Solution of Home Work # 2

6.3. Use Gauss-Seidel method to find the solution of the following equations

$$x_1 + x_1 x_2 = 10$$
$$x_1 + x_2 = 6$$

with the following initial estimates

(a) $x_1^{(0)} = 1$ and $x_2^{(0)} = 1$

(b) $x_1^{(0)} = 1$ and $x_2^{(0)} = 2$

Continue the iterations until $|\Delta x_1^{(k)}|$ and $|\Delta x_2^{(k)}|$ are less than 0.001.

Solving for x_1, and x_2 from the first and second equation respectively, results in

$$x_1 = \frac{10}{1 + x_2}$$
$$x_2 = 6 - x_1$$

(a) With initial estimates $x_1^{(0)} = 1$ and $x_2^{(0)} = 1$, the iterative sequence becomes

$$x_1^{(1)} = \frac{10}{1+1} = 5$$
$$x_2^{(1)} = 6 - 5 = 1$$
$$x_1^{(2)} = \frac{10}{1+1} = 5$$
$$x_2^{(2)} = 6 - 5 = 5$$

(b) With initial estimates $x_1^{(0)} = 1$ and $x_2^{(0)} = 2$, the iterative sequence becomes

$$x_1^{(1)} = \frac{10}{1+2} = 3.3333$$
$$x_2^{(1)} = 6 - 3.3333 = 2.6666$$
$$x_1^{(2)} = \frac{10}{1 + 2.6666} = 2.7272$$
$$x_2^{(2)} = 6 - 2.7272 = 3.2727$$
$$x_1^{(3)} = \frac{10}{1 + 3.2727} = 2.3404$$
$$x_2^{(3)} = 6 - 2.3404 = 3.6596$$
$$x_1^{(4)} = \frac{10}{1 + 3.6596} = 2.1461$$
$$x_2^{(4)} = 6 - 2.1461 = 3.8539$$
$$x_1^{(5)} = \frac{10}{1 + 3.8539} = 2.0602$$

$$x_2^{(5)} = 6 - 2.0602 = 3.9398$$
$$x_1^{(6)} = \frac{10}{1 + 3.9398} = 2.0244$$
$$x_2^{(6)} = 6 - 2.0244 = 3.9756$$
$$x_1^{(7)} = \frac{10}{1 + 3.9756} = 2.0098$$
$$x_2^{(7)} = 6 - 2.00098 = 3.9902$$
$$x_1^{(8)} = \frac{10}{1 + 3.9902} = 2.0039$$
$$x_2^{(8)} = 6 - 2.0039 = 3.9961$$
$$x_1^{(9)} = \frac{10}{1 + 3.9961} = 2.0016$$

$$x_2^{(9)} = 6 - 2.0244 = 3.9984$$
$$x_1^{(10)} = \frac{10}{1 + 3.9984} = 2.0006$$
$$x_2^{(10)} = 6 - 2.0006 = 3.9994$$

6.6. In the power system network shown in Figure 51, bus 1 is a slack bus with $V_1 = 1.0\angle 0^\circ$ per unit and bus 2 is a load bus with $S_2 = 280$ MW + $j60$ Mvar. The line impedance on a base of 100 MVA is $Z = 0.02 + j0.04$ per unit.

(a) Using Gauss-Seidel method, determine V_2 . Use an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and perform four iterations.

(b) If after several iterations voltage at bus 2 converges to $V_2 = 0.90 - j0.10$, determine S_1 and the real and reactive power loss in the line.

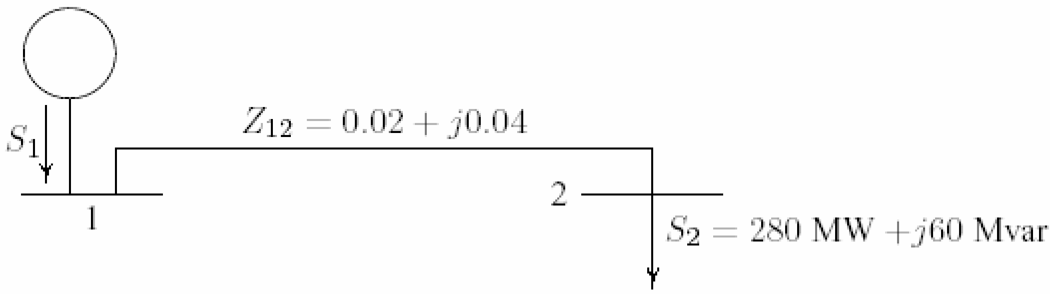


FIGURE 51
One-line diagram for Problem 6.6.

$$y_{12} = \frac{1}{0.02 + j0.04} = 10 - j29$$

The per unit load at bus 2 is

$$S_2 = -\frac{280 + j60}{100} = -2.8 - j0.60$$

Starting with an initial estimate of $V_2^{(0)} = 1.0 + j0.0$, the voltage at bus 2 computed from (6.28) for three iterations are

$$V_2^{(1)} = \frac{\frac{-2.8+j0.60}{1.00000-j0.00000} + (10-j20)(1)}{10-j20} = 0.92000 - j0.10000$$
$$V_2^{(2)} = \frac{\frac{-2.8+j0.60}{0.92000+j0.10000} + (10-j20)(1)}{10-j20} = 0.90238 - j0.09808$$
$$V_2^{(3)} = \frac{\frac{-2.8+j0.60}{0.90238-j0.09808} + (10-j20)(1)}{10-j20} = 0.90050 - j0.10000$$

(b) Assuming voltage at bus 2 converges to $V_2 = 0.9 - j0.1$, the line flows are computed as follows

$$I_{12} = y_{12}(V_1 - V_2) = (10 - j20)[(1 + j0) - (0.9 - j0.10] = 3.0 - j1.0$$

$$I_{21} = -I_{12} = -3.0 + j1.0$$

$$\begin{aligned} S_{12} = V_1 I_{12}^* = (1.0 + j0.0)(3.0 + j1.0) &= 3 + j1 \text{ pu} \\ &= 300 \text{ MW} + j100 \text{ Mvar} \\ S_{21} = V_2 I_{21}^* = (0.9 - j0.1)(-3.0 - j1.0) &= -2.8 - j0.6 \text{ pu} \\ &= -280 \text{ MW} - j60 \text{ Mvar} \end{aligned}$$

The line loss is

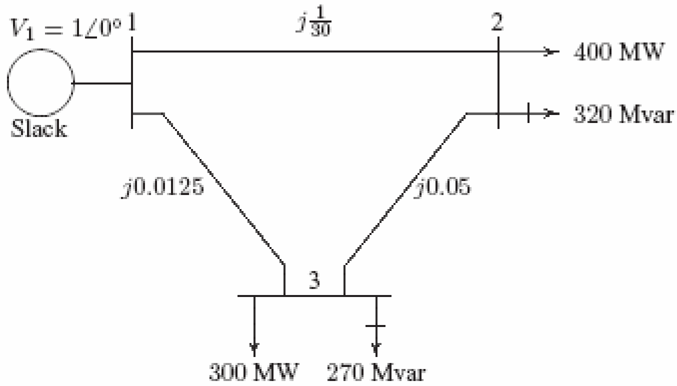
$$S_{L12} = S_{12} + S_{21} = (300 + j100) + (-280 - j60) = 20 \text{ MW} + j40 \text{ Mvar}$$

The slack bus real and reactive power are $P_1 = 300$ MW, and $Q_1 = 100$ Mvar.

6.7. Figure 6.6 shows the one-line diagram of a simple three-bus power system with generation at bus 1. The voltage at bus 1 is $V_1 = 1.0\angle 0^\circ$ per unit. The scheduled loads on buses 2 and 3 are marked on the diagram. Line impedances are marked in per unit on a 100 MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.
(a) Using Gauss-Seidel method and initial estimates of $V_2^{(0)} = 1.0 + j0$ and $V_3^{(0)} = 1.0 + j0$, determine V_2 and V_3. Perform two iterations.
(b) If after several iterations the bus voltages converge to

$$V_2 = 0.90 - j0.10 \text{ pu}$$
$$V_3 = 0.95 - j0.05 \text{ pu}$$

determine the line flows and line losses and the slack bus real and reactive power. Construct a power flow diagram and show the direction of the line flows.
(c) Check the power flow solution using the **lfgauss** and other required programs. (Refer to Example 6.9.) Use a power accuracy of 0.00001 and an acceleration factor of 1.0.



(a) Line impedances are converted to admittances

$$y_{12} = -j30$$
$$y_{13} = \frac{1}{j0.0125} = -j80$$
$$y_{23} = \frac{1}{j0.05} = -j20$$

At the P-Q buses, the complex loads expressed in per units are

$$S_2^{sch} = -\frac{(400 + j320)}{100} = -4.0 - j3.2 \ \text{pu}$$
$$S_3^{sch} = -\frac{(300 + j270)}{100} = -3.0 - j2.7 \ \text{pu}$$

For hand calculation, we use (6.28). Bus 1 is taken as reference bus (slack bus). Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.0 + j0.0$, V_2 and V_3 are computed from (6.28) as follows

$$\begin{aligned} V_2^{(1)} &= \frac{\frac{S_2^{sch*}}{V_2^{(0)*}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}} \\ &= \frac{\frac{-4.0+j3.2}{1.0-j0} + (-j30)(1.0 + j0) + (-j20)(1.0 + j0)}{-j50} \\ &= 0.936 - j0.08 \end{aligned}$$

and

$$V_3^{(1)} = \frac{\frac{S_3^{sch*}}{V_3^{(0)*}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}}$$

$$= \frac{\frac{-3.0+j2.7}{1-j0} + (-j80)(1.0 + j0) + (-j20)(0.936 - j0.08)}{-j100}$$

$$= 0.9602 - j0.046$$

For the second iteration we have

$$V_2^{(2)} = \frac{\frac{-4.0+j3.2}{0.936+j0.08} + (-j30)(1.0 + j0) + (-j20)(0.9602 - j0.046)}{-j50}$$

$$= 0.9089 - j0.0974$$

and

$$V_3^{(2)} = \frac{\frac{-3.0+j2.7}{0.9602+j0.046} + (-j80)(1.0 + j0) + (-j20)(0.9089 - j0.0974)}{(-j100)}$$

$$= 0.9522 - j0.0493$$

The process is continued and a solution is converged with an accuracy of 5×10^{-5} per unit in seven iterations as given below.

$$\begin{aligned}
V_2^{(3)} &= 0.9020 - j0.0993 & \qquad V_3^{(3)} &= 0.9505 - j0.0498 \\
V_2^{(4)} &= 0.9004 - j0.0998 & \qquad V_3^{(4)} &= 0.9501 - j0.0500 \\
V_2^{(5)} &= 0.9001 - j0.1000 & \qquad V_3^{(5)} &= 0.9500 - j0.0500 \\
V_2^{(6)} &= 0.9000 - j0.1000 & \qquad V_3^{(6)} &= 0.9500 - j0.0500 \\
V_2^{(7)} &= 0.9000 - j0.1000 & \qquad V_3^{(7)} &= 0.9500 - j0.0500
\end{aligned}$$

The final solution is

$$\begin{aligned}
V_2 &= 0.90 - j0.10 = 0.905554\angle{-6.34^\circ} \quad \text{pu} \\
V_3 &= 0.95 - j0.05 = 0.9513\angle{-3.0128^\circ} \quad \text{pu}
\end{aligned}$$

(b) With the knowledge of all bus voltages, the slack bus power is obtained from (6.27)

$$\begin{aligned}
P_1 - jQ_1 &= V_1^*[V_1(y_{12} + y_{13}) - (y_{12}V_2 + y_{13}V_3)] \\
&= 1.0[1.0(-j30 - j80) - (-j30)(0.9 - j0.1) - \\
&\quad (-j80)(0.95 - j0.05)] \\
&= 7.0 - j7.0
\end{aligned}$$

or the slack bus real and reactive powers are $P_1 = 7.0$ pu = 700 MW and $Q_1 = 7.0$ pu $=$ 700 Mvar.

To find the line flows, first the line currents are computed. With line charging capacitors neglected, the line currents are

$$
\begin{aligned}
I_{12} &= y_{12}(V_1 - V_2) = (-j30)[(1.0 + j0) - (0.90 - j0.10)] = 3.0 - j3.0 \\
I_{21} &= -I_{12} = -3.0 + j3.0 \\
I_{13} &= y_{13}(V_1 - V_3) = (-j80)[(1.0 + j0) - (0.95 - j.05)] = 4.0 - j4.0 \\
I_{31} &= -I_{13} = -4.0 + j4.0 \\
I_{23} &= y_{23}(V_2 - V_3) = (-j20)[(0.90 - j0.10) - (0.95 - j.05)] = -1.0 + j1.0 \\
I_{32} &= -I_{23} = 1.0 - j1.0
\end{aligned}
$$

The line flows are

$$
\begin{aligned}
S_{12} = V_1 I_{12}^* &= (1.0 + j0.0)(3.0 + j3) = 3.0 + j3.0 \ \text{ pu} \\
&= 300 \ \text{ MW} + j300 \ \text{ Mvar}
\end{aligned}
$$

$$
\begin{aligned}
S_{21} = V_2 I_{21}^* &= (0.90 - j0.10)(-3 - j3) = -3.0 - j2.4 \ \text{ pu} \\
&= -300 \ \text{ MW} - j240 \ \text{ Mvar} \\
S_{13} = V_1 I_{13}^* &= (1.0 + j0.0)(4.0 + j4.0) = 4.0 + j4.0 \ \text{ pu} \\
&= 400 \ \text{ MW} + j400 \ \text{ Mvar} \\
S_{31} = V_3 I_{31}^* &= (0.95 - j0.05)(-4.0 - j4.0) = -4.0 - j3.6 \ \text{ pu} \\
&= -400 \ \text{ MW} - j360 \ \text{ Mvar} \\
S_{23} = V_2 I_{23}^* &= (0.90 - j0.10)(-1.0 - j1.0) = -1.0 - j0.80 \ \text{ pu} \\
&= -100 \ \text{ MW} - j80 \ \text{ Mvar} \\
S_{32} = V_3 I_{32}^* &= (0.95 - j0.05)(1 + j1) = 1.0 + j0.9 \ \text{ pu} \\
&= 100 \ \text{ MW} + j90 \ \text{ Mvar}
\end{aligned}
$$

and the line losses are

$$
\begin{aligned}
S_{L\,12} &= S_{12} + S_{21} = 0.0 \ \text{ MW} + j60 \ \text{ Mvar} \\
S_{L\,13} &= S_{13} + S_{31} = 0.0 \ \text{ MW} + j40 \ \text{ Mvar} \\
S_{L\,23} &= S_{23} + S_{32} = 0.0 \ \text{ MW} + j10 \ \text{ Mvar}
\end{aligned}
$$

The power flow diagram is shown in Figure 6.7, where real power direction is indicated by $\rightarrow$ and the reactive power direction is indicated by $\mapsto$. The values within parentheses are the real and reactive losses in the line.

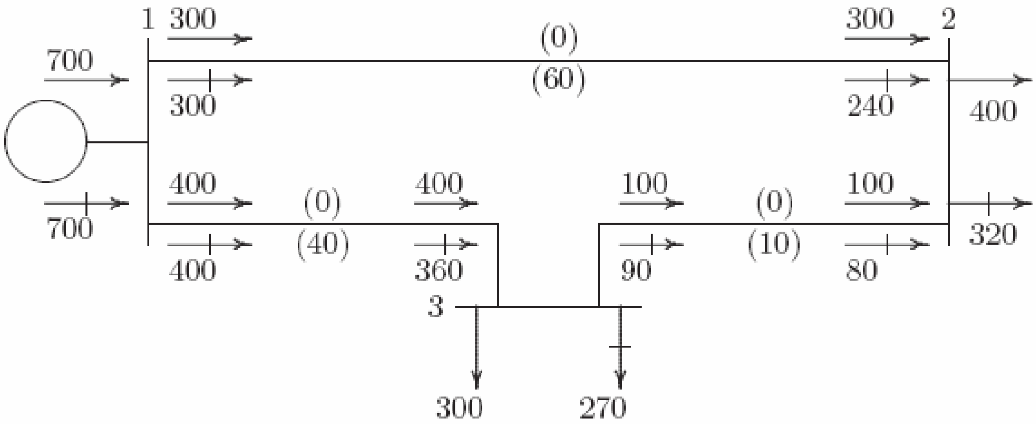


FIGURE 53
Power flow diagram of Problem 6.7 (powers in MW and Mvar).