King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE 360: Home Work #1
Due Date (Feb. 17th)

Key Solutions

Q1) Problem 1-6

SOLUTION This core can be divided up into five regions. Let $\mathcal{R}_1$ be the reluctance of the left-hand portion of the core, $\mathcal{R}_2$ be the reluctance of the left-hand air gap, $\mathcal{R}_3$ be the reluctance of the right-hand portion of the core, $\mathcal{R}_4$ be the reluctance of the right-hand air gap, and $\mathcal{R}_5$ be the reluctance of the center leg of the core. Then the total reluctance of the core is

$$\mathcal{R}_{TOT} = \mathcal{R}_5 + \frac{(\mathcal{R}_1 + \mathcal{R}_2)(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}$$

$$\mathcal{R}_1 = \frac{l_1}{\mu_r \mu_0 A_1} = \frac{1.11 \text{ m}}{(1500)(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.05 \text{ m})} = 168 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_2 = \frac{l_2}{\mu_0 A_2} = \frac{0.0007 \text{ m}}{(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.05 \text{ m})(1.05)} = 152 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_3 = \frac{l_3}{\mu_r \mu_0 A_3} = \frac{1.11 \text{ m}}{(1500)(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.05 \text{ m})} = 168 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_4 = \frac{l_4}{\mu_0 A_4} = \frac{0.0005 \text{ m}}{(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.05 \text{ m})(1.05)} = 108 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_5 = \frac{l_5}{\mu_r \mu_0 A_5} = \frac{0.37 \text{ m}}{(1500)(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.05 \text{ m})} = 56.1 \text{ kA} \cdot \text{t/Wb}$$

The total reluctance is

$$\mathcal{R}_{TOT} = \mathcal{R}_5 + \frac{(\mathcal{R}_1 + \mathcal{R}_2)(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} = 56.1 + \frac{(168+152)(168+108)}{168+152+168+108} = 204 \text{ kA} \cdot \text{t/Wb}$$

The total flux in the core is equal to the flux in the center leg:

$$\phi_{center} = \phi_{TOT} = \frac{\mathcal{F}}{\mathcal{R}_{TOT}} = \frac{(300 \text{ t})(1.0 \text{ A})}{204 \text{ kA} \cdot \text{t/Wb}} = 0.00147 \text{ Wb}$$

The fluxes in the left and right legs can be found by the "flux divider rule", which is analogous to the current divider rule.

$$\phi_{left} = \frac{(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} \phi_{TOT} = \frac{(168+108)}{168+152+168+108}(0.00147 \text{ Wb}) = 0.00068 \text{ Wb}$$

$$\phi_{right} = \frac{(\mathcal{R}_1 + \mathcal{R}_2)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} \phi_{TOT} = \frac{(168+152)}{168+152+168+108}(0.00147 \text{ Wb}) = 0.00079 \text{ Wb}$$

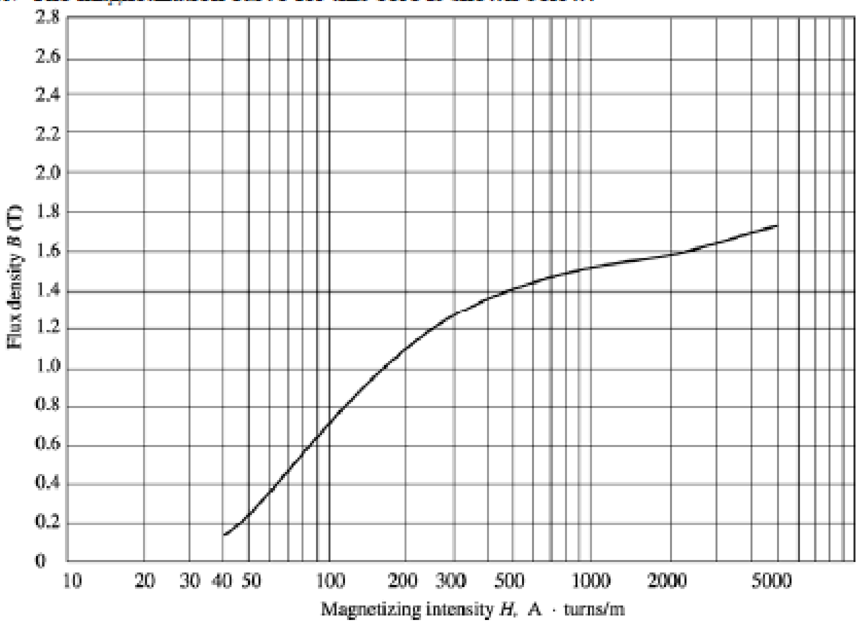
The flux density in the air gaps can be determined from the equation $\phi = BA$:

$$B_{left} = \frac{\phi_{left}}{A_{eff}} = \frac{0.00068 \text{ Wb}}{(0.07 \text{ cm})(0.05 \text{ cm})(1.05)} = 0.185 \text{ T}$$

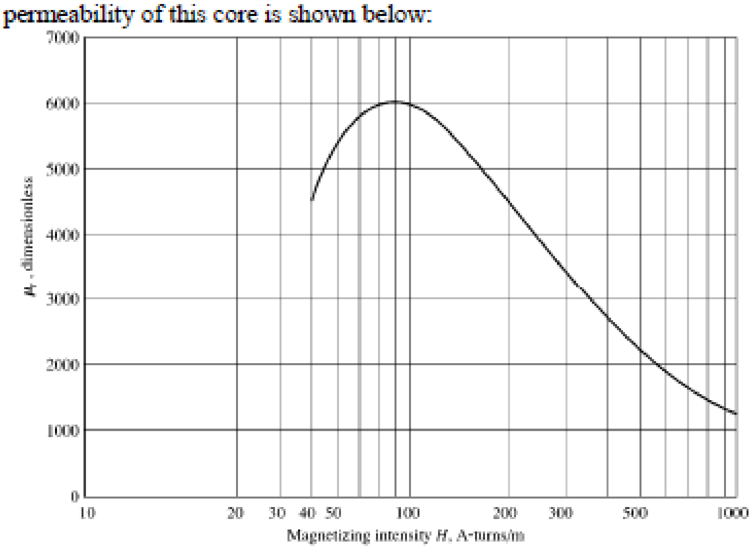
$$B_{right} = \frac{\phi_{right}}{A_{eff}} = \frac{0.00079 \text{ Wb}}{(0.07 \text{ cm})(0.05 \text{ cm})(1.05)} = 0.215 \text{ T}$$

Q2) Problem 1-17

SOLUTION The magnetization curve for this core is shown below:



The relative permeability of this core is shown below:



> **Note:** This is a design problem, and the answer presented here is not unique. Other values could be selected for the flux density in part *(a)*, and other numbers of turns could be selected in part *(c)*. These other answers are also correct if the proper steps were followed, and if the choices were reasonable.

(a) From Figure 1-10*c*, a reasonable maximum flux density would be about 1.2 T. Notice that the saturation effects become significant for higher flux densities.

(b) At a flux density of 1.2 T, the total flux in the core would be

$$\phi = BA = (1.2 \text{ T})(0.05 \text{ m})(0.05 \text{ m}) = 0.0030 \text{ Wb}$$

(c) The total reluctance of the core is:

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_{\text{stator}} + \mathcal{R}_{\text{air gap 1}} + \mathcal{R}_{\text{rotor}} + \mathcal{R}_{\text{air gap 2}}$$

At a flux density of 1.2 T, the relative permeability μ_r of the stator is about 3800, so the stator reluctance is

$$\mathcal{R}_{\text{stator}} = \frac{l_{\text{stator}}}{\mu_{\text{stator}} A_{\text{stator}}} = \frac{0.60 \text{ m}}{(3800)\left(4\pi \times 10^{-7} \text{ H/m}\right)(0.05 \text{ m})(0.05 \text{ m})} = 50.3 \text{ kA} \cdot \text{t/Wb}$$

At a flux density of 1.2 T, the relative permeability μ_r of the rotor is 3800, so the rotor reluctance is

$$\mathcal{R}_{\text{rotor}} = \frac{l_{\text{rotor}}}{\mu_{\text{stator}} A_{\text{rotor}}} = \frac{0.05 \text{ m}}{(3800)\left(4\pi \times 10^{-7} \text{ H/m}\right)(0.05 \text{ m})(0.05 \text{ m})} = 4.2 \text{ kA} \cdot \text{t/Wb}$$

The reluctance of both air gap 1 and air gap 2 is

$$\mathcal{R}_{\text{air gap 1}} = \mathcal{R}_{\text{air gap 2}} = \frac{l_{\text{air gap}}}{\mu_{\text{air gap}} A_{\text{air gap}}} = \frac{0.0005 \text{ m}}{\left(4\pi \times 10^{-7} \text{ H/m}\right)\left(0.0018 \text{ m}^2\right)} = 221 \text{ kA} \cdot \text{t/Wb}$$

Therefore, the total reluctance of the core is

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_{\text{stator}} + \mathcal{R}_{\text{air gap 1}} + \mathcal{R}_{\text{rotor}} + \mathcal{R}_{\text{air gap 2}}$$

$$\mathcal{R}_{\text{TOT}} = 50.3 + 221 + 4.2 + 221 = 496 \text{ kA} \cdot \text{t/Wb}$$

The required MMF is

$$\mathcal{F}_{\text{TOT}} = \phi \mathcal{R}_{\text{TOT}} = (0.003 \text{ Wb})(496 \text{ kA} \cdot \text{t/Wb}) = 1488 \text{ A} \cdot \text{t}$$

Since $\mathcal{F} = Ni$, and the current is limited to 1 A, one possible choice for the number of turns is $N = 2000$. This would allow the desired flux density to be achieved with a current of about 0.74 A.

Q3) A coil of 500 turns and resistance 20 Ω is wound uniformly on an iron ring of mean circumference of 50 cm and cross section 4 cm2. It is connected to a 24-volt DC supply. Under these conditions, the relative permeability of iron is 800. Calculate the values of:

(a) the magnetomotive force
(b) the magnetic field intensity
(c) the total flux in the iron
(d) the reluctance of the ring

Solution:

$$I = \frac{24}{20} = 1.2 \text{ A}$$

a) $F = NI = 1.2 * 500 = 600 \text{ AT}$

b) $H = \frac{F}{l} = \frac{600}{0.5} = 1200 \text{ AT/m}$

c) $B = \mu H = \mu_o \mu_r H = 4\pi * 10^{-7} * 800 * 1200 = 1.206 \text{ T}$

$\Phi = BA = 1.206 * 4 * 10^{-4} = 0.483 \text{ mWb}$

d) $\mathcal{R} = \frac{l}{\mu A} = \frac{0.5}{4\pi * 10^{-7} * 800 * 4 * 10^{-4}} = 1.243 * 10^{6} \text{ AT/Wb}$

Q4) The total core loss for a specimen of magnetic sheet steel is found to be 1800 W at 60 Hz. If the flux density is kept constant and the frequency of the supply increases 50%, the total core loss is found to be 3000 W. Compute the separate hysteresis and eddy-current losses at both frequencies.

Solution:

For Constant B

$P_h \propto f$ i.e. $P_h = Af$ where A is Constant

$P_e \propto f^2$ i.e $P_e = Bf^2$ where B is Constant

at 60 Hz

$$1800 = A * 60 + B * (60)^2 \longrightarrow ①$$

at 90 Hz

$$3000 = A * 90 + B * (90)^2 \longrightarrow ②$$

① by 3 and ② by ② and Subtract

$$3 * 1800 - 2 * 3000 = 3B * (60)^2 - 2B * (90)^2$$

$$-600 = -5400 B$$

So, $B = 0.111$ sub. in ①

$A = 23.333$

Hence; At 60 Hz

$P_h = 23.333 * 60 = 1400$ W

$P_e = 0.111 * (60)^2 = 400$ W

At 90 Hz

$P_h = 23.333 * 90 = 2100$ W

$P_e = 0.111 * (90)^2 = 900$ W