11.5. Two synchronous generators represented by a constant voltage behind transient reactance are connected by a pure reactance $X = 0.3$ per unit, as shown in Figure 91. The generator inertia constants are $H_1 = 4.0$ MJ/MVA and $H_2 = 6$ MJ/MVA, and the transient reactances are $X_1' = 0.16$ and $X_2' = 0.20$ per unit. The system is operating in the steady state with $E_1' = 1.2$, $P_{m1} = 1.5$ and $E_2' = 1.1$, $P_{m2} = 1.0$ per unit. Denote the relative power angle between the two machines by $\delta = \delta_1 - \delta_2$. Referring to Problem 11.4, reduce the two-machine system to an equivalent one-machine against an infinite bus. Find the inertia constant of the equivalent machine, the mechanical input power, and the amplitude of its power angle curve, and obtain the equivalent swing equation in terms of δ.

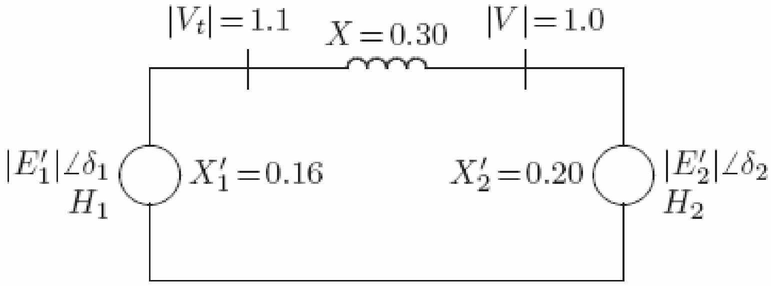

FIGURE 91
System of Problem 11.5.

Referring to Problem 11.4, the equivalent parameters are

$$H = \frac{(4)(6)}{4+6} = 2.4 \ \text{MJ/MVA}$$

$$P_m = \frac{(6)(1.50) - (4)(1)}{4+6} = 0.5 \ \text{pu}$$

$$P_{e1} = \frac{|E_1||E_2|}{X}\sin(\delta_1 - \delta_2) = \frac{(1.2)(1.1)}{0.66}\sin\delta = 2\sin\delta$$

Since $P_{e2} = -P_{e1}$, we have

$$P_e = \frac{(6)(2\sin\delta) + (4)(2\sin\delta)}{4+6} = 2\sin\delta$$

Therefore, the equivalent swing equation is

$$\frac{2.4}{(180)(60)}\frac{d^2\delta}{dt^2} = 0.5 - 2\sin\delta$$

or

$$\frac{d^2\delta}{dt^2} = 4500(0.5 - 2\sin\delta) \quad \text{where } \delta \text{ is in degrees}$$

11.6. A 60-Hz synchronous generator has a transient reactance of 0.2 per unit and an inertia constant of 5.66 MJ/MVA. The generator is connected to an infinite bus through a transformer and a double circuit transmission line, as shown in Figure 92. Resistances are neglected and reactances are expressed on a common MVA base and are marked on the diagram. The generator is delivering a real power of 0.77 per unit to bus bar 1. Voltage magnitude at bus 1 is 1.1. The infinite bus voltage $V = 1.0\angle 0°$ per unit. Determine the generator excitation voltage and obtain the swing equation as given by (pr11.36).

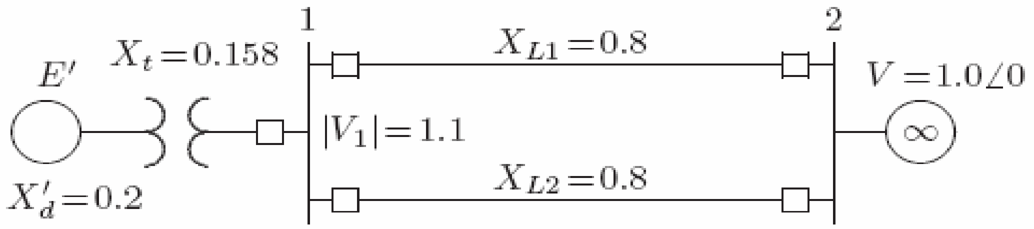

FIGURE 92
System of Problem 11.6.

$$P = \frac{|V_1||V_2|}{X_L}\sin\delta_1$$
$$0.77 = \frac{(1.1)(1.0)}{0.4}\sin\delta_1$$

or

$$\delta_1 = 16.26°$$

$$I = \frac{V_1 - V_2}{jX_L} = \frac{1.1\angle 16.26° - 1.0\angle 0°}{j0.4} = 0.77 - j0.14$$
$$= 0.7826\angle{-10.305°} \text{ pu}$$

The total reactance is $X = 0.2 + 0.158 + 0.4 = 0.758$, and the generator excitation voltage is

$$E' = 1.0 + j0.758(0.77 - j0.14) = 1.25\angle 27.819°$$

from (11.36) the swing equation with δ in radians is

$$\frac{5.66}{60\pi}\frac{d^2\delta}{dt^2} = 0.77 - \frac{(1.25)(1)}{0.758}\sin\delta$$
$$0.03\frac{d^2\delta}{dt^2} = 0.77 - 1.65\sin\delta$$

11.7. A three-phase fault occurs on the system of Problem 11.6 at the sending end of the transmission lines. The fault occurs through an impedance of 0.082 per unit. Assume the generator excitation voltage remains constant at $E' = 1.25$ per unit. Obtain the swing equation during the fault.

The impedance network with fault at bus 1, and with with $Z_f = j0.082$ is shown in Figure 93.

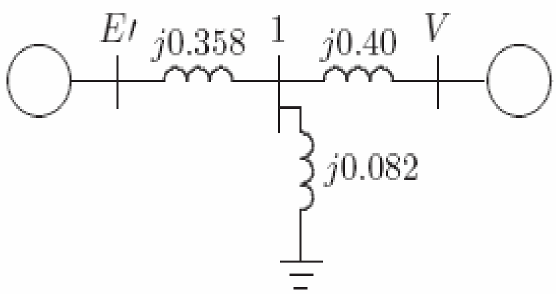

FIGURE 93
Impedance network with fault at bus 1.

Transforming the Y-connected circuit in Figure 93 into an equivalent Δ, the transfer reactance between E' and V is

$$X = \frac{(0.358)(0.082) + (0.358)(0.4) + (0.4)(0.082)}{0.082} = 2.5 \text{ pu}$$

$$P_{2\,max} = \frac{(1.25)(1)}{2.5} = 0.5$$

Therefore, the swing equation during fault with δ in radians is

$$0.03\frac{d^2\delta}{dt^2} = 0.77 - 0.5\sin\delta$$