King Fahd University of Petroleum & Minerals
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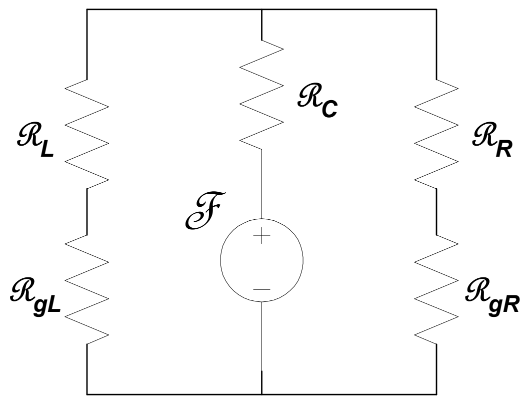
EE-360 Problem Session Solution #1, 2014, 132

Solution 1:

$$\mathcal{R}_{tot} = \mathcal{R}_c + \frac{\left(\mathcal{R}_L + \mathcal{R}_{gL}\right)\left(\mathcal{R}_R + \mathcal{R}_{gR}\right)}{\mathcal{R}_L + \mathcal{R}_{gL} + \mathcal{R}_R + \mathcal{R}_{gR}}$$



$$\mathcal{R}_L = \frac{l_L}{\mu A_L} = \frac{l_L}{\mu_0 \mu_r A_L} = \frac{1.2m}{2000 \times 4\pi 10^{-7}(0.07m)(0.1m)} = 68.2kA \cdot t\ /wb$$

$$\mathcal{R}_{gL} = \frac{l_{gL}}{\mu A_{gL}} = \frac{l_{gL}}{\mu_0 A_{gL}} = \frac{0.0005m}{4\pi 10^{-7}(0.07m)(0.1m)(1.05)} = 54.1kA \cdot t\ /wb$$

$$\mathcal{R}_R = \frac{l_R}{\mu A_R} = \frac{l_R}{\mu_0 \mu_r A_R} = \frac{1.2m}{2000 \times 4\pi 10^{-7}(0.07m)(0.1m)} = 68.2kA \cdot t\ /wb$$

$$\mathcal{R}_{gR} = \frac{l_{gR}}{\mu A_{gR}} = \frac{l_{gR}}{\mu_0 A_{gR}} = \frac{0.0007m}{4\pi 10^{-7}(0.07m)(0.1m)(1.05)} = 75.8kA \cdot t\ /wb$$

$$\mathcal{R}_c = \frac{l_c}{\mu A_c} = \frac{l_c}{\mu_0 \mu_r A_c} = \frac{0.4m}{2000 \times 4\pi 10^{-7}(0.07m)(0.1m)} = 22.7kA \cdot t\ /wb$$

$$\mathcal{R}_{tot} = 22.7 + \frac{\left(68.2 + 54.1\right)\left(68.2 + 75.8\right)}{68.2 + 54.1 + 68.2 + 75.8} = 88.8kA \cdot t\ /wb$$

$$\phi_{total} = \phi_c = \frac{\mathscr{F}}{\mathscr{R}_{tot}} = \frac{Ni}{\mathscr{R}_{tot}} = \frac{300t \times 1A}{88.8kA \cdot t\ /wb} = 0.0034\ wb$$

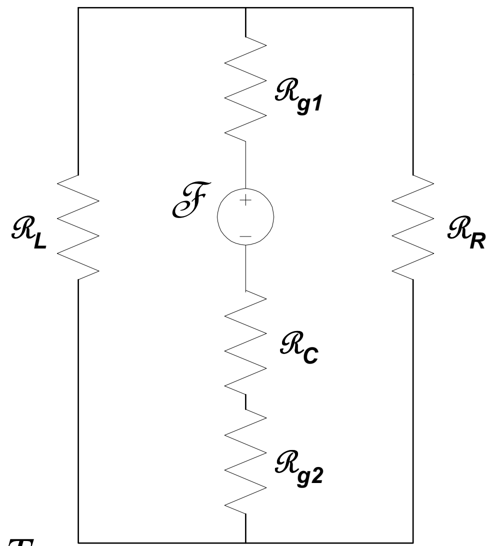
$$\phi_L = \phi_{tot} \times \frac{\mathscr{R}_R + \mathscr{R}_{gR}}{\mathscr{R}_L + \mathscr{R}_{gL} + \mathscr{R}_R + \mathscr{R}_{gR}} = 0.0018\ wb$$

$$\phi_R = \phi_{tot} \times \frac{\mathscr{R}_L + \mathscr{R}_{gL}}{\mathscr{R}_L + \mathscr{R}_{gL} + \mathscr{R}_R + \mathscr{R}_{gR}} = 0.0016\ wb$$

$$B_{gL} = \frac{\phi_L}{A_{gL}} = \frac{0.0018}{0.07 \times 0.1 \times 1.05} = 0.245\ T$$

$$B_{gR} = \frac{\phi_R}{A_{gR}} = \frac{0.0016}{0.07 \times 0.1 \times 1.05} = 0.22\ T$$

Solution 2:



$\phi_c = 0.02\ wb$

$N = 1200\ t$

$A = 200\ cm^2$

$$B_C = B_g = \frac{\phi_C}{A} = \frac{0.02\ wb}{200 \times 10^{-4} mm^2} = 1\ T$$

$$B_L = B_R = \frac{\phi_c / 2}{A} = \frac{0.01\ wb}{200 \times 10^{-4} mm^2} = 0.5\ T$$

Using the cast steel curve to find H from the values of B above:

$$H_C = 800 \ A \cdot t \ / \ m$$

$$H_L = H_R = 280 \ A \cdot t \ / \ m$$

$$H_g = \frac{B_g}{\mu_0} = \frac{1 \ T}{4\pi \times 10^{-7}} = 795.77 \times 10^3 \ A \cdot t \ / \ m$$

$$\mathscr{F}_{g_1} = \mathscr{F}_{g2} = H_g l_g = 795.77 \times 10^3 \times 0.001 = 795.77 \ A \cdot t$$

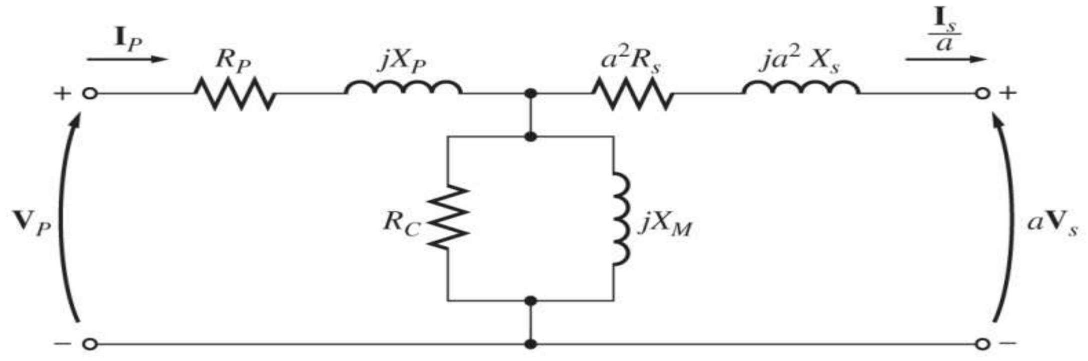
$$\mathscr{F}_C = H_C l_C = 800 \times 0.1 = 80 \ A \cdot t$$

$$\mathscr{F}_L = \mathscr{F}_R = H_L l_L = 280 \times 0.2 = 56 \ A \cdot t$$

$$\mathscr{F}_{tot} = 2 \times \mathscr{F}_g + \mathscr{F}_C + \mathscr{F}_L = 1727.54 \ A \cdot t$$

$$I = \frac{\mathscr{F}_{tot}}{N} = \frac{1727.54 \ A \cdot t}{1200 \ t} = 1.44 \ A$$

Solution 3:



$$I_S = \frac{S}{V_S} = \frac{150\ kVA}{240\ V} = 625\angle -36.8^\circ A$$

$$\frac{I_S}{a} = \frac{625\angle -36.8^\circ A}{10} = 62.5\angle -36.8^\circ A = 50 - j\,37.5\ A$$

$$aV_S = 2400\angle 0^\circ\ V = 2400 + j\,0\ V$$

$$a^2R_2 = 0.2\ \Omega \text{ and } a^2X_2 = 0.45\ \Omega$$

Therefore,

$$E_1 = (2400 + j\,0\ V) + (50 - j\,37.5\ A)(0.2 + j\,0.45)$$

$$= 2427 + j\,15 = 2427\angle 0.35^\circ\ V$$

$$I_m = \frac{E_1}{X_m} = \frac{2427\angle 0.35^\circ}{1550\angle 90^\circ} = 1.56\angle -89.65^\circ = 0.0095 - j\,1.56\ A$$

$$I_c = \frac{E_1}{R_c} = \frac{2427\angle 0.35^\circ}{10000} = 0.2427 - j\,0\ A$$

$$I_o = I_m + I_c = 0.25 - j\,1.56\ A$$

$$I_p = I_o + \frac{I_S}{a} = 50.25 - j\,39.06 = 63.65\angle -37.850^\circ\ A$$

Therefore, the primary voltage is

$$V_P = E_1 + I_P \times (R_1 + jX_1)$$
$$= \left(2427 + j15\right) + \left(50.25 - j39.06\right)\left(0.2 + j0.45\right)$$
$$= 2455 + J30 = 2455\angle 0.7^{\circ}\ V$$

$$VR = \frac{V_P / a - V_{S,fL}}{V_{S,fL}} \times 100\% = \frac{2455/10 - 240}{240} \times 100\% = 2.3\%$$

$$P_i = I_c^2 R_c = 0.2427^2 \times 10000 = 589\ W$$
$$P_{cu} = I_P^2 R_1 + I_S^2 R_2 = 63.65^2 \times 0.2 + 62.5^2 \times 0.2 = 1592\ W$$
$$P_{loss} = P_i + P_{cu} = 589 + 1592 = 2181\ W$$

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\% = \frac{150kVA \times 0.8}{150kVA \times 0.8 + 2.181kW} \times 100\% = 98.2\%$$

Solution 4:

(a) From O/C test result:

$$R_{cs} = \frac{V_{o/c}^2}{P_{o/c}} = \frac{120^2}{80} = 180\ \Omega$$
$$R_{cp} = a^2 R_c = (450/120)^2 \times 180\ \Omega = 2530\ \Omega$$

$$I_{cs} = \frac{V_{o/c}}{R_c^{'}} = \frac{120\ V}{180\ \Omega} = 0.667\ A$$

$$I_{ms} = \sqrt{I_o^2 - I_{cs}^2} = \sqrt{4.2^2 - 0.667^2} = 4.15\ A$$

$$X_{ms} = \frac{V_{o/c}}{I_m^{'}} = \frac{120}{4.15} = 28.92\ \Omega$$

$$X_{mp} = a^2 X_m^{'} = (450/120)^2 \times 28.92\ \Omega = 406.7\ \Omega$$

From S/C test result:

$$Z_{eq} = \frac{V_{\mathrm{s/c}}}{I_{\mathrm{s/c}}} = \frac{9.65\ V}{22.2\ A} = 0.435\ \Omega$$

$$R_{eq} = \frac{P_{s/c}}{I_{s/c}^2} = \frac{120\ W}{22.2^2\ A} = 0.243\ \Omega$$

$$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2} = \sqrt{0.435^2 - 0.243^2} = 0.361\ \Omega$$

$$\theta_{sc} = \tan^{-1}\left(\frac{X_{eq}}{R_{eq}}\right) = \tan^{-1}\left(\frac{0.361}{0.234}\right) = 57^{\circ}$$

(b) At full load:

$$I_{\mathrm{S\_FL}} = \frac{S}{V_{S\_rated}} = \frac{10\ kVA}{120\ V} = 83.33\ A$$

At power factor PF = 0.8 leading,

$$I_{\mathrm{S}} = 83.33\angle 36.9^{\circ}\ A$$

$$\frac{V_P}{a} = V_S + I_S \times (R_{eq} + jX_{eq})$$

$$= 120\angle 0^\circ + (83.33\angle 36.9^\circ)(0.243) + (83.33\angle 36.9^\circ)(j\,0.361)$$

$$= 120 + 20.25\angle 36.9^\circ + 30.08\angle 126.9^\circ$$

$$= 120 + 16.2 + j\,12.16 - 18.06 + j\,24.05$$

$$= 118.14 + j\,36.21 = 123.56\angle 17^\circ \;\; V$$

$$VR = \frac{V_P / a - V_{S,fL}}{V_{S,fL}} \times 100\% = \frac{123.56 - 120}{120} \times 100\% = 2.96\%$$

(C) Efficiency at half load:

$$P_{core} = \frac{(V_p / a)^2}{R_{cs}} = \frac{123.56^2}{180} = 84.8 \;\; W$$

$$P_{cu} = \left(0.5 I_S\right)^2 R_{eq} = 41.66^2 \times 0.243 = 421.84 \;\; W$$

$$\eta = \frac{0.5 P_{out}}{0.5 P_{out} + P_{core} + 0.5 P_{cu}} \times 100\%$$

$$= \frac{0.5 \times 10kVA \times 0.8}{0.5 \times 10kVA \times 0.8 + 84.8 + 421.84} \times 100\% = 88.75\%$$