Home Work #4: Optimal Dispatch of Generation

7.5. Find the minimum value of the function

$$f(x, y) = x^2 + y^2$$

subject to the equality constraint

$$g(x, y) = x^2 - 6x - y^2 + 17 = 0$$

Forming the Lagrangian function, we obtain

$$\mathcal{L} = x^2 + y^2 + \lambda(x^2 - 6x - y^2 + 17)$$

The resulting necessary conditions for constrained local maxima of $\mathcal{L}$ are

$$\frac{\partial \mathcal{L}}{\partial x} = 2x + \lambda(2x - 6) = 0$$
$$\frac{\partial \mathcal{L}}{\partial y} = 2y - 2\lambda y = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = x^2 - 6x - y^2 + 17 = 0$$

From the second condition $\lambda = 1$. Substituting in the first condition

$$2x + (1)(2x - 6) = 0 \quad \text{or} \quad x = 1.5$$

Substituting for x in the third condition results in $y = 3.20156$. Thus the minimum value of the function is

$$f(\hat{x}, \hat{y}) = (1.5)^2 + (3.20156)^2 = 12.5$$

7.7. The fuel-cost functions in $/h for two 800 MW thermal plants are given by

$$C_1 = 400 + 6.0P_1 + 0.004P_1^2$$
$$C_2 = 500 + \ \beta P_2 + \quad \gamma P_2^2$$

where P_1 and P_2 are in MW.

(a) The incremental cost of power λ is $8/MWh when the total power demand is 550 MW. Neglecting losses, determine the optimal generation of each plant.

(b) The incremental cost of power λ is $10/MWh when the total power demand is 1300 MW. Neglecting losses, determine the optimal generation of each plant.
(c) From the results of (a) and (b) find the fuel-cost coefficients β and γ of the second plant.

$$\frac{dC_1}{dP_1} = 6 + 0.008P_1 = \lambda$$
$$\frac{dC_2}{dP_2} = \beta + 2\gamma P_2 = \lambda$$

(a) For $\lambda = 8$, and $P_D = 550$ MW, we have

$$P_1 = \frac{8-6}{0.008} = 250 \text{ MW}$$
$$P_2 = P_D - P_1 = 500 - 250 = 300 \text{ MW}$$

(b) For $\lambda = 10$, and $P_D = 1300$ MW, we have

$$P_1 = \frac{10-6}{0.008} = 500 \text{ MW}$$
$$P_2 = P_D - P_1 = 1300 - 500 = 800 \text{ MW}$$

(c) The incremental cost of power for plant 2 are given by

$$\beta + 2\gamma(300) = 8$$
$$\beta + 2\gamma(800) = 10$$

Solving the above equations, we find $\beta = 6.8$, and $\gamma = 0.002$

7.8. The fuel-cost functions in \$/h for three thermal plants are given by

$$C_1 = 350 + 7.20P_1 + 0.0040P_1^2$$
$$C_2 = 500 + 7.30P_2 + 0.0025P_2^2$$
$$C_3 = 600 + 6.74P_3 + 0.0030P_3^2$$

where P_1, P_2, and P_3 are in MW. The governors are set such that generators share the load equally. Neglecting line losses and generator limits, find the total cost in \$/h when the total load is

(i) $P_D = 450$ MW
(ii) $P_D = 745$ MW
(iii) $P_D = 1335$ MW

(i) For $P_D = 450$ MW, $P_1 = P_2 = P_3 = \frac{450}{3} = 150$ MW. The total fuel cost is

$$C_t = 350 + 7.20(150) + 0.004(150)^2 + 500 + 7.3(150) + 0.0025(150)^2 + 600 + 6.74(150) + 0.003(150)^2 = 4,849.75 \text{ \$/h}$$

(ii) For $P_D = 745$ MW, $P_1 = P_2 = P_3 = \frac{745}{3}$ MW. The total fuel cost is

$$C_t = 350 + 7.20\left(\frac{745}{3}\right) + 0.004\left(\frac{745}{3}\right)^2 + 500 + 7.3\left(\frac{745}{3}\right) + 0.0025\left(\frac{745}{3}\right)^2 + 600 + 6.74\left(\frac{745}{3}\right) + 0.003\left(\frac{745}{3}\right)^2 = 7,310.46 \text{ \$/h}$$

(iii) For $P_D = 1335$ MW, $P_1 = P_2 = P_3 = 445$ MW. The total fuel cost is

$$C_t = 350 + 7.20(445) + 0.004(445)^2 + 500 + 7.3(445) + 0.0025(445)^2 + 600 + 6.74(445) + 0.003(445)^2 = 12,783.04 \text{ \$/h}$$

7.11. The fuel-cost function in \$/h of two thermal plants are

$$C_1 = 320 + 6.2P_1 + 0.004P_1^2$$
$$C_2 = 200 + 6.0P_2 + 0.003P_2^2$$

where P_1 and P_2 are in MW. Plant outputs are subject to the following limits (in MW)

$$50 \le P_1 \le 250$$
$$50 \le P_2 \le 350$$

The per-unit system real power loss with generation expressed in per unit on a 100-MVA base is given by

$$P_{L(pu)} = 0.0125P_{1(pu)}^2 + 0.00625P_{2(pu)}^2$$

The total load is 412.35 MW. Determine the optimal dispatch of generation. Start with an initial estimate of $\lambda = 7$ \$/MWh. Use the **dispatch** program to check your results.

In the cost function P_i is expressed in MW. Therefore, the real power loss in terms of MW generation is

$$P_L = \left[0.0125\left(\frac{P_1}{100}\right)^2 + 0.00625\left(\frac{P_2}{100}\right)^2\right] \times 100 \ \ \text{MW}$$
$$= 0.000125P_1^2 + 0.0000625P_2^2 \ \ \text{MW}$$

For the numerical solution using the gradient method, assume the initial value of $\lambda^{(1)} = 7.0$. From coordination equations, given by (7.70), $P_1^{(1)}$, and $P_2^{(1)}$ are

$$P_1^{(1)} = \frac{7.0 - 6.2}{2(0.004 + 7.0 \times 0.000125)} = 82.05128 \ \ \text{MW}$$
$$P_2^{(1)} = \frac{7.0 - 6.0}{2(0.003 + 7.0 \times 0.0000625)} = 145.4545 \ \ \text{MW}$$

The real power loss is

$$P_L^{(1)} = 0.000125(82.05228)^2 + 0.0000625(145.4545)^2 = 2.16386$$

Since $P_D = 412.35$ MW, the error $\Delta P^{(1)}$ from (7.68) is

$$\Delta P^{(1)} = 412.35 + 2.16386 - (82.05128 + 145.4545) = 187.008$$

From (7.71)

$$\sum_{i=1}^{2}\left(\frac{\partial P_i}{\partial \lambda}\right)^{(1)} = \frac{0.004 + 0.000125 \times 6.2}{2(0.004 + 7.0 \times 0.000125)^2} + \frac{0.003 + 0.0000625 \times 6.0}{2(0.003 + 7.0 \times 0.0000625)^2}$$
$$= 243.2701$$

From (7.65)

$$\Delta\lambda^{(1)} = \frac{187.008}{243.2701} = 0.7687$$

Therefore, the new value of λ is

$$\lambda^{(2)} = 7.0 + 0.7687 = 7.7687$$

Continuing the process, for the second iteration, we have

$$P_1^{(2)} = \frac{7.7687 - 6.2}{2(0.004 + 7.7687 \times 0.000125)} = 157.7848 \quad \text{MW}$$
$$P_2^{(2)} = \frac{7.7687 - 6.0}{2(0.003 + 7.7687 \times 0.0000625)} = 253.7229 \ \ \text{MW}$$

The real power loss is

$$P_L^{(2)} = 0.000125(157.7848)^2 + 0.0000625(253.7229)^2 = 7.1355$$

Since $P_D = 412.35$ MW, the error $\Delta P^{(2)}$ from (7.68) is

$$\Delta P^{(2)} = 412.35 + 7.1355 - (157.7848 + 253.7229) = 7.9778$$

From (7.71)

$$\sum_{i=1}^{2} \left(\frac{\partial P_i}{\partial \lambda}\right)^{(2)} = \frac{0.004 + 0.000125 \times 6.2}{2(0.004 + 7.7687 \times 0.000125)^2} + \frac{0.003 + 0.0000625 \times 6.0}{2(0.003 + 7.7687 \times 0.0000625)^2}$$
$$= 235.514$$

From (7.65)

$$\Delta\lambda^{(2)} = \frac{7.9778}{235.514} = 0.0339$$

Therefore, the new value of λ is

$$\lambda^{(2)} = 7.7687 + 0.0339 = 7.8026$$

For the third iteration, we have

$$P_1^{(3)} = \frac{7.8026 - 6.2}{2(0.004 + 7.8026 \times 0.000125)} = 161.0548 \quad \text{MW}$$
$$P_2^{(3)} = \frac{7.8026 - 6.0}{2(0.003 + 7.8026 \times 0.0000625)} = 258.4252 \ \ \text{MW}$$

The real power loss is

$$P_L^{(3)} = 0.000125(161.0548)^2 + 0.0000625(258.4252)^2 = 7.4163$$

Since $P_D = 412.35$ MW, the error $\Delta P^{(3)}$ from (7.68) is

$$\Delta P^{(3)} = 412.35 + 7.4163 - (161.0548 + 258.4252) = 0.2863$$

From (7.71)

$$\sum_{i=1}^{2} \left(\frac{\partial P_i}{\partial \lambda}\right)^{(3)} = \frac{0.004 + 0.000125 \times 6.2}{2(0.004 + 7.8026 \times 0.000125)^2} + \frac{0.003 + 0.0000625 \times 6.0}{2(0.003 + 7.8026 \times 0.0000625)^2}$$
$$= 235.18$$

From (7.65)

$$\Delta\lambda^{(3)} = \frac{0.2683}{235.18} = 0.0012$$

Therefore, the new value of λ is

$$\lambda^{(3)} = 7.8026 + 0.0011 = 7.8038$$

Continuing the process, the optimal dispatch after six iteration converges to $P_1 =$ 161.1765 MW, and $P_2 =$ 258.6003 MW, with an incremental production cost of $\lambda = 7.8038$\$/MWh.