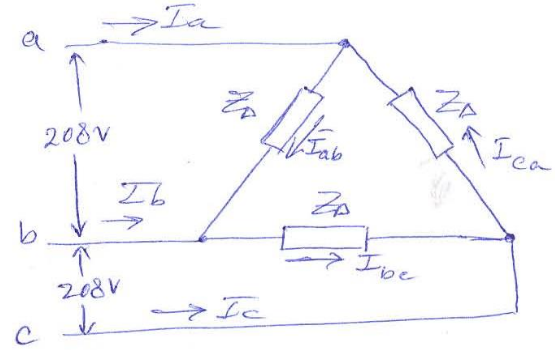
Problem Session #1

Three-Phase Circuits

Problem 1:

$\overline{V}_{an} = 120\angle 0^\circ \Rightarrow V_{ab} = \sqrt{3}\,120\angle 30^\circ$



$\overline{V}_{ab} = 208\angle 30^\circ$ V
$\overline{V}_{bc} = 208\angle -90^\circ$ V
$\overline{V}_{ca} = 208\angle +150$ V
} phase voltages

(a)
$\overline{I}_{ab} = \dfrac{\overline{V}_{ab}}{Z_\Delta} = \dfrac{208\angle 30^\circ}{20\angle -30} = 10.4\angle 60^\circ$ A
$\overline{I}_{bc} = \dfrac{\overline{V}_{bc}}{Z_\Delta} = \dfrac{208\angle -90^\circ}{20\angle -30} = 10.4\angle -60^\circ$ A
$\overline{I}_{ca} = \dfrac{\overline{V}_{ca}}{Z_\Delta} = \dfrac{208\angle 150^\circ}{20\angle -30} = 10.4\angle 180^\circ$ A
} phase currents

(b)
$\overline{I}_a = \sqrt{3}\,\overline{I}_{ab}\angle -30^\circ = \sqrt{3}\,10.4\angle 60^\circ - 30^\circ = 18\angle 30^\circ$ A
$\overline{I}_b = 18\angle -90^\circ$ A
$\overline{I}_c = 18\angle 150^\circ$ A
} line currents

(c) $PF = \cos(-30^\circ) = 0.866$ (leading)

(d) $S_{3ph} = 3\,V_{ph}\cdot I_{ph} = \sqrt{3}\,V_L I_L = \sqrt{3}\;208 \times 18 = 6485$ VA

$P_{3\phi} = 3\,V_{ph}\cdot I_{ph}\cdot\cos\phi = \sqrt{3}\,V_L I_L \cos\phi = 3 \times 208 \times 10.4 \times 0.866$

$P_{3\phi} = 5.616$ KW

$Q_{3\phi} = \sqrt{S_{3\phi}^2 - P_{3\phi}^2} = 3.243$ kVAr

Problem 2:

Part(I) The switch is open: $Z_\Delta = 10\angle 30^\circ = 8.66 + j5\,\Omega$

$Z_Y = 2.5\angle 36.9^\circ = 2 + j1.5\,\Omega$

For the Δ-connected load ⇒ $V_{ph} = 480\,v$

$$I_{ph_\Delta} = \frac{V_{ph}}{Z_\Delta} = \frac{480}{10} = 48\ A$$

For the Y-connected load ⇒ $V_{ph} = \frac{480}{\sqrt{3}} = 277.1\,V$

$$I_{ph_Y} = \frac{277.1}{2.5} = 110.85\,A$$

(a)

$$P_\Delta = 3 I_{ph}^2 R_\Delta = 3(48)^2(8.66) = 59.86\ KW$$

$$Q_\Delta = 3 I_{ph}^2 X_\Delta = 3(48)^2(5) = 34.56\ KVAR$$

$$P_Y = 3 I_{ph}^2 R_Y = 3(110.85)^2 \times 2 = 73.73\ KW$$

$$Q_Y = 3 I_{ph}^2 X_Y = 3(110.85)^2 \times 1.5 = 55.29\ KVAR$$

(b)

$$P_T = P_\Delta + P_Y = (59.86 + 73.73) = 133.59\ KW$$

$$Q_T = Q_\Delta + Q_Y = (34.56 + 55.29) = 89.85\ KVAR$$

(c)

$$PF_\Delta = \cos\theta_\Delta = \cos 30^\circ = 0.866 \text{ lagging}$$

$$PF_Y = \cos\theta_Y = \cos 36.9^\circ = 0.80 \text{ lagging}$$

$$PF_T = \cos\theta_T, \quad \theta_T = \tan^{-1}\left(\frac{Q_T}{P_T}\right) = \tan^{-1}\left(\frac{89.85}{133.59}\right) = 33.9^\circ$$

$$PF_T = 0.829 \text{ lagging}$$

(d)

$$P_T = \sqrt{3} V_L I_L \cos\theta_T \Rightarrow I_L = \frac{P_{Tot}}{\sqrt{3} V_L \cos\theta_T} = \frac{133.59 \times 10^3}{\sqrt{3}\ 480\,(0.829)}$$

$I_{tot} = 193.65\,A$ total line current supplied by the utility

Continue .... Problem 2

$$Z_c = 3 \angle -90^\circ \ \Omega = -j3\Omega \Rightarrow X_c = 3\Omega$$

$$P_c = 0.0 \text{ W}, \quad Q_c = \frac{3 V_{ph}^2}{X_c}, \quad V_{ph} = \frac{480}{\sqrt{3}} = 277.1 \text{ V}$$

$$Q_c = \frac{3(277.1)^2}{3} \Rightarrow Q_c = 76.8 \text{ KVAR supplied}$$

(a) $P_{tot.} = 133.59$ KW supplied by the utility
(the Cap. does't absorb power)

$$Q_{tot} = Q_{load} - Q_c = (89.85 - 76.8) = 13.05 \text{ KVAR}$$
supplied by the utility

(b)
$$PF_{tot} = \cos\theta_{tot} \Rightarrow \theta_{tot.} = \tan^{-1}\frac{Q_{tot}}{P_{tot}} = \tan^{-1}\left(\frac{13.05}{133.59}\right)$$

$$\theta_{tot} = 5.6^\circ \Rightarrow PF_{tot} = \cos(5.6^\circ) = 0.995 \text{ lagging}$$

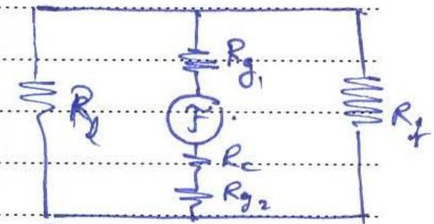
$$I_L = \frac{P_{tot}}{\sqrt{3}\, V_L \cos\theta_{tot}} = \frac{133.59 \times 10^3}{\sqrt{3}\ 480\ 0.995} = 161.45 \text{ A}$$

$$\boxed{I_{L_{tot}} = 161.45 \text{ A}}$$

Problem 3:

$\phi = 0.02\ Wb$ & $N = 1200$

$A = 200\ Cm^2$



$$B_c = \frac{20\times10^{-3}}{200\times10^{-4}} = 1\ \text{Tesla}$$

$$H_c = 800\ A.t/m \ / \ H_g = \frac{1}{4\pi\,10^{-7}} = 795.77\times10^{3}\ A.t/m$$

$$\mathcal{F}_c = H_c\, l_c = 800\times0.1 = 80\ A\cdot t$$

$$\mathcal{F}_{g_1} = H_{g_1} l_{g_1} = 795.77\times10^{+3}\times10^{-3} = 795.77\ A\cdot t = \mathcal{F}_{g_2}$$

$$B_l = B_r = 0.5\ T \Rightarrow H_l = H_r = 280\ A\text{-}t/m$$

$$\mathcal{F}_l = \mathcal{F}_r = 280\times0.2 = 56\ A\cdot t$$

$$\mathcal{F}_t = 2\times\mathcal{F}_g + \mathcal{F}_c + \mathcal{F}_l = 1727.54\ A\cdot t$$

$$I = \frac{\mathcal{F}_t}{N} = \frac{1727.54}{1200} = \underline{1.44\ A}$$

Problem 4: Let $\mathcal{R}_1$ be the reluctance of the left-hand portion of the core, $\mathcal{R}_2$ be the reluctance of the left-hand air gap, $\mathcal{R}_3$ be the reluctance of the right-hand portion of the core, $\mathcal{R}_4$ be the reluctance of the right hand air gap, and $\mathcal{R}_5$ be the reluctance of the center leg of the core. The total reluctance of the coil is

$$\mathcal{R}_{tot} = \mathcal{R}_5 + \frac{(\mathcal{R}_1+\mathcal{R}_2)(\mathcal{R}_3+\mathcal{R}_4)}{\mathcal{R}_1+\mathcal{R}_2+\mathcal{R}_3+\mathcal{R}_4}$$

$$\mathcal{R}_1 = \frac{l_1}{\mu_1 A_1} = \frac{l_1}{\mu_r \mu_0 A_1} = \frac{1.2\ m}{2000 \times 4\pi 10^{-7}\ (0.07m)(0.10m)} = 68.2\ K\ A\text{-}t/Wb$$

$$\mathcal{R}_2 = \frac{l_2}{\mu A_2} = \frac{l_2}{\mu_0 A_2} = \frac{0.0005}{4\pi 10^{-7}\ (0.07)(0.1)(1.05)} = 54.1\ KA\text{-}t/Wb$$

$$\mathcal{R}_3 = \frac{l_3}{\mu A_3} = \frac{l_3}{\mu_r \mu_0 A_3} = \frac{1.2}{2000 \times 4\pi 10^{-7}\ (0.07)(0.1)} = 68.2\ KA\text{-}t/Wb$$

$$\mathcal{R}_4 = \frac{l_4}{\mu A_4} = \frac{l_4}{\mu_0 A_4} = \frac{0.0007\ m}{4\pi 10^{-7}\ (0.07m)(0.1m)(1.05)} = 75.8\ KA\text{-}t/Wb$$

$$\mathcal{R}_5 = \frac{l_5}{\mu A_5} = \frac{l_5}{\mu_r \mu_0 A_5} = \frac{0.4\ m}{2000 \times 4\pi 10^{-7}\ (0.07m)(0.1)} = 22.7\ KA\text{-}t/Wb$$

$$\mathcal{R}_{tot} = 22.7 + \frac{(68.2+54.1)(68.2+75.8)}{68.2+54.1+68.2+75.8} = 88.8\ KA\text{-}t/Wb$$

$$\phi_{tot} = \frac{\mathcal{F}}{\mathcal{R}} = \frac{Ni}{\mathcal{R}_{tot}} = \frac{(300t)(1A)}{88.8\ KA\text{-}t/Wb} = \boxed{0.0034\ Wb}$$

This is the flux in the center leg.

$$\phi_{left} = \frac{\mathcal{R}_3+\mathcal{R}_4}{\mathcal{R}_1+\mathcal{R}_2+\mathcal{R}_3+\mathcal{R}_4}\phi_{tot} = 0.0018\ Wb$$

$$\phi_{right} = \frac{\mathcal{R}_1+\mathcal{R}_2}{\mathcal{R}_1+\mathcal{R}_2+\mathcal{R}_3+\mathcal{R}_4}\phi_{tot} = 0.0016\ Wb$$

$$B_{g\,left} = \frac{\phi_{left}}{A \times 1.05} = \frac{0.0018}{0.007 \times 1.05} = 0.245\ T$$

$$B_{g\,right} = \frac{\phi_{right}}{A \times 1.05} = \frac{0.0016}{0.007 \times 1.05} = 0.22\ T$$