EE 360 HW # 6 Solution

Q1) (a) A 40 km line is a short one. Hence

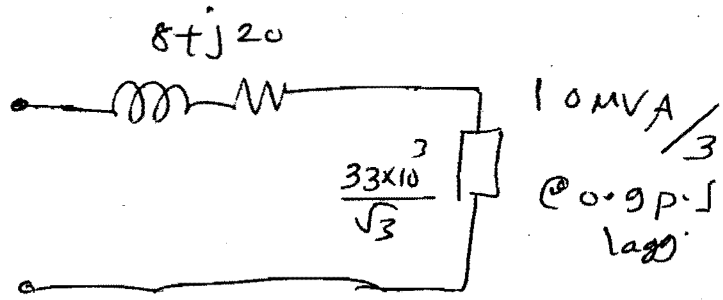
$A = 1 = D$

$B = Z$

$C = 0$

$A = (0.2 + j0.5)\,40 = \underline{8 + j20\ \Omega}$

(b)



$$\begin{bmatrix} \vec{V}_s \\ \vec{I}_s \end{bmatrix} = \begin{bmatrix} 1 & 8+j20 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \vec{V}_R \\ \vec{I}_R \end{bmatrix}$$

For lagg p.f

$\vec{V}_R = \frac{33 \times 10^3}{\sqrt{3}} \angle 0$

$\vec{I}_R = \frac{10 \times 10^6 / 3}{\frac{33 \times 10^3}{\sqrt{3}}} \angle -\cos^{-1} 0.9$

$\vec{I}_R = 174.95 \angle -25.8^\circ$ A

$\Rightarrow \vec{V}_s = \frac{33 \times 10^3}{\sqrt{3}} \angle 0 + (8+j20)(174.95 \angle -25.8^\circ)$

$+ (21.54 \angle 68.2)(174.95 \angle -25.8)$

$= 19052.5 + 3768.4 \angle 42.4^\circ$

$= 19052.5 + 2782.8 + j2541$

$= 21835.3 + j2541$

$\boxed{\vec{V}_s = 21.98 \angle 6.63\ \text{kV}}$

For Leading p.f

$\vec{I}_R = 174.95 \angle +25.8^\circ$

$\vec{V}_s = 19052.5 + 3768.4 \angle 94$

$= (19052.5 - 262.8) + j3759.2$

$= 18789.6 + j3759.2$

$\boxed{\vec{V}_s = 19.16 \angle 11.3\ \text{kV}}$

Q2)

Solution For a load of 40 MW, at 220 kV and 0.9 power factor lagging, the receiving-end current is given by

$$\mathbf{I}_R = \frac{40{,}000}{\sqrt{3}(220)(0.9)} \angle -\cos^{-1} 0.9 = 116.6 \angle -25.8^\circ$$

The sending-end voltage is found as follows:

$$\begin{aligned}\mathbf{V}_S &= \mathbf{V}_R + \mathbf{Z}\mathbf{I}_R \\ &= (220/\sqrt{3}) \times 10^3 \angle 0^\circ + (35 + j140)(116.6 \angle -25.8^\circ) \\ &= 127{,}000 \angle 0^\circ + (144.3 \angle 76^\circ)(116.6 \angle -25.8^\circ) \\ &= 138.4 \angle 5.4^\circ \text{ kV} \quad \text{(line-to-neutral)} \\ &= 239.7 \angle 35.4^\circ \text{ kV} \quad \text{(line-to-line)}\end{aligned}$$

Since there is no shunt branch, the sending-end current is the same as the receiving-end current. Thus,

$$\mathbf{I}_S = \mathbf{I}_R = 116.6 \angle -25.8^\circ$$

The sending-end power factor is

$$\text{PF}_S = \cos[5.4^\circ - (-25.8^\circ)] = 0.86 \text{ lagging}$$

The percent voltage regulation is computed as

$$\begin{aligned}\text{Voltage regulation} &= \frac{V_{R,nl} - V_{R,fl}}{V_{R,fl}} 100\% \\ &= \frac{239.7 - 220}{220} 100\% = 8.95\%\end{aligned}$$

The sending-end real power is

$$\begin{aligned}P_S = 3V_S I_S \text{PF}_S &= 3(138.4 \times 10^3)(116.6)(0.86) \\ &= 41.6 \times 10^6 \text{ W} = 41.6 \text{ MW}\end{aligned}$$

Therefore, the efficiency of the line is found as follows:

$$\text{Efficiency} = (P_R/P_S)100\% = (40/41.6)100\% = 96.2\%$$

Q3)

Solution For a load of 40 MW, at 220 kV and 0.9 power factor lagging, the receiving-end current is given by

$$\mathbf{I}_R = \frac{40{,}000}{\sqrt{3}(220)(0.9)} \angle -\cos^{-1} 0.9 = 116.6\angle -25.8^\circ \text{ A}$$

For the nominal π equivalent circuit, the ABCD parameters are computed as follows:

$$\begin{aligned}
A = D &= ZY/2 + 1 \\
&= (35 + j140)(930 \times 10^{-6}\angle 90^\circ)/2 + 1 \\
&= 0.935 + j0.0163 = 0.935\angle 1^\circ \\
B = Z &= 35 + j140 = 144.3\angle 76^\circ \ \Omega \\
C &= Y(ZY/4 + 1) \\
&= (930 \times 10^{-6}\angle 90^\circ)[(35 + j140)(930 \times 10^{-6}\angle 90^\circ)/4 + 1] \\
&= (-7.57 + j899.7) \times 10^{-6} = 900 \times 10^{-6}\angle 90.5^\circ \text{ S}
\end{aligned}$$

Thus, the sending-end voltage and current are given by

$$\begin{aligned}
\mathbf{V}_S &= A\mathbf{V}_R + B\mathbf{I}_R \\
&= (0.935\angle 1^\circ)(127{,}000\angle 0^\circ) + (144.3\angle 76^\circ)(116.6\angle -25.8^\circ) \\
&= 130.4\angle 6.6^\circ \text{ kV} \quad \text{(line-to-neutral)} \\
&= 225.8\angle 36.6^\circ \text{ kV} \quad \text{(line-to-line)}
\end{aligned}$$

$$\begin{aligned}
\mathbf{I}_S &= C\mathbf{V}_R + D\mathbf{I}_R \\
&= (900 \times 10^{-6}\angle 90.5^\circ)(127{,}000\angle 0^\circ) + (0.935\angle 1^\circ)(116.6\angle -25.8^\circ) \\
&= 97.97 + j68.57 = 119.6\angle 35^\circ \text{ A}
\end{aligned}$$

The sending-end power factor is

$$PF_S = \cos(6.6^\circ - 35.0^\circ) = 0.88 \text{ leading}$$

At no load, the receiving-end voltage is $1/A$ times the sending-end voltage; thus, the voltage regulation is computed as follows:

$$\begin{aligned}
\text{Voltage regulation} &= \frac{V_S/A - V_{R,fl}}{V_{R,fl}} 100\% \\
&= \frac{225.8/0.935 - 220}{220} 100\% = 9.77\%
\end{aligned}$$

The sending-end real power is

$$\begin{aligned}
P_S = 3V_S I_S PF_S &= 3(130.4 \times 10^3)(119.6)(0.88) \\
&= 41.17 \times 10^6 \text{ W} = 41.17 \text{ MW}
\end{aligned}$$

Therefore, the efficiency of the line is found as follows:

$$\text{Efficiency} = (P_R/P_S)100\% = (40/41.17)100\% = 97.2\%$$