{V}_{\phi,\text{load}} - 277\angle 0° \text{ V}\right) + (0.4\angle -36.87°)\mathbf{V}_{\phi,\text{load}} + (0.6\angle 20°)\mathbf{V}_{\phi,\text{load}} = 0$$

$$(5.955\angle -53.34°)\ \mathbf{V}_{\phi,\text{load}} = 1508\angle -60.6°$$

$$\mathbf{V}_{\phi,\text{load}} = 253.2\angle -7.3° \text{ V}$$

Therefore, the line voltage at the loads is $V_L \sqrt{3}\ V_\phi = 439 \text{ V}$.

(b) The voltage drop in the transmission lines is

$$\Delta\mathbf{V}_{\text{line}} = \mathbf{V}_{\phi,\text{gen}} - \mathbf{V}_{\phi,\text{load}} = 277\angle 0° \text{ V} - 253.2\angle -7.3° = 41.3\angle 52° \text{ V}$$

(c) The real and reactive power of each load is

$$P_1 = 3\frac{V_\phi^2}{Z}\cos\theta = 3\frac{(253.2 \text{ V})^2}{2.5\ \Omega}\cos\ 36.87° = 61.6 \text{ kW}$$

$$Q_1 = 3\frac{V_\phi^2}{Z}\sin\theta = 3\frac{(253.2 \text{ V})^2}{2.5\ \Omega}\sin 36.87° = 46.2 \text{ kvar}$$

$$P_2 = 3\frac{V_\phi^2}{Z}\cos\theta = 3\frac{(253.2 \text{ V})^2}{1.67\ \Omega}\cos\ (-20°) = 108.4 \text{ kW}$$

$$Q_2 = 3\frac{V_\phi^2}{Z}\sin\theta = 3\frac{(253.2 \text{ V})^2}{1.67\ \Omega}\sin\ (-20°) = -39.5 \text{ kvar}$$

(d) The line current is

$$\mathbf{I}_{\text{line}} = \frac{\Delta\mathbf{V}_{\text{line}}}{Z_{\text{line}}} = \frac{41.3\angle 52° \text{ V}}{0.09 + j0.16\ \Omega} = 225\angle -8.6° \text{A}$$

Therefore, the loses in the transmission line are

$$P_{\text{line}} = 3I_{\text{line}}^{\ 2}R_{\text{line}} = 3\ (225 \text{ A})^2(0.09\ \Omega) = 13.7 \text{ kW}$$

$$Q_{\text{line}} = 3I_{\text{line}}^{\ 2}X_{\text{line}} = 3\ (225 \text{ A})^2(0.16\ \Omega) = 24.3 \text{ kvar}$$

(e) The real and reactive power supplied by the generator is

$$P_{\text{gen}} = P_{\text{line}} + P_1 + P_2 = 13.7 \text{ kW} + 61.6 \text{ kW} + 108.4 \text{ kW} = 183.7 \text{ kW}$$

$$Q_{\text{gen}} = Q_{\text{line}} + Q_1 + Q_2 = 24.3 \text{ kvar} + 46.2 \text{ kvar} - 39.5 \text{ kvar} = 31 \text{ kvar}$$

The power factor of the generator is

$$\text{PF} = \cos\left[\tan^{-1}\frac{Q_{\text{gen}}}{P_{\text{gen}}}\right] = \cos\left[\tan^{-1}\frac{31 \text{ kvar}}{183.7 \text{ kW}}\right] = 0.986 \text{ lagging}$$

A-3. Figure PA-2 shows a one-line diagram of a simple power system containing a single 480 V generator and three loads. Assume that the transmission lines in this power system are lossless, and answer the following questions.

(a) Assume that Load 1 is Y-connected. What are the phase voltage and currents in that load?

(b) Assume that Load 2 is Δ-connected. What are the phase voltage and currents in that load?

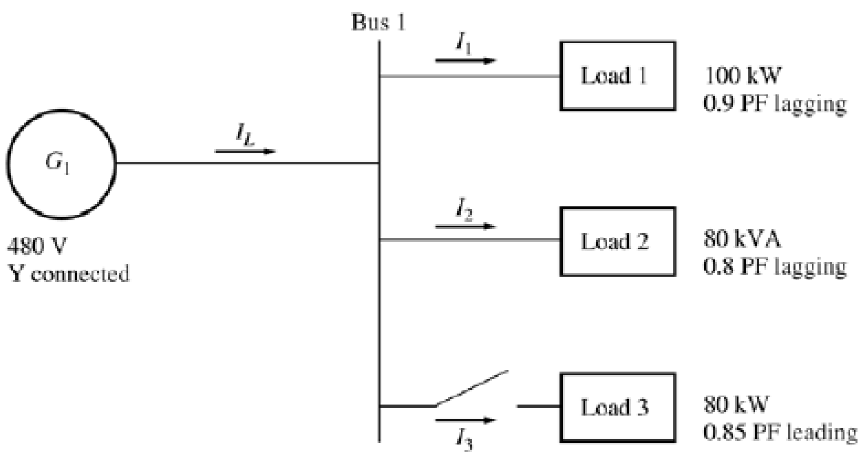
(c) What real, reactive, and apparent power does the generator supply when the switch is open?

(d) What is the total line current I_L when the switch is open?

(e) What real, reactive, and apparent power does the generator supply when the switch is closed?

(f) What is the total line current I_L when the switch is closed?

(g) How does the total line current I_L compare to the sum of the three individual currents $I_1 + I_2 + I_3$? If they are not equal, why not?



SOLUTION Since the transmission lines are lossless in this power system, the full voltage generated by G_1 will be present at each of the loads.

(a) Since this load is Y-connected, the phase voltage is

$$V_{\phi 1} = \frac{480\text{ V}}{\sqrt{3}} = 277\text{ V}$$

The phase current can be derived from the equation $P = 3V_\phi I_\phi \cos\theta$ as follows:

$$I_{\phi 1} = \frac{P}{3V_\phi \cos\theta} = \frac{100\text{ kW}}{3(277\text{ V})(0.9)} = 133.7\text{ A}$$

(b) Since this load is Δ-connected, the phase voltage is

$$V_{\phi 2} = 480\text{ V}$$

The phase current can be derived from the equation $S = 3V_\phi I_\phi$ as follows:

$$I_{\phi 2} = \frac{S}{3V_\phi} = \frac{80\text{ kVA}}{3(480\text{ V})} = 55.56\text{ A}$$

(c) The real and reactive power supplied by the generator when the switch is open is just the sum of the real and reactive powers of Loads 1 and 2.

$$P_1 = 100 \text{ kW}$$

$$Q_1 = P \tan\theta = P \tan\left(\cos^{-1} \text{PF}\right) = (100 \text{ kW})(\tan 25.84°) = 48.4 \text{ kvar}$$

$$P_2 = S\cos\theta = (80 \text{ kVA})(0.8) = 64 \text{ kW}$$

$$Q_2 = S\sin\theta = (80 \text{ kVA})(0.6) = 48 \text{ kvar}$$

$$P_G = P_1 + P_2 = 100 \text{ kW} + 64 \text{ kW} = 164 \text{ kW}$$

$$Q_G = Q_1 + Q_2 = 48.4 \text{ kvar} + 48 \text{ kvar} = 96.4 \text{ kvar}$$

(d) The line current when the switch is open is given by $I_L = \dfrac{P}{\sqrt{3}\, V_L \cos\theta}$, where $\theta = \tan^{-1}\dfrac{Q_G}{P_G}$.

$$\theta = \tan^{-1}\frac{Q_G}{P_G} = \tan^{-1}\frac{96.4 \text{ kvar}}{164 \text{ kW}} = 30.45°$$

$$I_L = \frac{P}{\sqrt{3}\, V_L \cos\theta} = \frac{164 \text{ kW}}{\sqrt{3}(480 \text{ V})\cos(30.45°)} = 228.8 \text{ A}$$

(e) The real and reactive power supplied by the generator when the switch is closed is just the sum of the real and reactive powers of Loads 1, 2, and 3. The powers of Loads 1 and 2 have already been calculated. The real and reactive power of Load 3 are:

$$P_3 = 80 \text{ kW}$$

$$Q_3 = P \tan\theta = P \tan\left(\cos^{-1} \text{PF}\right) = (80 \text{ kW})\left[\tan(-31.79°)\right] = -49.6 \text{ kvar}$$

$$P_G = P_1 + P_2 + P_3 = 100 \text{ kW} + 64 \text{ kW} + 80 \text{ kW} = 244 \text{ kW}$$

$$Q_G = Q_1 + Q_2 + Q_3 = 48.4 \text{ kvar} + 48 \text{ kvar} - 49.6 \text{ kvar} = 46.8 \text{ kvar}$$

(f) The line current when the switch is closed is given by $I_L = \dfrac{P}{\sqrt{3}\, V_L \cos\theta}$, where $\theta = \tan^{-1}\dfrac{Q_G}{P_G}$.

$$\theta = \tan^{-1}\frac{Q_G}{P_G} = \tan^{-1}\frac{46.8 \text{ kvar}}{244 \text{ kW}} = 10.86°$$

$$I_L = \frac{P}{\sqrt{3}\, V_L \cos\theta} = \frac{244 \text{ kW}}{\sqrt{3}(480 \text{ V})\cos(10.86°)} = 298.8 \text{ A}$$

(g) The total line current from the generator is 298.8 A. The line currents to each individual load are:

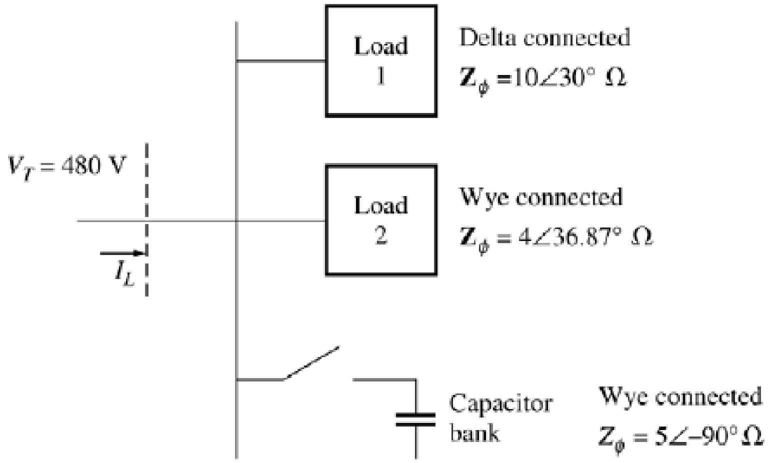
$$I_{L1} = \frac{P_1}{\sqrt{3}\, V_L \cos\theta_1} = \frac{100 \text{ kW}}{\sqrt{3}(480 \text{ V})(0.9)} = 133.6 \text{ A}$$

$$I_{L2} = \frac{S_2}{\sqrt{3}\, V_L} = \frac{80 \text{ kVA}}{\sqrt{3}(480 \text{ V})} = 96.2 \text{ A}$$

$$I_{L3} = \frac{P_3}{\sqrt{3}\, V_L \cos\theta_3} = \frac{80 \text{ kW}}{\sqrt{3}(480 \text{ V})(0.85)} = 113.2 \text{ A}$$

The sum of the three individual line currents is 343 A, while the current supplied by the generator is 298.8 A. These values are *not* the same, because the three loads have different impedance angles. Essentially, Load 3 is supplying some of the reactive power being consumed by Loads 1 and 2, so that it does not have to come from the generator.

A-6. Figure PA-4 shows a small 480-V distribution system. Assume that the lines in the system have zero impedance.



(a) If the switch shown is open, find the real, reactive, and apparent powers in the system. Find the total current supplied to the distribution system by the utility.

(b) Repeat part *(a)* with the switch closed. What happened to the total current supplied? Why?

SOLUTION

(a) With the switch open, the power supplied to each load is

$$P_1 = 3\frac{V_\phi^2}{Z}\cos\theta = 3\frac{(480\text{ V})^2}{10\,\Omega}\cos 30° = 59.86\text{ kW}$$

$$Q_1 = 3\frac{V_\phi^2}{Z}\sin\theta = 3\frac{(480\text{ V})^2}{10\,\Omega}\sin 30° = 34.56\text{ kvar}$$

$$P_2 = 3\frac{V_\phi^2}{Z}\cos\theta = 3\frac{(277\text{ V})^2}{4\,\Omega}\cos 36.87° = 46.04\text{ kW}$$

$$Q_2 = 3\frac{V_\phi^2}{Z}\sin\theta = 3\frac{(277\text{ V})^2}{4\,\Omega}\sin 36.87° = 34.53\text{ kvar}$$

$$P_{\text{TOT}} = P_1 + P_2 = 59.86\text{ kW} + 46.04\text{ kW} = 105.9\text{ kW}$$

$$Q_{\text{TOT}} = Q_1 + Q_2 = 34.56\text{ kvar} + 34.53\text{ kvar} = 69.09\text{ kvar}$$

The apparent power supplied by the utility is

$$S_{\text{TOT}} = \sqrt{P_{\text{TOT}}^2 + Q_{\text{TOT}}^2} = 126.4\text{ kVA}$$

The power factor supplied by the utility is

$$\text{PF} = \cos\left[\tan^{-1}\frac{Q_{\text{TOT}}}{P_{\text{TOT}}}\right] = \cos\left[\tan^{-1}\frac{69.09\text{ kvar}}{105.9\text{ kW}}\right] = 0.838\text{ lagging}$$

The current supplied by the utility is

$$I_L = \frac{P_{\text{TOT}}}{\sqrt{3}\,V_T\,\text{PF}} = \frac{105.9\text{ kW}}{\sqrt{3}\,(480\text{ V})\,(0.838)} = 152\text{ A}$$

(b) With the switch closed, P_3 is added to the circuit. The real and reactive power of P_3 is

$$P_3 = 3\frac{V_\phi^{\ 2}}{Z}\cos\theta = 3\frac{(277\text{ V})^2}{5\,\Omega}\cos(\text{-}90°) = 0\text{ kW}$$

$$P_3 = 3\frac{V_\phi^{\ 2}}{Z}\sin\theta = 3\frac{(277\text{ V})^2}{5\,\Omega}\sin(\text{-}90°) = -46.06\text{ kvar}$$

$$P_{\text{TOT}} = P_1 + P_2 + P_3 = 59.86\text{ kW} + 46.04\text{ kW} + 0\text{ kW} = 105.9\text{ kW}$$

$$Q_{\text{TOT}} = Q_1 + Q_2 + Q_3 = 34.56\text{ kvar} + 34.53\text{ kvar} - 46.06\text{ kvar} = 23.03\text{ kvar}$$

The apparent power supplied by the utility is

$$S_{\text{TOT}} = \sqrt{P_{\text{TOT}}^{\ 2} + Q_{\text{TOT}}^{\ 2}} = 108.4\text{ kVA}$$

The power factor supplied by the utility is

$$\text{PF} = \cos\left[\tan^{-1}\frac{Q_{\text{TOT}}}{P_{\text{TOT}}}\right] = \cos\left[\tan^{-1}\frac{23.03\text{ kVAR}}{105.9\text{ kW}}\right] = 0.977\text{ lagging}$$

The current supplied by the utility is

$$I_L = \frac{P_{\text{TOT}}}{\sqrt{3}\ V_T\ \text{PF}} = \frac{105.9\text{ kW}}{\sqrt{3}\ (480\text{ V})\ (0.977)} = 130.4\text{ A}$$

(c) The total current supplied by the power system drops when the switch is closed because the capacitor bank is supplying some of the reactive power being consumed by loads 1 and 2.