Solution of Home Work #3

6.5. Use Newton-Raphson method and hand calculation to find the solution of the following equations:

$$x_1^2 - 2x_1 - x_2 = 3$$
$$x_1^2 + x_2^2 = 41$$

(a) Start with the initial estimates of $x_1^{(0)} = 2$, $x_2^{(0)} = 3$. Perform three iterations.
(b) Write a *MATLAB* program to find one of the solutions of the above equations by Newton-Raphson method. The program should prompt the user to input the initial estimates. Run the program with the above initial estimates.

Taking partial derivatives of the above equations results in the Jacobian matrix

$$J = \begin{bmatrix} 2x_1 - 2 & -1 \\ 2x_1 & 2x_2 \end{bmatrix}$$

(a) Starting with initial estimates $x_1^{(0)} = 2$, and $x_2^{(0)} = 3$, the analytical solution given by the Newton-Raphson method is

$$\Delta C^{(0)} = \begin{bmatrix} 3 - [(2)^2 - 2(2) - 3] \\ 41 - [(2)^2 + (3)^2] \end{bmatrix} = \begin{bmatrix} 6 \\ 28 \end{bmatrix}$$

$$J = \begin{bmatrix} 2(2) - 2 & -1 \\ 2(2) & 2(3) \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 4 & 6 \end{bmatrix}$$

$$\Delta X^{(0)} = \begin{bmatrix} 2 & -1 \\ 4 & 6 \end{bmatrix}^{-1} \begin{bmatrix} 6 \\ 28 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$X^{(0)} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

For the second iteration, we have

$$\Delta C^{(1)} = \begin{bmatrix} 3 - [(6)^2 - 2(6) - 5] \\ 41 - [(6)^2 + (5)^2] \end{bmatrix} = \begin{bmatrix} -16 \\ -20 \end{bmatrix}$$

$$J^{(1)} = \begin{bmatrix} 2(6) - 2 & -1 \\ 2(6) & 2(5) \end{bmatrix} = \begin{bmatrix} 10 & -1 \\ 12 & 10 \end{bmatrix}$$

$$\Delta X^{(1)} = \begin{bmatrix} 10 & -1 \\ 12 & 10 \end{bmatrix}^{-1} \begin{bmatrix} -16 \\ -20 \end{bmatrix} = \begin{bmatrix} -1.6071 \\ -0.0714 \end{bmatrix}$$

$$X^{(2)} = \begin{bmatrix} 6 \\ 5 \end{bmatrix} + \begin{bmatrix} -1.6071 \\ -0.0714 \end{bmatrix} = \begin{bmatrix} 4.3929 \\ 4.9286 \end{bmatrix}$$

and for the third iteration, we have

$$\Delta C^{(2)} = \begin{bmatrix} 3 - [(4.3929)^2 - 2(4.3929) - 4.9286] \\ 41 - [(4.3929)^2 + (4.9286)^2] \end{bmatrix} = \begin{bmatrix} -2.5829 \\ -2.5880 \end{bmatrix}$$

$$J^{(2)} = \begin{bmatrix} 2(4.3929) - 2 & -1 \\ 2(4.3929) & 2(4.9286) \end{bmatrix} = \begin{bmatrix} 6.7857 & -1.0000 \\ 8.7857 & 10.000 \end{bmatrix}$$

$$\Delta X^{(1)} = \begin{bmatrix} 6.7857 & -1.0000 \\ 8.7857 & 10.000 \end{bmatrix}^{-1} \begin{bmatrix} -2.5829 \\ -2.5880 \end{bmatrix} = \begin{bmatrix} -0.3706 \\ 0.0678 \end{bmatrix}$$

$$X^{(3)} = \begin{bmatrix} 4.3929 \\ 4.9286 \end{bmatrix} + \begin{bmatrix} -0.3706 \\ 0.0678 \end{bmatrix} = \begin{bmatrix} 4.0222 \\ 4.9964 \end{bmatrix}$$

The summary of the iterations is

Iter	DC	Jacobin matrix		Dx	x
1	6	2	-1	4	6
	28	4	6	2	5
2	-16.0000	10.0000	-1.0000	-1.6071	4.3929
	-20.0000	12.0000	10.0000	-0.0714	4.9286
3	-2.5829	6.7857	-1.0000	-0.3706	4.0222
	-2.5880	8.7857	9.8571	0.0678	4.9964
4	-0.1374	6.0444	-1.0000	-0.0221	4.0001
	-0.1420	8.0444	9.9928	0.0036	5.0000
5	-0.0005	6.0002	-1.0000	-0.0001	4.0000
	-0.0005	8.0002	10.0000	0.0000	5.0000

6.12. Figure 60 shows the one-line diagram of a simple three-bus power system with generation at buses 1 and 2. The voltage at bus 1 is $V = 1.0\angle 0^\circ$ per unit. Voltage magnitude at bus 2 is fixed at 1.05 pu with a real power generation of 400 MW. A load consisting of 500 MW and 400 Mvar is taken from bus 3. Line admittances are marked in per unit on a 100 MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.

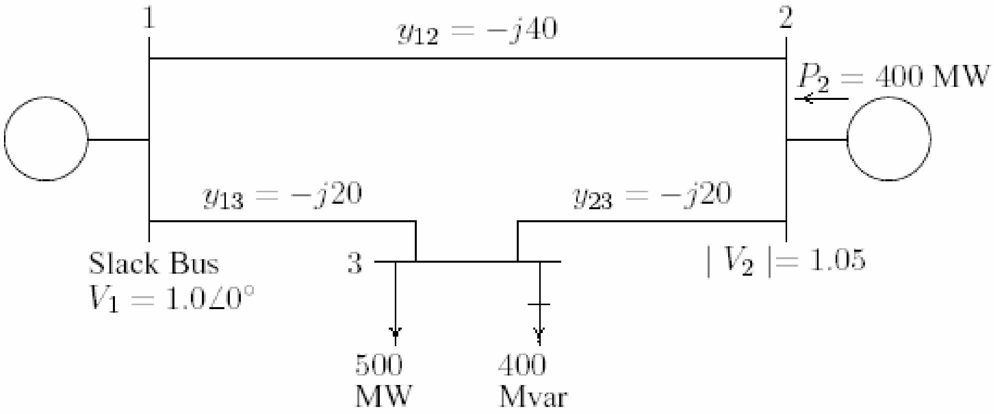

FIGURE 60
One-line diagram for problem 6.12.

(a) Show that the expression for the real power at bus 2 and real and reactive power at bus 3 are

$$P_2 = 40|V_2||V_1|\cos(90^\circ - \delta_2 + \delta_1) + 20|V_2||V_3|\cos(90^\circ - \delta_2 + \delta_3)$$
$$P_3 = 20|V_3||V_1|\cos(90^\circ - \delta_3 + \delta_1) + 20|V_3||V_2|\cos(90^\circ - \delta_3 + \delta_2)$$
$$Q_3 = -20|V_3||V_1|\sin(90^\circ - \delta_3 + \delta_1) - 20|V_3||V_2|\sin(90^\circ - \delta_3 + \delta_2) + 40|V_3|^2$$

(b) Using Newton-Raphson method, start with the initial estimates of $V_2^{(0)} = 1.05 + j0$ and $V_3^{(0)} = 1.0 + j0$, and keeping $|V_2| = 1.05$ pu, determine the phasor values of V_2 and V_3. Perform two iterations.

By inspection, the bus admittance matrix in polar form is

$$Y_{bus} = \begin{bmatrix} 60\angle -\frac{\pi}{2} & 40\angle \frac{\pi}{2} & 20\angle \frac{\pi}{2} \\ 40\angle \frac{\pi}{2} & 60\angle -\frac{\pi}{2} & 20\angle \frac{\pi}{2} \\ 20\angle \frac{\pi}{2} & 20\angle \frac{\pi}{2} & 40\angle -\frac{\pi}{2} \end{bmatrix}$$

(a) The power flow equation with voltages and admittances expressed in polar form is

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

Substituting the elements of the bus admittance matrix in the above equations for P_2, P_3, and Q_3 will result in the given equations.

(b) Elements of the Jacobian matrix are obtained by taking partial derivatives of the given equations with respect to δ_2, δ_3 and $|V_3|$.

$$\frac{\partial P_2}{\partial \delta_2} = 40|V_2||V_1|\sin(\frac{\pi}{2} - \delta_2 + \delta_1) + 20|V_2||V_3|\sin(\frac{\pi}{2} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial \delta_3} = -20|V_2||V_3|\sin(\frac{\pi}{2} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial |V_3|} = 20|V_2|\cos(\frac{\pi}{2} - \delta_2 + \delta_3)$$

$$\frac{\partial P_3}{\partial \delta_2} = -20|V_3||V_2|\sin(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial \delta_3} = 20|V_3||V_1|\sin(\frac{\pi}{2} - \delta_3 + \delta_1) + 20|V_3||V_2|\sin(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial |V_3|} = 20|V_1|\cos(\frac{\pi}{2} - \delta_3 + \delta_1) + 20|V_2|\cos(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_3}{\partial \delta_2} = -20|V_3||V_2|\cos(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_3}{\partial \delta_3} = 20|V_3||V_1|\cos(\frac{\pi}{2} - \delta_3 + \delta_1) + 20|V_3||V_2|\cos(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_3}{\partial |V_3|} = -20|V_1|\sin(\frac{\pi}{2} - \delta_3 + \delta_1) - 20|V_2|\sin(\frac{\pi}{2} - \delta_3 + \delta_2) + 80|V_3|$$

The load and generation expressed in per units are

$$P_2^{sch} = \frac{400}{100} = 4.0 \text{ pu}$$

$$S_3^{sch} = -\frac{(500 + j400)}{100} = -5.0 - j4.0 \text{ pu}$$

The slack bus voltage is $V_1 = 1.0\angle 0$ pu, and the bus 2 voltage magnitude is $|V_2| = 1.05$ pu. Starting with an initial estimate of $|V_3^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, and $\delta_3^{(0)} = 0.0$, the power residuals are

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = 4.0 - (0) = 4.0$$
$$\Delta P_3^{(0)} = P_3^{sch} - P_3^{(0)} = -5.0 - (0) = -5.0$$
$$\Delta Q_3^{(0)} = Q_3^{sch} - Q_3^{(0)} = -4.0 - (-1.0) = -3.0$$

Evaluating the elements of the Jacobian matrix with the initial estimate, the set of linear equations in the first iteration becomes

$$\begin{bmatrix} -2.8600 \\ 1.4384 \\ -0.2200 \end{bmatrix} = \begin{bmatrix} 63 & -21 & 0 \\ -21 & 41 & 0 \\ 0 & 0 & 39 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta|V_3^{(0)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, the new bus voltages in the first iteration are

$$\begin{aligned} \Delta\delta_2^{(0)} &= 0.0275 & \delta_2^{(1)} &= 0 + 0.0275 = 0.0275 \text{ radian} = 1.5782^\circ \\ \Delta\delta_3^{(0)} &= -0.1078 & \delta_3^{(1)} &= 0 + (-0.1078) = -0.1078 \text{ radian} = -6.1790^\circ \\ \Delta|V_3^{(0)}| &= -0.0769 & |V_3^{(1)}| &= 1 + (-0.0769) = 0.9231 \text{ pu} \end{aligned}$$

For the second iteration, we have

$$\begin{bmatrix} 0.2269 \\ -0.3965 \\ -0.5213 \end{bmatrix} = \begin{bmatrix} 61.1913 & -19.2072 & 2.8345 \\ -19.2072 & 37.5615 & -4.9871 \\ 2.6164 & -4.6035 & 33.1545 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta\delta_3^{(1)} \\ \Delta|V_3^{(1)}| \end{bmatrix}$$

and

$$\begin{aligned} \Delta\delta_2^{(1)} &= 0.0006 & \delta_2^{(2)} &= 0.0275 + 0.0006 = 0.0281 \text{ radian} = 1.61^\circ \\ \Delta\delta_3^{(1)} &= -0.0126 & \delta_3^{(2)} &= -0.1078 + (-0.0126) = -0.1204 \text{ radian} = -6.898^\circ \\ \Delta|V_3^{(1)}| &= -0.0175 & |V_3^{(2)}| &= 0.9231 + (-0.0175) = 0.9056 \text{ pu} \end{aligned}$$

6.13. For Problem 6.12:

(a) Obtain the power flow solution using the fast decoupled algorithm. Perform two iterations.
(b)Check the power flow solution for Problem 6.12 using the **decouple** and other required programs. Assume the regulated bus (bus # 2) reactive power limits are between 0 and 600 Mvar.

(a) In this system, bus 1 is the slack bus and the corresponding bus susceptance matrix for evaluation of phase angles $\Delta\delta_2$ and $\Delta\delta_3$ form the bus admittance matrix in Problem 6.12 is

$$B' = \begin{bmatrix} -60 & 20 \\ 20 & -40 \end{bmatrix}$$

The inverse of the above matrix is

$$[B']^{-1} = \begin{bmatrix} -0.02 & -0.01 \\ -0.01 & -0.03 \end{bmatrix}$$

The expressions for real power at bus 2 and 3 and the reactive power at bus 3 are given in Problem 6.12. The slack bus voltage is $V_1 = 1.0\angle 0$ pu, and the bus 2 voltage magnitude is $|V_3| = 1.05$ pu. Starting with an initial estimate of $|V_3^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, and $\delta_3^{(0)} = 0.0$, the power residuals are computed from (6.63) and (6.64)

$$\begin{aligned}
\Delta P_2^{(0)} &= P_2^{sch} - P_2^{(0)} = 4 - (0) = 4 \\
\Delta P_3^{(0)} &= P_3^{sch} - P_3^{(0)} = -5 - (0) = -5 \\
\Delta Q_3^{(0)} &= Q_3^{sch} - Q_3^{(0)} = -4 - (-1) = -3
\end{aligned}$$

The fast decoupled power flow algorithm given by (6.77) becomes

$$\begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \end{bmatrix} = -\begin{bmatrix} -0.02 & -0.0 \\ -0.01 & -0.03 \end{bmatrix} \begin{bmatrix} \frac{4}{1.05} \\ \frac{-5}{1.0} \end{bmatrix} = \begin{bmatrix} .0262 \\ -0.1119 \end{bmatrix}$$

Since bus 2 is a regulated bus, the corresponding row and column of B' are eliminated and we get

$$B'' = [-40]$$

From (6.78), we have

$$\Delta|V_3| = -\left[\frac{-1}{40}\right]\left[\frac{-3}{1.0}\right] = -0.075$$

The new bus voltages in the first iteration are

$$\begin{aligned}
\Delta\delta_2^{(0)} &= 0.0262 \qquad \delta_2^{(1)} = 0 + (0.0262) = 0.0262 \ \text{ radian} = 1.5006^\circ \\
\Delta\delta_3^{(0)} &= -0.1119 \qquad \delta_3^{(1)} = 0 + (-0.1119) = -0.1119 \ \text{ radian} = -6.4117^\circ \\
\Delta|V_3^{(0)}| &= -0.075 \qquad |V_2^{(1)}| = 1 + (-0.075) = 0.925 \ \text{ pu}
\end{aligned}$$

For the second iteration, the power residuals are

$$\begin{aligned}
\Delta P_2^{(1)} &= P_2^{sch} - P_2^{(1)} = 4 - (3.7739) = 0.2261 \\
\Delta P_3^{(1)} &= P_3^{sch} - P_3^{(1)} = -5 - (-4.7399) = -.2601 \\
\Delta Q_3^{(1)} &= Q_3^{sch} - Q_3^{(1)} = -4 - (-3.3994) = -0.6006
\end{aligned}$$

$$\begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta\delta_3^{(1)} \end{bmatrix} = -\begin{bmatrix} -0.02 & -0.0 \\ -0.01 & -0.03 \end{bmatrix} \begin{bmatrix} \frac{0.2261}{1.05} \\ \frac{-0.2601}{0.925} \end{bmatrix} = \begin{bmatrix} 0.0015 \\ -0.0063 \end{bmatrix}$$

From (6.78), we have

$$\Delta|V_3| = -\left[\frac{-1}{40}\right]\left[\frac{-0.6006}{0.925}\right] = -0.0162$$

The new bus voltages in the second iteration are

$$\begin{aligned} \Delta\delta_2^{(1)} &= 0.0015 & \delta_2^{(2)} &= 0.0262 + (0.0015) = 0.0277 \text{ radian} = 1.5863^\circ \\ \Delta\delta_3^{(1)} &= -0.0063 & \delta_3^{(2)} &= -0.1119 + (-0.0063) = -0.1182 \text{ radian} = -6.7716^\circ \\ \Delta|V_3^{(1)}| &= -0.0162 & |V_3^{(2)}| &= 0.925 + (-0.0162) = 0.9088 \text{ pu} \end{aligned}$$