KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

ELECTRICAL ENGINEERING DEPARTMENT

EE 463 – Term 121

HW # 2: Load Flow

Key Solutions

6.1. A power system network is shown in Figure 47. The generators at buses 1 and 2 are represented by their equivalent current sources with their reactances in per unit on a 100-MVA base. The lines are represented by π model where series reactances and shunt reactances are also expressed in per unit on a 100 MVA base. The loads at buses 3 and 4 are expressed in MW and Mvar.

(a) Assuming a voltage magnitude of 1.0 per unit at buses 3 and 4, convert the loads to per unit impedances. Convert network impedances to admittances and obtain the bus admittance matrix by inspection.

(b) Use the function **Y = ybus(zdata)** to obtain the bus admittance matrix. The function argument **zdata** is a matrix containing the line bus numbers, resistance and reactance. (See Example 6.1.)

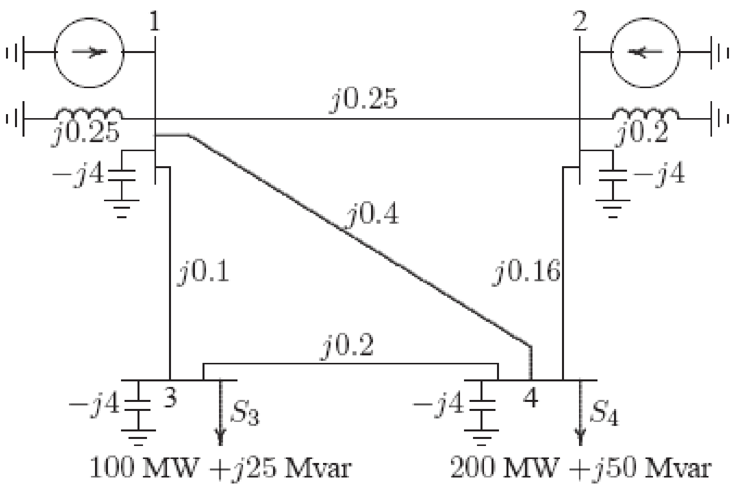

FIGURE 47
One-line diagram for Problem 6.1.

The load impedance in per unit is found from

$$Z = \frac{|V_{L-L}|^2}{S_L^*}\ \Omega \quad \& \quad Z_B = \frac{|V_B|^2}{S_B^*}\ \Omega \quad \text{or} \quad Z = \frac{|V_{pu}|^2}{S_{pu}^*}\ \text{pu}$$

$$Z_3 = \frac{(1.0)^2}{1 - j0.25} = 0.9412 + j0.2353\ \text{pu}$$

$$Z_4 = \frac{(1.0)^2}{2 - j0.5} = 0.4706 + j0.11765\ \text{pu}$$

Converting all impedances to admittances results in the admittance diagram shown in Figure 48

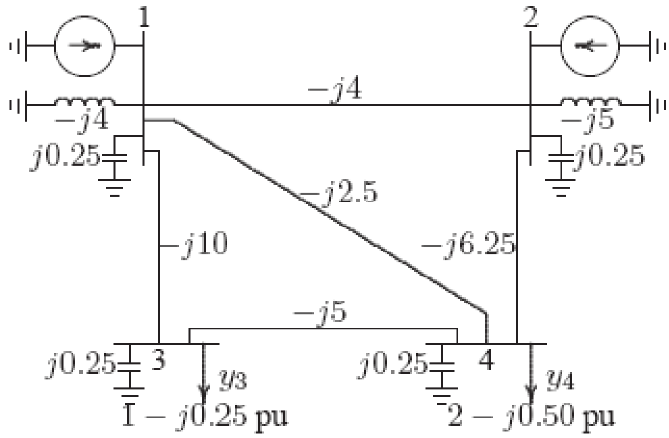

FIGURE 48
The admittance diagram for problem 6.1.

The self admittances are

$$Y_{11} = -j4 + j0.25 - j4 - j10 - j2.5 = -j20.25$$
$$Y_{22} = -j5 + j0.25 - j4 - j6.25 = -j15$$
$$Y_{33} = (1 - j0.25) + j0.25 - j10 - j5 = 1 - j15$$
$$Y_{44} = (2 - j0.5) + j0.25 - j2.5 - j6.25 - j5 = 2 - j14$$

Therefore, the bus admittance matrix is

$$Y_{bus} = \begin{bmatrix} -j20.25 & j4 & j10 & j2.5 \\ j4 & -j15 & 0 & j6.25 \\ j10 & 0 & 1 - j15 & j5 \\ j2.5 & j6.25 & j5 & 2 - j14 \end{bmatrix}$$

From the impedance diagram the following data is constructed for use with the function **Y = ybus(Z)**

```
z = [0   1    0        0.25
     0   1    0       -4.0
     0   2    0        0.2
     0   2    0       -4.0
     0   3    0       -4.0
     0   3    0.9412  0.2353
     0   4    0       -4.0
     0   4    0.4706  0.1176
     1   2    0        0.25
     1   3    0        0.10
     1   4    0        0.40
     2   4    0        0.16
     3   4    0        0.20];
   Y=ybus(z)
```

The result is

```
Y =
     0 -20.25i     0 + 4.00i     0 +10.00i     0 + 2.50i
     0 + 4.00i     0 -15.00i     0             0 + 6.25i
     0 +10.00i     0             1 -15.00i     0 + 5.00i
     0 + 2.50i     0 + 6.25i     0 + 5.00i     2 -14.00i
```

6.7. Figure 6.6 shows the one-line diagram of a simple three-bus power system with generation at bus 1. The voltage at bus 1 is $V_1 = 1.0\angle 0°$ per unit. The scheduled loads on buses 2 and 3 are marked on the diagram. Line impedances are marked in per unit on a 100 MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.
(a) Using Gauss-Seidel method and initial estimates of $V_2^{(0)} = 1.0 + j0$ and $V_3^{(0)} = 1.0 + j0$, determine V_2 and V_3. Perform two iterations.
(b) If after several iterations the bus voltages converge to

$$V_2 = 0.90 - j0.10 \text{ pu}$$
$$V_3 = 0.95 - j0.05 \text{ pu}$$

determine the line flows and line losses and the slack bus real and reactive power. Construct a power flow diagram and show the direction of the line flows.
(c) Check the power flow solution using the **lfgauss** and other required programs. (Refer to Example 6.9.) Use a power accuracy of 0.00001 and an acceleration factor of 1.0.

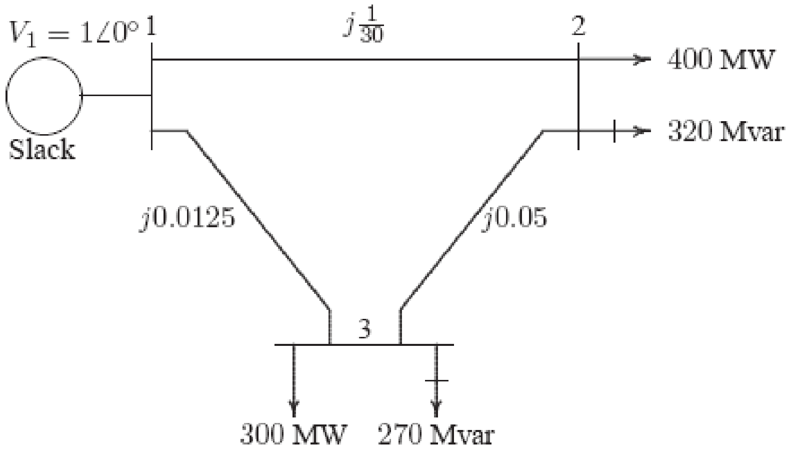

FIGURE 52
One-line diagram for Problem 6.7.

(a) Line impedances are converted to admittances

$$y_{12} = -j30$$
$$y_{13} = \frac{1}{j0.0125} = -j80$$
$$y_{23} = \frac{1}{j0.05} = -j20$$

At the P-Q buses, the complex loads expressed in per units are

$$S_2^{sch} = -\frac{(400 + j320)}{100} = -4.0 - j3.2 \text{ pu}$$
$$S_3^{sch} = -\frac{(300 + j270)}{100} = -3.0 - j2.7 \text{ pu}$$

For hand calculation, we use (6.28). Bus 1 is taken as reference bus (slack bus). Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.0 + j0.0$, V_2 and V_3 are computed from (6.28) as follows

$$\begin{aligned} V_2^{(1)} &= \frac{\frac{S_2^{sch*}}{V_2^{(0)*}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}} \\ &= \frac{\frac{-4.0+j3.2}{1.0-j0} + (-j30)(1.0 + j0) + (-j20)(1.0 + j0)}{-j50} \\ &= 0.936 - j0.08 \end{aligned}$$

and

$$\begin{aligned} V_3^{(1)} &= \frac{\frac{S_3^{sch*}}{V_3^{(0)*}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}} \\ &= \frac{\frac{-3.0+j2.7}{1-j0} + (-j80)(1.0 + j0) + (-j20)(0.936 - j0.08)}{-j100} \\ &= 0.9602 - j0.046 \end{aligned}$$

For the second iteration we have

$$\begin{aligned} V_2^{(2)} &= \frac{\frac{-4.0+j3.2}{0.936+j0.08} + (-j30)(1.0 + j0) + (-j20)(0.9602 - j0.046)}{-j50} \\ &= 0.9089 - j0.0974 \end{aligned}$$

and

$$\begin{aligned} V_3^{(2)} &= \frac{\frac{-3.0+j2.7}{0.9602+j0.046} + (-j80)(1.0 + j0) + (-j20)(0.9089 - j0.0974)}{(-j100)} \\ &= 0.9522 - j0.0493 \end{aligned}$$

The process is continued and a solution is converged with an accuracy of 5×10^{-5} per unit in seven iterations as given below.

$$
\begin{aligned}
V_2^{(3)} &= 0.9020 - j0.0993 & V_3^{(3)} &= 0.9505 - j0.0498 \\
V_2^{(4)} &= 0.9004 - j0.0998 & V_3^{(4)} &= 0.9501 - j0.0500 \\
V_2^{(5)} &= 0.9001 - j0.1000 & V_3^{(5)} &= 0.9500 - j0.0500 \\
V_2^{(6)} &= 0.9000 - j0.1000 & V_3^{(6)} &= 0.9500 - j0.0500 \\
V_2^{(7)} &= 0.9000 - j0.1000 & V_3^{(7)} &= 0.9500 - j0.0500
\end{aligned}
$$

The final solution is

$$
\begin{aligned}
V_2 &= 0.90 - j0.10 = 0.905554\angle{-6.34^\circ} \quad \text{pu} \\
V_3 &= 0.95 - j0.05 = 0.9513\angle{-3.0128^\circ} \quad \text{pu}
\end{aligned}
$$

(b) With the knowledge of all bus voltages, the slack bus power is obtained from (6.27)

$$
\begin{aligned}
P_1 - jQ_1 &= V_1^*[V_1(y_{12} + y_{13}) - (y_{12}V_2 + y_{13}V_3)] \\
&= 1.0[1.0(-j30 - j80) - (-j30)(0.9 - j0.1) - \\
&\quad (-j80)(0.95 - j0.05)] \\
&= 7.0 - j7.0
\end{aligned}
$$

or the slack bus real and reactive powers are $P_1 = 7.0$ pu = 700 MW and $Q_1 = 7.0$ pu $= 700$ Mvar.

To find the line flows, first the line currents are computed. With line charging capacitors neglected, the line currents are

$$
\begin{aligned}
I_{12} &= y_{12}(V_1 - V_2) = (-j30)[(1.0 + j0) - (0.90 - j0.10)] = 3.0 - j3.0 \\
I_{21} &= -I_{12} = -3.0 + j3.0 \\
I_{13} &= y_{13}(V_1 - V_3) = (-j80)[(1.0 + j0) - (0.95 - j.05)] = 4.0 - j4.0 \\
I_{31} &= -I_{13} = -4.0 + j4.0 \\
I_{23} &= y_{23}(V_2 - V_3) = (-j20)[(0.90 - j0.10) - (0.95 - j.05)] = -1.0 + j1.0 \\
I_{32} &= -I_{23} = 1.0 - j1.0
\end{aligned}
$$

The line flows are

$$
\begin{aligned}
S_{12} = V_1 I_{12}^* &= (1.0 + j0.0)(3.0 + j3) = 3.0 + j3.0 \ \text{pu} \\
&= 300 \ \text{MW} + j300 \ \text{Mvar}
\end{aligned}
$$

$$
\begin{aligned}
S_{21} = V_2 I_{21}^* &= (0.90 - j0.10)(-3 - j3) = -3.0 - j2.4 \ \text{pu} \\
&= -300 \ \text{MW} - j240 \ \text{Mvar} \\
S_{13} = V_1 I_{13}^* &= (1.0 + j0.0)(4.0 + j4.0) = 4.0 + j4.0 \ \text{pu} \\
&= 400 \ \text{MW} + j400 \ \text{Mvar} \\
S_{31} = V_3 I_{31}^* &= (0.95 - j0.05)(-4.0 - j4.0) = -4.0 - j3.6 \ \text{pu} \\
&= -400 \ \text{MW} - j360 \ \text{Mvar} \\
S_{23} = V_2 I_{23}^* &= (0.90 - j0.10)(-1.0 - j1.0) = -1.0 - j0.80 \ \text{pu} \\
&= -100 \ \text{MW} - j80 \ \text{Mvar} \\
S_{32} = V_3 I_{32}^* &= (0.95 - j0.05)(1 + j1) = 1.0 + j0.9 \ \text{pu} \\
&= 100 \ \text{MW} + j90 \ \text{Mvar}
\end{aligned}
$$

and the line losses are

$$
\begin{aligned}
S_{L\,12} &= S_{12} + S_{21} = 0.0 \ \text{MW} + j60 \ \text{Mvar} \\
S_{L\,13} &= S_{13} + S_{31} = 0.0 \ \text{MW} + j40 \ \text{Mvar} \\
S_{L\,23} &= S_{23} + S_{32} = 0.0 \ \text{MW} + j10 \ \text{Mvar}
\end{aligned}
$$

The power flow diagram is shown in Figure 6.7, where real power direction is indicated by $\rightarrow$ and the reactive power direction is indicated by $\mapsto$. The values within parentheses are the real and reactive losses in the line.

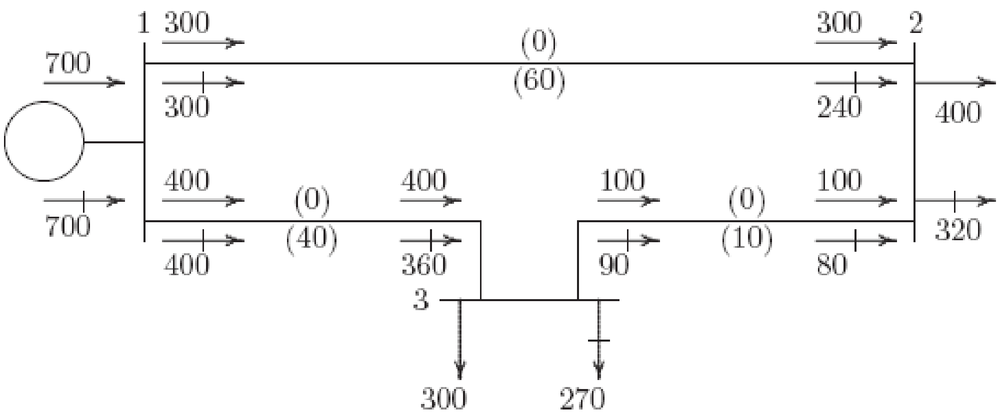

FIGURE 53
Power flow diagram of Problem 6.7 (powers in MW and Mvar).

(c) The power flow program **lfgauss** is used to obtain the solution, with the following statements:

```
clear
basemva = 100;  accuracy = 0.000001; accel = 1.1; maxiter = 100;
```

```
%      Problem   6.7(c)
%      Bus  Bus Voltage Angle -Load---   -Generator-- Injected
%       No code Mag. Degree MW  MVAR  MW  MVAR Qmin Qmax  Mvar
busdata=[1   1  1.0   0.0  0.0  0.0   0.0  0.0   0   0    0
         2   0  1.0   0.0  400  320   0.0  0.0   0   0    0
         3   0  1.0   0.0  300  270   0.0  0.0   0   0    0];

%                                   Line code
%        Bus bus   R    X   1/2 B   = 1 for lines
%         nl  nr pu    pu     pu    >1 or <1 tr. tap at bus nl
linedata=[1   2  0.0  1/30    0.0       1
          1   3  0.0  0.0125  0.0       1
          2   3  0.0  0.050   0.0       1];
disp('Problem 6.7(c)')
lfybus                        % form the bus admittance matrix
lfgauss            % Load flow solution by Gauss-Seidel method
busout          % Prints the power flow solution on the screen
lineflow      % Computes and displays the line flow and losses
```

The above statements are saved in the file **ch6p7c.m**. Run the program to obtain the solution.

6.12. Figure 60 shows the one-line diagram of a simple three-bus power system with generation at buses 1 and 2. The voltage at bus 1 is $V = 1.0\angle 0°$ per unit. Voltage magnitude at bus 2 is fixed at 1.05 pu with a real power generation of 400 MW. A load consisting of 500 MW and 400 Mvar is taken from bus 3. Line admittances are marked in per unit on a 100 MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.

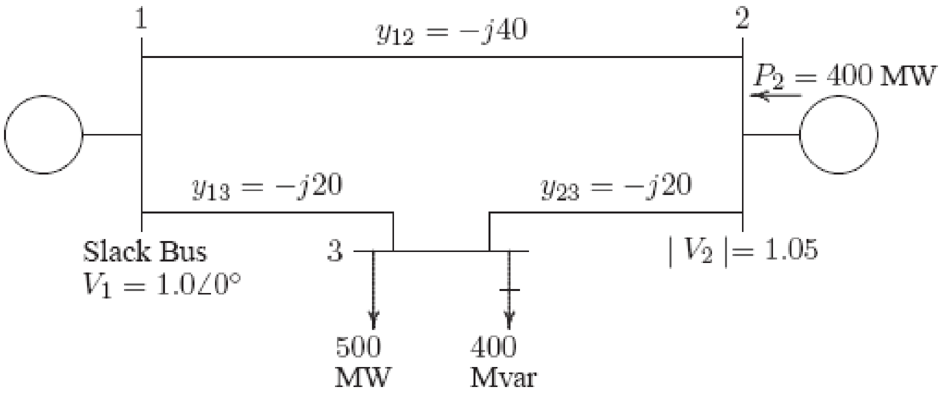

FIGURE 60
One-line diagram for problem 6.12.

(a) Show that the expression for the real power at bus 2 and real and reactive power at bus 3 are

$$P_2 = 40|V_2||V_1|\cos(90° - \delta_2 + \delta_1) + 20|V_2||V_3|\cos(90° - \delta_2 + \delta_3)$$
$$P_3 = 20|V_3||V_1|\cos(90° - \delta_3 + \delta_1) + 20|V_3||V_2|\cos(90° - \delta_3 + \delta_2)$$
$$Q_3 = -20|V_3||V_1|\sin(90° - \delta_3 + \delta_1) - 20|V_3||V_2|\sin(90° - \delta_3 + \delta_2) + 40|V_3|^2$$

(b) Using Newton-Raphson method, start with the initial estimates of $V_2^{(0)} = 1.05 + j0$ and $V_3^{(0)} = 1.0 + j0$, and keeping $|V_2| = 1.05$ pu, determine the

phasor values of V_2 and V_3. Perform two iterations.

(c) Check the power flow solution for Problem 6.12 using the **lfnewton** and other required programs. Assume the regulated bus (bus # 2) reactive power limits are between 0 and 600 Mvar.

By inspection, the bus admittance matrix in polar form is

$$Y_{bus} = \begin{bmatrix} 60\angle -\frac{\pi}{2} & 40\angle \frac{\pi}{2} & 20\angle \frac{\pi}{2} \\ 40\angle \frac{\pi}{2} & 60\angle -\frac{\pi}{2} & 20\angle \frac{\pi}{2} \\ 20\angle \frac{\pi}{2} & 20\angle \frac{\pi}{2} & 40\angle -\frac{\pi}{2} \end{bmatrix}$$

(a) The power flow equation with voltages and admittances expressed in polar form is

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$
$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

Substituting the elements of the bus admittance matrix in the above equations for P_2, P_3, and Q_3 will result in the given equations.

(b) Elements of the Jacobian matrix are obtained by taking partial derivatives of the given equations with respect to δ_2, δ_3 and $|V_3|$.

$$\frac{\partial P_2}{\partial \delta_2} = 40|V_2||V_1|\sin(\frac{\pi}{2} - \delta_2 + \delta_1) + 20|V_2||V_3|\sin(\frac{\pi}{2} - \delta_2 + \delta_3)$$
$$\frac{\partial P_2}{\partial \delta_3} = -20|V_2||V_3|\sin(\frac{\pi}{2} - \delta_2 + \delta_3)$$
$$\frac{\partial P_2}{\partial |V_3|} = 20|V_2|\cos(\frac{\pi}{2} - \delta_2 + \delta_3)$$
$$\frac{\partial P_3}{\partial \delta_2} = -20|V_3||V_2|\sin(\frac{\pi}{2} - \delta_3 + \delta_2)$$
$$\frac{\partial P_3}{\partial \delta_3} = 20|V_3||V_1|\sin(\frac{\pi}{2} - \delta_3 + \delta_1) + 20|V_3||V_2|\sin(\frac{\pi}{2} - \delta_3 + \delta_2)$$
$$\frac{\partial P_3}{\partial |V_3|} = 20|V_1|\cos(\frac{\pi}{2} - \delta_3 + \delta_1) + 20|V_2|\cos(\frac{\pi}{2} - \delta_3 + \delta_2)$$
$$\frac{\partial Q_3}{\partial \delta_2} = -20|V_3||V_2|\cos(\frac{\pi}{2} - \delta_3 + \delta_2)$$
$$\frac{\partial Q_3}{\partial \delta_3} = 20|V_3||V_1|\cos(\frac{\pi}{2} - \delta_3 + \delta_1) + 20|V_3||V_2|\cos(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_3}{\partial |V_3|} = -20|V_1|\sin(\frac{\pi}{2} - \delta_3 + \delta_1) - 20|V_2|\sin(\frac{\pi}{2} - \delta_3 + \delta_2) + 80|V_3|$$

The load and generation expressed in per units are

$$P_2^{sch} = \frac{400}{100} = 4.0 \text{ pu}$$
$$S_3^{sch} = -\frac{(500 + j400)}{100} = -5.0 - j4.0 \text{ pu}$$

The slack bus voltage is $V_1 = 1.0\angle 0$ pu, and the bus 2 voltage magnitude is $|V_2| = 1.05$ pu. Starting with an initial estimate of $|V_3^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, and $\delta_3^{(0)} = 0.0$, the power residuals are

$$\begin{aligned}
\Delta P_2^{(0)} &= P_2^{sch} - P_2^{(0)} = 4.0 - (0) = 4.0 \\
\Delta P_3^{(0)} &= P_3^{sch} - P_3^{(0)} = -5.0 - (0) = -5.0 \\
\Delta Q_3^{(0)} &= Q_3^{sch} - Q_3^{(0)} = -4.0 - (-1.0) = -3.0
\end{aligned}$$

Evaluating the elements of the Jacobian matrix with the initial estimate, the set of linear equations in the first iteration becomes

$$\begin{bmatrix} -2.8600 \\ 1.4384 \\ -0.2200 \end{bmatrix} = \begin{bmatrix} 63 & -21 & 0 \\ -21 & 41 & 0 \\ 0 & 0 & 39 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta|V_3^{(0)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, the new bus voltages in the first iteration are

$$\begin{aligned}
\Delta\delta_2^{(0)} &= 0.0275 \qquad \delta_2^{(1)} = 0 + 0.0275 = 0.0275 \text{ radian} = 1.5782^\circ \\
\Delta\delta_3^{(0)} &= -0.1078 \qquad \delta_3^{(1)} = 0 + (-0.1078) = -0.1078 \text{ radian} = -6.1790^\circ \\
\Delta|V_3^{(0)}| &= -0.0769 \qquad |V_3^{(1)}| = 1 + (-0.0769) = 0.9231 \text{ pu}
\end{aligned}$$

For the second iteration, we have

$$\begin{bmatrix} 0.2269 \\ -0.3965 \\ -0.5213 \end{bmatrix} = \begin{bmatrix} 61.1913 & -19.2072 & 2.8345 \\ -19.2072 & 37.5615 & -4.9871 \\ 2.6164 & -4.6035 & 33.1545 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta\delta_3^{(1)} \\ \Delta|V_3^{(1)}| \end{bmatrix}$$

and

$$\begin{aligned}
\Delta\delta_2^{(1)} &= 0.0006 \qquad \delta_2^{(2)} = 0.0275 + 0.0006 = 0.0281 \text{ radian} = 1.61^\circ \\
\Delta\delta_3^{(1)} &= -0.0126 \qquad \delta_3^{(2)} = -0.1078 + (-0.0126) = -0.1204 \text{ radian} = -6.898^\circ \\
\Delta|V_3^{(1)}| &= -0.0175 \qquad |V_3^{(2)}| = 0.9231 + (-0.0175) = 0.9056 \text{ pu}
\end{aligned}$$

(c) The power flow program **lfnewton** is used to obtain the solution, with the following statements:

```
clear
basemva = 100;  accuracy = 0.000001;  maxiter = 10;

%      Problem   6.12(c)
%      Bus  Bus Voltage Angle -Load---   -Generator-- Injected
%       No code Mag. Degree MW  MVAR  MW  MVAR Qmin Qmax  Mvar
busdata=[1   1  1.0   0.0  0.0  0.0   0.0  0.0   0   0    0
         2   2  1.05  0.0    0    0   400  0.0  600  0    0
         3   0  1.0   0.0  500  400   0.0  0.0   0   0    0];

%                                    Line code
%        Bus bus  R    X    1/2 B   = 1 for lines
%         nl  nr pu   pu      pu    >1 or <1 tr. tap at bus nl
linedata=[1   2  0.0  0.025   0.0       1
          1   3  0.0  0.05    0.0       1
          2   3  0.0  0.05    0.0       1];
disp('Problem 6.12(c)')
lfybus                          % form the bus admittance matrix
lfnewton          % Power flow solution by Gauss-Seidel method
busout          % Prints the power flow solution on the screen
lineflow      % Computes and displays the line flow and losses
```

The above statements are saved in the file **ch6p12c.m**. Run the program to obtain the solution.