EE 460
Solution of Home Work # 3

13.5 $\Gamma_R = \frac{4-1}{4+1} = 0.6 \qquad \Gamma_S = \frac{\frac{1}{3}-1}{\frac{1}{3}+1} = -0.5$

$$E_G(S) = \frac{E}{S}$$

$$V(x,S) = \frac{E}{S}\left[\frac{1}{\frac{1}{3}+1}\right]\frac{\left[e^{-\frac{Sx}{v}} + 0.6e^{S\left(\frac{x}{v}-2\tau\right)}\right]}{1-(0.6)(-0.5)e^{-2S\tau}}$$

$$V(x,S) = \frac{3E}{4S}\left[\frac{e^{-\frac{Sx}{v}} + 0.6e^{S\left(\frac{x}{v}-2\tau\right)}}{1+0.3e^{-2S\tau}}\right]$$

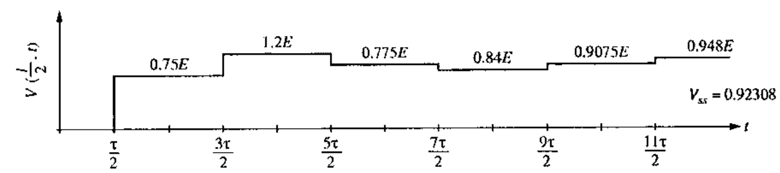
$$V(x,S) = \frac{3E}{4S}\left[e^{-\frac{Sx}{v}} + 0.6e^{S\left(\frac{x}{v}-2\tau\right)}\right]\left[1-0.3e^{-2S\tau}+(0.3)^2e^{-4S\tau}\ldots\right]$$

$$V(x,S) = \frac{3E}{4S}\left[e^{-\frac{Sx}{v}} + 0.6e^{S\left(\frac{x}{v}-2\tau\right)} - 0.3e^{-S\left(\frac{x}{v}+2\tau\right)} - 0.18e^{S\left(\frac{x}{v}-4\tau\right)}\right.$$
$$\left. + 0.09e^{-S\left(\frac{x}{v}+4\tau\right)} + 0.054e^{S\left(\frac{x}{v}-6\tau\right)}\ldots\right]$$

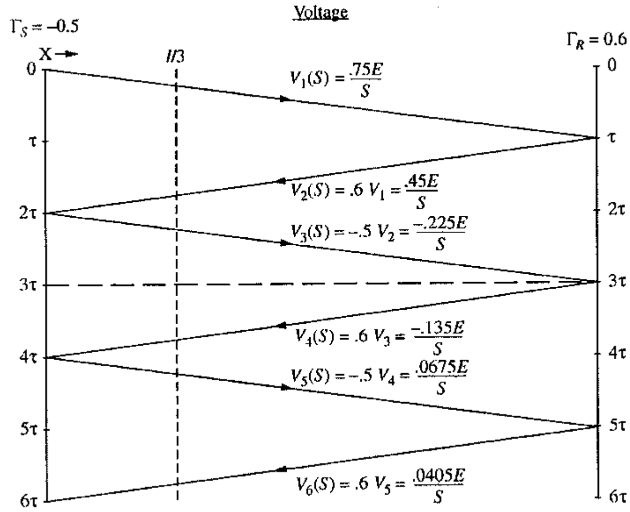
$$V(x,t) = \frac{3E}{4}\left[U_{-1}\left(t-\frac{x}{v}\right) + 0.6U_{-1}\left(t+\frac{x}{v}-2\tau\right)\right.$$
$$-0.3U_{-1}\left(t-\frac{x}{v}-2\tau\right) - 0.18U_{-1}\left(t+\frac{x}{v}-4\tau\right)$$
$$\left.+0.09U_{-1}\left(t-\frac{x}{v}-4\tau\right) + 0.054U_{-1}\left(t+\frac{x}{v}-6\tau\right)\cdots\right]$$

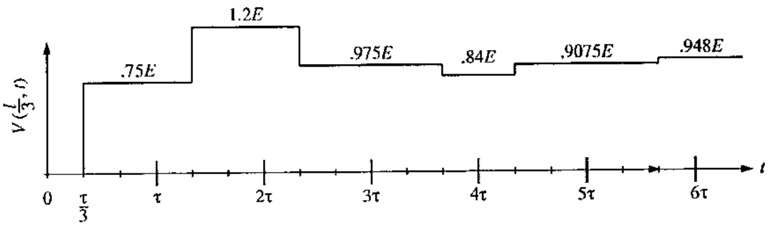
At the center of the line, where $x = \frac{l}{2}$:

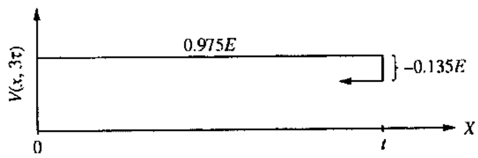
$$V\left(\frac{l}{2},t\right) = \frac{3E}{4}\left[U_{-1}\left(t-\frac{\tau}{2}\right) + 0.6U_{-1}\left(t-\frac{3\tau}{2}\right) - 0.3U_{-1}\left(t-\frac{5}{2}\tau\right)\right.$$
$$\left.-0.18U_{-1}\left(t-\frac{7\tau}{2}\right) + 0.09U_{-1}\left(t-\frac{9\tau}{2}\right) + 0.054U_{-1}\left(t-\frac{11\tau}{2}\right)\ldots\right]$$



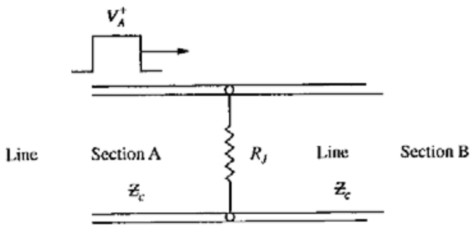
13.9







13.13



For a voltage wave V_A^+ arriving at the junction:

$KVL:\quad V_A^+ + V_A^- = V_B^+ \quad (1)$

$KCL:\quad I_A^+ + I_A^- = I_B^+ + \frac{V_B^+}{R_J}$

$$\frac{V_A^+}{Z_c} - \frac{V_A^-}{Z} = \frac{V_B^+}{Z_c} + \frac{V_B^+}{R_J} = V_B^+\left(\frac{1}{Z_c} + \frac{1}{R_J}\right) = \frac{V_B^+}{Z_{eq}} \qquad (2)$$

where $Z_{eq} = \frac{R_J Z_c}{R_J + Z_c}$

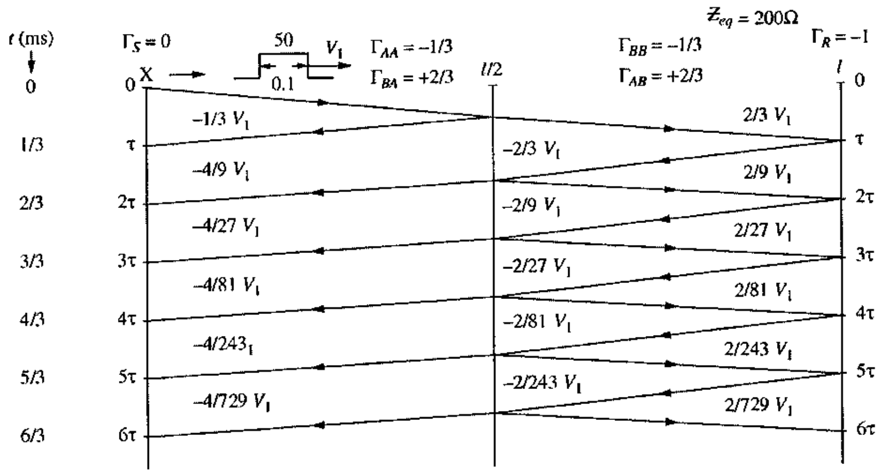
Solving (1) and (2):

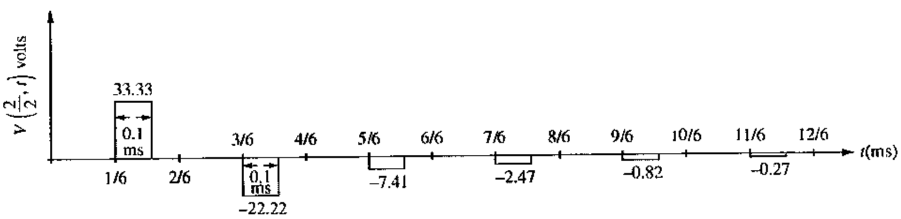
$$V_A^- = \left(\frac{\frac{Z_{eq}}{Z_c} - 1}{\frac{Z_{eq}}{Z_c} + 1}\right) V_A^+ = \Gamma_{AA} V_A^+ \qquad V_B^+ = \left(\frac{2\left(\frac{Z_{eq}}{Z_c}\right)}{\frac{Z_{eq}}{Z_c} + 1}\right) V_A^+ = \Gamma_{BA} V_A^+$$

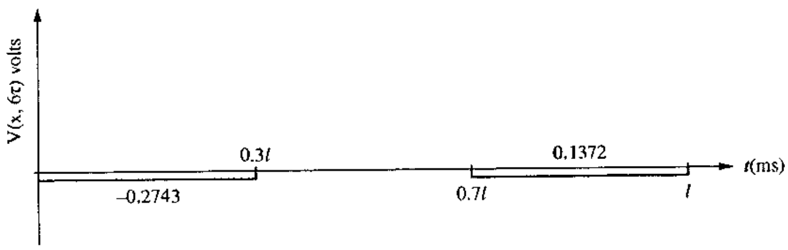
Since Line Sections A and B have the same characteristic impedance Z_c, $\Gamma_{BB} = \Gamma_{AA}$ and $\Gamma_{AB} = \Gamma_{BA}$.

$$\tau = \frac{\ell}{w} = \frac{100\times10^3}{3\times10^8} = \frac{1}{3}\text{ms}$$

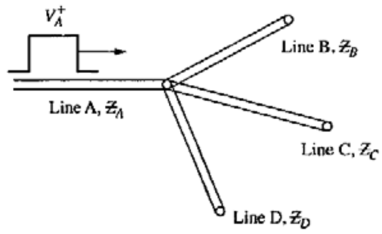
Voltage







13.14 For a voltage wave V_A^+ arriving at the junction from line A,



$$KVL \quad V_A^+ + V_A^- = V_B^+ \qquad (1)$$

$$V_B^+ = V_C^+ \qquad (2)$$

$$V_B^+ = V_D^+ \qquad (3)$$

$$KCL \quad I_A^+ + I_A^- = I_B^+ + I_C^+ + I_D^+$$

$$\frac{V_A^+}{Z_A} - \frac{V_A^-}{Z_A} = \frac{V_B^+}{Z_B} + \frac{V_C^+}{Z_C} + \frac{V_D^+}{Z_D} \qquad (4)$$

Using Eqs (2) and (3) in Eq (4):

$$\frac{V_A^+}{Z_A} - \frac{V_A^-}{Z_A} = V_B^+\left(\frac{1}{Z_B} + \frac{1}{Z_C} + \frac{1}{Z_D}\right) = \frac{V_B^+}{Z_{eq}} \qquad (5)$$

Where $Z_{eq} = Z_B \,//\, Z_C \,//\, Z_D = \dfrac{1}{\dfrac{1}{Z_B} + \dfrac{1}{Z_C} + \dfrac{1}{Z_D}}$

Solving Eqs (1) and (5):

$$V_A^- = \left[\frac{\dfrac{Z_{eq}}{Z_A} - 1}{\left(Z_{eq}/Z_A\right) + 1}\right] V_A^+ = \Gamma_{AA} V_A^+ \qquad V_B^+ = \left[\frac{2\left(Z_{eq}/Z_A\right)}{\left(Z_{eq}/Z_A\right) + 1}\right] V_A^+ = \Gamma_{BA} V_A^+$$

Also $V_C^+ = \Gamma_{CA} V_A^+ \qquad V_D^+ = \Gamma_{DA} V_A^+ \qquad \Gamma_{CA} = \Gamma_{DA} = \Gamma_{BA}$