

8.1. A sinusoidal voltage given by $v(t) = 390 \sin(315t + \alpha)$ is suddenly applied to a series RL circuit. $R = 32 \Omega$ and $L = 0.4$ H.

(a) The switch is closed at such a time as to permit no transient current. What value of α corresponds to this instant of closing the switch? Obtain the instantaneous expression for $i(t)$. Use *MATLAB* to plot $i(t)$ up to 80 ms in steps of 0.01 ms.

(b) The switch is closed at such a time as to permit maximum transient current. What value of α corresponds to this instant of closing the switch? Obtain the instantaneous expression for $i(t)$. Use *MATLAB* to plot $i(t)$ up to 80 ms in steps of 0.01 ms.

(c) What is the maximum value of current in part (b) and at what time does this occur after the switch is closed?

$$v(t) = 390 \sin(315t + \alpha)$$

$$i(t) = I_m \sin(315t + \alpha - \gamma) - I_m e^{-t/\tau} \sin(\alpha - \gamma)$$

(a) For no transient $\alpha = \gamma$

$$\gamma = \tan^{-1} \frac{(315)(0.4)}{32} = 75.75^\circ \Rightarrow \alpha = 75.75^\circ$$

$$Z = 32 + j(315)(0.4) = 32 + j126 = 130 \angle 75.75^\circ \Omega$$

$$I = \frac{390}{130} = 3 \text{ A} \Rightarrow i(t) = 3 \sin 315t$$

(b) For maximum transient current $\alpha - \gamma = -90^\circ$. Therefore, $\alpha = 75.75 - 90 = -14.25^\circ$, and $\tau = \frac{L}{R} = 0.0125$ sec, and the current is

$$i(t) = 3 \sin\left(315t - \frac{\pi}{2}\right) + 3e^{-80t}$$

(c)

$$\frac{di(t)}{dt} = (3)(315) \cos(315t - \frac{\pi}{2}) - 240e^{-80t} = 0$$

Use the command `[Imax, k] = max(i)`, `tmax = t(k)` to find the maximum value of current, and the corresponding time.

$$i_{max} = 4.371 \text{ A}$$
$$t_{max} = 0.0096 \text{ sec}$$

Save the following statements in a file named **ch8p1.m**, and run to obtain the plots.

```
disp('Problem 8.1')
R = 32; L = 0.4; tau= L/R;w=315; X=w*L;
Z=R+j*X; Vm = 390;
gama=angle(Z);
gamad=gama*180/pi;
Im=Vm/130;
disp('a')
alpha1= gama; alpha = alpha1*180/pi
t=0:.0001:.08;
i1=Im*sin(315*t+alpha1 -gama);
subplot(211), plot(t, i1), grid
xlabel('t, sec'), ylabel('i(t)')
[I1max,k] = max(i1)
tmax= t(k)
disp('b), (c)')
alpha2=gama - pi/2; alpha = alpha2*180/pi
i2=Im*sin(315*t+alpha2 -gama)-Im*exp(-t/tau).*sin(alpha2-gama);
subplot(212), plot(t, i2), grid
xlabel('t, sec'), ylabel('i(t)')
[I2max,k] = max(i2)
tmax= t(k)
```

9.1. The system shown in Figure 65 is initially on no load with generators operating at their rated voltage with their emfs in phase. The rating of the generators and the transformers and their respective percent reactances are marked on the diagram. All resistances are neglected. The line impedance is $j160 \Omega$. A three-phase balanced fault occurs at the receiving end of the transmission line. Determine the short-circuit current and the short-circuit MVA.

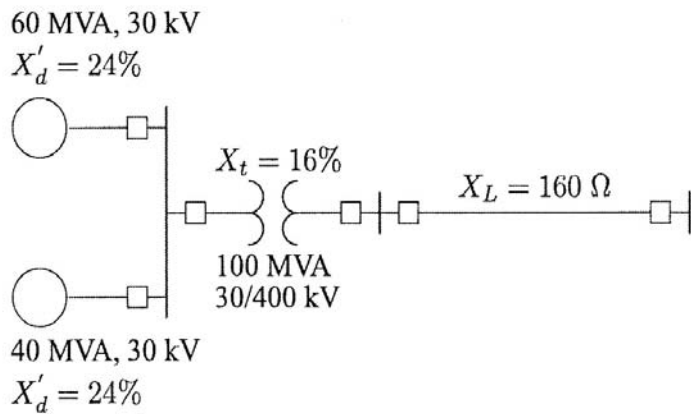


FIGURE 65
One-line diagram for Problem 9.1.

The base impedance for line is

$$Z_B = \frac{(400)^2}{100} = 1,600 \Omega$$

and the base current is

$$I_B = \frac{100,000}{\sqrt{3}(400)} = 144.3375 \text{ A}$$

The reactances on a common 100 MVA base are

$$X'_{dg1} = \frac{100}{60}(0.24) = 0.4 \text{ pu}$$

$$X'_{dg2} = \frac{100}{40}(0.24) = 0.6 \text{ pu}$$

$$X_t = \frac{100}{100}(0.16) = 0.16 \text{ pu}$$

$$X_{line} = \frac{160}{1600} = 0.1 \text{ pu}$$

The impedance diagram is as shown in Figure 66.

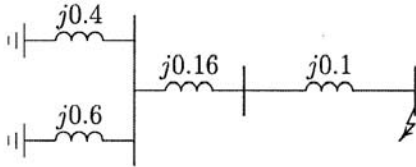


FIGURE 66

The impedance diagram for Problem 9.1.

Impedance to the point of fault is

$$X = j \frac{(0.4)(0.6)}{0.4 + 0.6} + j0.16 + j0.1 = j0.5 \text{ pu}$$

The fault current is

$$\begin{aligned} I_f &= \frac{1}{j0.5} = 2 \angle -90^\circ \text{ pu} \\ &= (144.3375)(2 \angle -90^\circ) = 288.675 \angle -90^\circ \text{ A} \end{aligned}$$

The Short-circuit MVA is

$$\text{SCMVA} = \sqrt{3}(400)(288.675)(10^{-3}) = 200 \text{ MVA}$$

9.2. The system shown in Figure 67 shows an existing plant consisting of a generator of 100 MVA, 30 kV, with 20 percent subtransient reactance and a generator of 50 MVA, 30 kV with 15 percent subtransient reactance, connected in parallel to a 30-kV bus bar. The 30-kV bus bar feeds a transmission line via the circuit breaker

C which is rated at 1250 MVA. A grid supply is connected to the station bus bar through a 500-MVA, 400/30-kV transformer with 20 percent reactance. Determine the reactance of a current limiting reactor in ohm to be connected between the grid system and the existing bus bar such that the short-circuit MVA of the breaker C does not exceed.

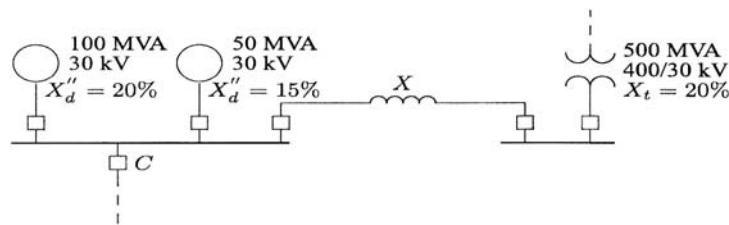


FIGURE 67
One-line diagram for Problem 9.2.

The base impedance for line is

$$Z_B = \frac{(30)^2}{100} = 9 \Omega$$

The reactances on a common 100 MVA base are

$$X''_{dg1} = \frac{100}{100}(0.2) = 0.2 \text{ pu}$$

$$X''_{dg2} = \frac{100}{50}(0.15) = 0.3 \text{ pu}$$

$$X_t = \frac{100}{500}(0.2) = 0.04 \text{ pu}$$

The impedance diagram is as shown in Figure 68.
Reactance to the point of fault is

$$X_f = \frac{S_B}{\text{SCMVA}} = \frac{100}{1250} = 0.08 \text{ pu}$$

Parallel reactance of the generators is

$$X_{||} = \frac{(0.2)(0.3)}{0.2 + 0.3} = 0.12 \text{ pu}$$

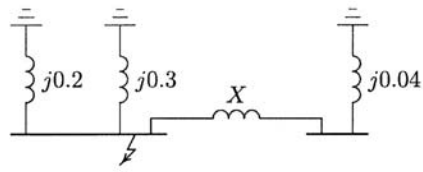


FIGURE 68
The impedance diagram for Problem 9.2.

From Figure 68, reactance to the point of fault is

$$\frac{(0.12)(X + 0.04)}{0.12 + (X + 0.04)} = 0.08$$

Solving for X , we get $X = 0.2$ pu., or

$$X_{\Omega} = (X)(Z_B) = (0.2)(9) = 1.8 \text{ } \Omega$$

9.5. The one-line diagram of a simple four-bus power system is shown in Figure 73. Each generator is represented by an emf behind the transient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A bolted three-phase fault occurs at bus 4.

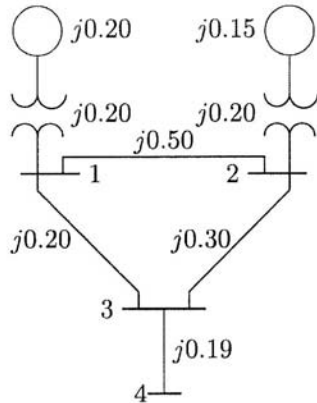


FIGURE 73
One-line diagram for Problem 9.5.

- Using Thévenin's theorem obtain the impedance to the point of fault and the fault current in per unit.
- Determine the bus voltages and line currents during fault.

(c) Repeat (a) and (b) for a fault at bus 2 with a fault impedance of $Z_f = j0.0225$.

(a) Converting the Δ formed by buses 123 to an equivalent Y as shown in Figure 74(a), we have

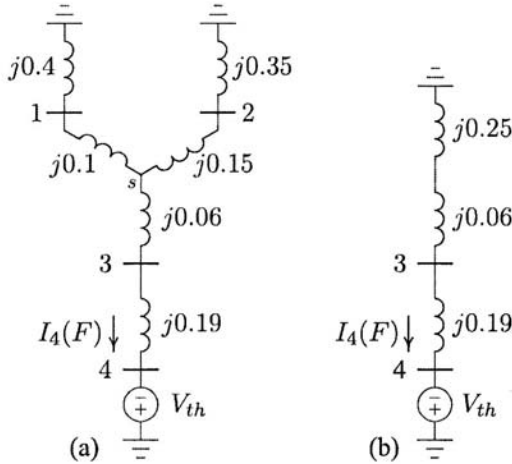


FIGURE 74
Reduction of Thévenin's equivalent network.

$$Z_{1s} = \frac{(j0.2)(j0.5)}{j1.0} = j0.10 \quad Z_{2s} = \frac{(j0.5)(j0.3)}{j1.0} = j0.15$$

$$Z_{3s} = \frac{(j0.2)(j0.3)}{j1.0} = j0.06$$

Combining the parallel branches, Thévenin's impedance is

$$Z_{33} = \frac{(j0.5)(j0.5)}{j0.5 + j0.5} + j0.06 + j0.19 = j0.5$$

The fault current is

$$I_4(F) = \frac{V_4(F)}{Z_{44}} = \frac{1.0}{j0.5} = -j2.0 \text{ pu}$$

With reference to Figure 74(a), the current divisions between the two generators are

$$I_{G1} = \frac{j0.5}{j0.5 + j0.5} I_4(F) = -j1.0 \text{ pu}$$

$$I_{G2} = \frac{j0.5}{j0.5 + j0.5} I_4(F) = -j1.0 \text{ pu}$$

(b) For the bus voltage changes from Figure 74(a), we get

$$\begin{aligned}\Delta V_1 &= 0 - (j0.4)(-j1.0) = -0.4 \text{ pu} \\ \Delta V_2 &= 0 - (j0.35)(-j1) = -0.35 \text{ pu} \\ \Delta V_3 &= 1 - (j0.19)(-j2) = -0.62 \text{ pu}\end{aligned}$$

The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, i.e.,

$$\begin{aligned}V_1(F) &= V_1(0) + \Delta V_1 = 1.0 - 0.4 = 0.60 \text{ pu} \\ V_2(F) &= V_2(0) + \Delta V_2 = 1.0 - 0.35 = 0.65 \text{ pu} \\ V_3(F) &= V_3(0) + \Delta V_3 = 1.0 - 0.62 = 0.38 \text{ pu} \\ V_4(F) &= 0 \text{ pu}\end{aligned}$$

The short circuit-currents in the lines are

$$\begin{aligned}I_{21}(F) &= \frac{V_2(F) - V_1(F)}{z_{12}} = \frac{0.65 - 0.6}{j0.5} = 0.1 \angle -90^\circ \text{ pu} \\ I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.60 - 0.38}{j0.2} = 1.1 \angle -90^\circ \text{ pu} \\ I_{23}(F) &= \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.65 - 0.38}{j0.3} = 0.9 \angle -90^\circ \text{ pu} \\ I_{34}(F) &= \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.38 - 0}{j0.19} = 2.0 \angle -90^\circ \text{ pu}\end{aligned}$$

(c) (a) Combining parallel branches between buses 1 and 2 results in the circuit shown in Figure 75(a). Combining the parallel branches, Thévenin's impedance is

$$Z_{22} = \frac{(j0.65)(j0.35)}{j0.65 + j0.35} = j0.2275$$

The fault current is

$$I_2(F) = \frac{V_4(F)}{Z_{44} + Z_f} = \frac{1.0}{j0.2275 + j0.0225} = -j4.0 \text{ pu}$$

With reference to Figure 75(a), the current divisions between the two generators are

$$\begin{aligned}I_{G1} &= \frac{j0.35}{j0.65 + j0.35} I_2(F) = -j1.4 \text{ pu} \\ I_{G2} &= \frac{j0.65}{j0.65 + j0.35} I_2(F) = -j2.6 \text{ pu}\end{aligned}$$

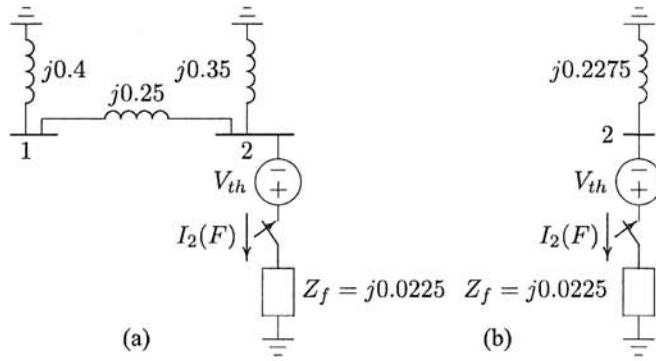


FIGURE 75
Reduction of Thévenin's equivalent network.

(c) (b) For the bus voltage changes from Figure 74(a), we get

$$\begin{aligned} \Delta V_1 &= 0 - (j0.4)(-j1.4) = -0.56 \text{ pu} \\ \Delta V_2 &= 0 - (j0.35)(-j2.6) = -0.91 \text{ pu} \\ \Delta V_3 &= -j0.56 - (j0.2)\left(-\frac{j1.4}{2}\right) = -0.70 \text{ pu} \\ \Delta V_4 &= \Delta V_3 = -0.70 \text{ pu} \end{aligned}$$

The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, i.e.,

$$\begin{aligned} V_1(F) &= V_1(0) + \Delta V_1 = 1.0 - 0.56 = 0.44 \text{ pu} \\ V_2(F) &= V_2(0) + \Delta V_2 = 1.0 - 0.91 = 0.09 \text{ pu} \\ V_3(F) &= V_3(0) + \Delta V_3 = 1.0 - 0.70 = 0.30 \text{ pu} \\ V_4(F) &= V_4(0) + \Delta V_4 = 1.0 - 0.70 = 0.30 \text{ pu} \end{aligned}$$

The short circuit-currents in the lines are

$$\begin{aligned} I_{12}(F) &= \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.44 - 0.09}{j0.5} = 0.7 \angle -90^\circ \text{ pu} \\ I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.44 - 0.30}{j0.2} = 0.7 \angle -90^\circ \text{ pu} \\ I_{32}(F) &= \frac{V_3(F) - V_2(F)}{z_{23}} = \frac{0.30 - 0.09}{j0.3} = 0.7 \angle -90^\circ \text{ pu} \\ I_{34}(F) &= \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.30 - 0.30}{j0.19} = 0 \text{ pu} \end{aligned}$$

[j0.05 j0.02 j0.05]

9.8. Obtain the bus impedance matrix for the network of Problem 9.4.

Add branch 1, $z_{10} = j0.05$ between node $q = 1$ and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{11} = z_{10} = j0.05$$

Next, add branch 2, $z_{20} = j0.075$ between node $q = 2$ and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{11} & 0 \\ 0 & Z_{22} \end{bmatrix} = \begin{bmatrix} j0.05 & 0 \\ 0 & j0.075 \end{bmatrix}$$

Add branch 3, $z_{13} = j0.3$ between the new node $q = 3$ and the existing node $p = 1$. According to rule 2, we get

$$\mathbf{Z}_{bus}^{(3)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{11} \\ Z_{21} & Z_{22} & Z_{21} \\ Z_{11} & Z_{12} & Z_{11} + z_{13} \end{bmatrix} = \begin{bmatrix} j0.05 & 0 & j0.05 \\ 0 & j0.075 & 0 \\ j0.05 & 0 & j0.35 \end{bmatrix}$$

Add link 4, $z_{12} = j0.75$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\begin{aligned} \mathbf{Z}_{bus}^{(4)} &= \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12} - Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22} - Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32} - Z_{31} \\ Z_{21} - Z_{11} & Z_{22} - Z_{12} & Z_{23} - Z_{13} & Z_{44} \end{bmatrix} \\ &= \begin{bmatrix} j0.05 & 0 & j0.05 & -j0.05 \\ 0 & j0.075 & 0 & j0.075 \\ j0.05 & 0 & j0.35 & -j0.05 \\ -j0.05 & j0.075 & -j0.05 & Z_{44} \end{bmatrix} \end{aligned}$$

From (9.58)

$$Z_{44} = z_{12} + Z_{11} + Z_{22} - 2Z_{12} = j0.75 + j0.05 + j0.075 - 2(j0) = j0.875$$

and

$$\begin{aligned} \frac{\Delta \mathbf{Z} \Delta \mathbf{Z}^T}{Z_{44}} &= \frac{1}{j0.875} \begin{bmatrix} -j0.05 \\ j0.075 \\ -j0.05 \end{bmatrix} \begin{bmatrix} -j0.05 & j0.075 & -j0.05 \end{bmatrix} \\ &= \begin{bmatrix} j0.002857 & -j0.004286 & j0.002857 \\ -j0.004286 & j0.006428 & -j0.004286 \\ j0.002857 & -j0.004286 & j0.002857 \end{bmatrix} \end{aligned}$$

From (9.59), the new bus impedance matrix is

$$\begin{aligned} \mathbf{Z}_{bus}^{(4)} &= \begin{bmatrix} j0.05 & 0 & j0.05 \\ 0 & j0.075 & 0 \\ j0.05 & 0 & j0.35 \end{bmatrix} - \begin{bmatrix} j0.002857 & -j0.004286 & j0.002857 \\ -j0.004286 & j0.006428 & -j0.004286 \\ j0.002857 & -j0.004286 & j0.002857 \end{bmatrix} \\ &= \begin{bmatrix} j0.047143 & j0.004286 & j0.047143 \\ j0.004286 & j0.068571 & j0.004286 \\ j0.047143 & j0.004286 & j0.347142 \end{bmatrix} \end{aligned}$$

Add link 5, $z_{23} = j0.45$ between node $q = 3$ and node $p = 2$. From (9.57), we have

$$\begin{aligned} \mathbf{Z}_{bus}^{(5)} &= \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{13} - Z_{12} \\ Z_{21} & Z_{22} & Z_{23} & Z_{23} - Z_{22} \\ Z_{31} & Z_{32} & Z_{33} & Z_{33} - Z_{32} \\ Z_{31} - Z_{21} & Z_{32} - Z_{22} & Z_{33} - Z_{23} & Z_{44} \end{bmatrix} \\ &= \begin{bmatrix} j0.047143 & j0.004286 & j0.047143 & j0.042857 \\ j0.004286 & j0.068571 & j0.004286 & -j0.064286 \\ j0.047143 & j0.004286 & j0.347142 & j0.342857 \\ j0.042857 & -j0.064286 & j0.342857 & Z_{44} \end{bmatrix} \end{aligned}$$

From (9.58)

$$Z_{44} = z_{23} + Z_{22} + Z_{33} - 2Z_{23} = j0.45 + j0.068571 + j0.347142 - 2(j0.004286) = j0.85714$$

and

$$\begin{aligned} \frac{\Delta \mathbf{Z} \Delta \mathbf{Z}^T}{Z_{44}} &= \frac{1}{j0.85714} \begin{bmatrix} j0.042857 \\ -j0.064286 \\ j0.342857 \end{bmatrix} [j0.042857 \quad -j0.064286 \quad j0.342857] \\ &= \begin{bmatrix} j0.002143 & -j0.003214 & j0.017143 \\ -j0.003214 & j0.004821 & -j0.025714 \\ j0.017143 & -j0.025714 & j0.137143 \end{bmatrix} \end{aligned}$$

From (9.59), the new bus impedance matrix is

$$\begin{aligned} \mathbf{Z}_{bus} &= \begin{bmatrix} j0.047143 & j0.004286 & j0.047143 \\ j0.004286 & j0.068571 & j0.004286 \\ j0.047143 & j0.004286 & j0.347142 \end{bmatrix} - \\ &\quad \begin{bmatrix} j0.002143 & -j0.003214 & j0.017143 \\ -j0.003214 & j0.004821 & -j0.025714 \\ j0.017143 & -j0.025714 & j0.137142 \end{bmatrix} = \begin{bmatrix} j0.0450 & j0.00750 & j0.0300 \\ j0.0075 & j0.06375 & j0.0300 \\ j0.0300 & j0.03000 & j0.210 \end{bmatrix} \end{aligned}$$

9.11. The per unit bus impedance matrix for the power system of Problem 9.5 is given by

$$Z_{bus} = j \begin{bmatrix} 0.240 & 0.140 & 0.200 & 0.200 \\ 0.140 & 0.2275 & 0.175 & 0.175 \\ 0.200 & 0.175 & 0.310 & 0.310 \\ 0.200 & 0.1750 & 0.310 & 0.500 \end{bmatrix}$$

- A bolted three-phase fault occurs at bus 4. Using the bus impedance matrix calculate the fault current, bus voltages, and line currents during fault.
- Repeat (a) for a three-phase fault at bus 2 with a fault impedance of $Z_f = j0.0225$.
- Check your result using the **Zbuild** and **symfault** programs.

From (9.22), for a solid fault at bus 4 the fault current is

$$I_4(F) = \frac{V_4(0)}{Z_{44}} = \frac{1.0}{j0.5} = -j2 \text{ pu}$$

From (9.23), bus voltages during the fault are

$$\begin{aligned} V_1(F) &= V_1(0) - Z_{14}I_4(F) = 1.0 - (j0.200)(-j2) = 0.60 \text{ pu} \\ V_2(F) &= V_2(0) - Z_{24}I_4(F) = 1.0 - (j0.175)(-j2) = 0.65 \text{ pu} \\ V_3(F) &= V_3(0) - Z_{34}I_4(F) = 1.0 - (j0.310)(-j2) = 0.38 \text{ pu} \\ V_4(F) &= V_4(0) - Z_{44}I_4(F) = 1.0 - (j0.500)(-j2) = 0 \text{ pu} \end{aligned}$$

From (9.25), the short circuit currents in the lines are

$$\begin{aligned} I_{21}(F) &= \frac{V_2(F) - V_1(F)}{z_{12}} = \frac{0.65 - 0.60}{j0.5} = -j0.1 \text{ pu} \\ I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.60 - 0.38}{j0.2} = -j1.1 \text{ pu} \\ I_{23}(F) &= \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.65 - 0.38}{j0.3} = -j0.9 \text{ pu} \\ I_{34}(F) &= \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.38 - 0}{j0.19} = -j2 \text{ pu} \end{aligned}$$

(c) From (9.22), for a fault at bus 2 with fault impedance $Z_f = j0.0225$ per unit, the fault current is

$$I_2(F) = \frac{V_2(0)}{Z_{22} + Z_f} = \frac{1.0}{j0.2275 + j0.0225} = -j4 \text{ pu}$$

From (9.23), bus voltages during the fault are

$$\begin{aligned} V_1(F) &= V_1(0) - Z_{12}I_2(F) = 1.0 - (j0.140)(-j4) = 0.44 \text{ pu} \\ V_2(F) &= V_2(0) - Z_{22}I_2(F) = 1.0 - (j0.2275)(-j4) = 0.09 \text{ pu} \\ V_3(F) &= V_3(0) - Z_{32}I_2(F) = 1.0 - (j0.175)(-j4) = 0.30 \text{ pu} \\ V_4(F) &= V_4(0) - Z_{42}I_2(F) = 1.0 - (j0.175)(-j4) = 0.30 \text{ pu} \end{aligned}$$

From (9.25), the short circuit currents in the lines are

$$\begin{aligned} I_{12}(F) &= \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.44 - 0.09}{j0.5} = -j0.7 \text{ pu} \\ I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.44 - 0.30}{j0.2} = -j0.7 \text{ pu} \\ I_{32}(F) &= \frac{V_3(F) - V_2(F)}{z_{23}} = \frac{0.30 - 0.09}{j0.3} = -j0.7 \text{ pu} \\ I_{34}(F) &= \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.30 - 0.30}{j0.19} = 0 \text{ pu} \end{aligned}$$

To check the result, we form the line data for the network of Problem 9.11, and we use the following commands

```

zdata=[0 1 0 0.40
        0 2 0 0.35
        1 2 0 0.50
        1 3 0 0.20
        2 3 0 0.30
        3 4 0 0.19];
Zbus=zbuild(zdata)
symfault(zdata, Zbus)

```

The result is

```

Zbus =
  0 + 0.2400i    0 + 0.1400i    0 + 0.2000i    0 + 0.2000i
  0 + 0.1400i    0 + 0.2275i    0 + 0.1750i    0 + 0.1750i
  0 + 0.2000i    0 + 0.1750i    0 + 0.3100i    0 + 0.3100i
  0 + 0.2000i    0 + 0.1750i    0 + 0.3100i    0 + 0.5000i

```

Enter Faulted Bus No. -> 4

Enter Fault Impedance Zf = R + j*X in complex form (for bolted fault enter 0). Zf = 0

Balanced three-phase fault at bus No. 4
Total fault current = 2.0000 per unit

Bus Voltages during fault in per unit

Bus No.	Voltage Magnitude	Angle degrees
1	0.6000	0.0000
2	0.6500	0.0000
3	0.3800	0.0000
4	0.0000	0.0000

Line currents for fault at bus No. 4

From Bus	To Bus	Current Magnitude	Angle degrees
G	1	1.0000	-90.0000
1	3	1.1000	-90.0000
G	2	1.0000	-90.0000
2	1	0.1000	-90.0000
2	3	0.9000	-90.0000
3	4	2.0000	-90.0000
4	F	2.0000	-90.0000

Another fault location? Enter 'y' or 'n' within single quote->'y'
Enter Faulted Bus No. -> 2

Enter Fault Impedance $Z_f = R + jX$ in
complex form (for bolted fault enter 0). $Z_f = j*0.0225$
Balanced three-phase fault at bus No. 2
Total fault current = 4.0000 per unit

Bus Voltages during fault in per unit

Bus No.	Voltage Magnitude	Angle degrees
1	0.4400	0.0000
2	0.0900	0.0000
3	0.3000	0.0000
4	0.3000	0.0000

Line currents for fault at bus No. 2

From Bus	To Bus	Current Magnitude	Angle degrees
G	1	1.4000	-90.0000
1	2	0.7000	-90.0000
1	3	0.7000	-90.0000
G	2	2.6000	-90.0000
2	F	4.0000	-90.0000
3	2	0.7000	-90.0000

9.12. The per unit bus impedance matrix for the power system shown in Figure 78 is given by

$$Z_{bus} = j \begin{bmatrix} 0.150 & 0.075 & 0.140 & 0.135 \\ 0.075 & 0.1875 & 0.090 & 0.0975 \\ 0.140 & 0.090 & 0.2533 & 0.210 \\ 0.135 & 0.0975 & 0.210 & 0.2475 \end{bmatrix}$$

A three-phase fault occurs at bus4 through a fault impedance of $Z_f = j0.0025$ per unit. Using the bus impedance matrix calculate the fault current, bus voltages and line currents during fault. Check your result using the **Zbuild** and **symfault** programs.

From (9.22), for a fault at bus 2 with fault impedance $Z_f = j0.0025$ per unit, the fault current is

$$I_4(F) = \frac{V_4(0)}{Z_{44} + Z_f} = \frac{1.0}{j0.2475 + j0.0025} = -j4 \text{ pu}$$

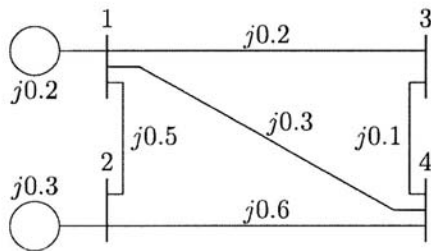


FIGURE 78
One-line diagram for Problem 9.12.

From (9.23), bus voltages during the fault are

$$\begin{aligned} V_1(F) &= V_1(0) - Z_{14}I_4(F) = 1.0 - (j0.135)(-j4) = 0.46 \text{ pu} \\ V_2(F) &= V_2(0) - Z_{24}I_4(F) = 1.0 - (j0.0975)(-j4) = 0.61 \text{ pu} \\ V_3(F) &= V_3(0) - Z_{34}I_4(F) = 1.0 - (j0.210)(-j4) = 0.16 \text{ pu} \\ V_4(F) &= V_4(0) - Z_{44}I_4(F) = 1.0 - (j0.2475)(-j4) = 0.01 \text{ pu} \end{aligned}$$

From (9.25), the short circuit currents in the lines are

$$\begin{aligned} I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.46 - 0.16}{j0.2} = -j1.5 \text{ pu} \\ I_{14}(F) &= \frac{V_1(F) - V_4(F)}{z_{14}} = \frac{0.46 - 0.01}{j0.3} = -j1.5 \text{ pu} \\ I_{21}(F) &= \frac{V_2(F) - V_1(F)}{z_{12}} = \frac{0.61 - 0.46}{j0.5} = -j0.3 \text{ pu} \\ I_{24}(F) &= \frac{V_2(F) - V_4(F)}{z_{24}} = \frac{0.61 - 0.01}{j0.6} = -j1.0 \text{ pu} \\ I_{34}(F) &= \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.16 - 0.01}{j0.10} = -j1.5 \text{ pu} \end{aligned}$$

To check the result, we form the line data for the network of Problem 9.12, and we use the following commands

```
zdata=[0  1  0  0.2
        0  2  0  0.3
        1  2  0  0.5
        1  3  0  0.2
        1  4  0  0.3
        2  4  0  0.6
        3  4  0  0.1];
```

```
Zbus=zbuild(zdata)
symfault(zdata, Zbus)
```

The result is

```
Zbus =
  0 + 0.1500i    0 + 0.0750i    0 + 0.1400i    0 + 0.1350i
  0 + 0.0750i    0 + 0.1875i    0 + 0.0900i    0 + 0.0975i
  0 + 0.1400i    0 + 0.0900i    0 + 0.2533i    0 + 0.2100i
  0 + 0.1350i    0 + 0.0975i    0 + 0.2100i    0 + 0.2475i
```

Enter Faulted Bus No. -> 4

Enter Fault Impedance $Z_f = R + jX$ in complex form (for bolted fault enter 0). $Z_f = j*0.0025$

Balanced three-phase fault at bus No. 4
Total fault current = 4.0000 per unit

Bus Voltages during fault in per unit

Bus No.	Voltage Magnitude	Angle degrees
1	0.4600	0.0000
2	0.6100	0.0000
3	0.1600	0.0000
4	0.0100	0.0000

Line currents for fault at bus No. 4

From Bus	To Bus	Current Magnitude	Angle degrees
G	1	2.7000	-90.0000
1	3	1.5000	-90.0000
1	4	1.5000	-90.0000
G	2	1.3000	-90.0000
2	1	0.3000	-90.0000
2	4	1.0000	-90.0000
3	4	1.5000	-90.0000
4	F	4.0000	-90.0000

Another fault location? Enter 'y' or 'n' within single quote->'n'

A three-phase fault occurs at bus 3 through a fault impedance of $Z_f = j0.19$ per unit. Using the bus impedance matrix calculate the fault current, bus voltages, and line currents during fault. Check your result using the **Zbuild** and **symfault** programs.

From (9.22), for a fault at bus 3 with fault impedance $Z_f = j0.19$ per unit, the fault current is

$$I_3(F) = \frac{V_3(0)}{Z_{33} + Z_f} = \frac{1.0}{j0.21 + j0.19} = -j2.5 \text{ pu}$$

From (9.23), bus voltages during the fault are

$$V_1(F) = V_1(0) - Z_{13}I_3(F) = 1.0 - (j0.03)(-j2.5) = 0.925 \text{ pu}$$

$$V_2(F) = V_2(0) - Z_{23}I_3(F) = 1.0 - (j0.03)(-j2.5) = 0.925 \text{ pu}$$

$$V_3(F) = V_3(0) - Z_{33}I_3(F) = 1.0 - (j0.21)(-j2.5) = 0.475 \text{ pu}$$

From (9.25), the short circuit currents in the lines are

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.925 - 0.925}{j0.75} = 0 \text{ pu}$$

$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.925 - 0.475}{j0.3} = -j1.5 \text{ pu}$$

$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.925 - 0.475}{j0.45} = -j1.0 \text{ pu}$$

To check the result, we form the line data for the network of Problem 9.4, and we use the following commands

```
zdata=[0  1  0  0.05
        0  2  0  0.075
        1  2  0  0.75
        1  3  0  0.30
        2  3  0  0.45];
Zbus=zbuild(zdata)
symfault(zdata, Zbus)
```

The result is

Zbus =

```
0 + 0.0450i    0 + 0.0075i    0 + 0.0300i
0 + 0.0075i    0 + 0.0638i    0 + 0.0300i
0 + 0.0300i    0 + 0.0300i    0 + 0.2100i
```

9.10. The per unit bus impedance matrix for the power system of Problem 9.4 is given by

$$Z_{bus} = j \begin{bmatrix} 0.0450 & 0.0075 & 0.0300 \\ 0.0075 & 0.06375 & 0.0300 \\ 0.0300 & 0.0300 & 0.2100 \end{bmatrix}$$

A three-phase fault occurs at bus 3 through a fault impedance of $Z_f = j0.19$ per unit. Using the bus impedance matrix calculate the fault current, bus voltages, and line currents during fault. Check your result using the **Zbuild** and **symfault** programs.

From (9.22), for a fault at bus 3 with fault impedance $Z_f = j0.19$ per unit, the fault current is

$$I_3(F) = \frac{V_3(0)}{Z_{33} + Z_f} = \frac{1.0}{j0.21 + j0.19} = -j2.5 \text{ pu}$$

From (9.23), bus voltages during the fault are

$$\begin{aligned} V_1(F) &= V_1(0) - Z_{13}I_3(F) = 1.0 - (j0.03)(-j2.5) = 0.925 \text{ pu} \\ V_2(F) &= V_2(0) - Z_{23}I_3(F) = 1.0 - (j0.03)(-j2.5) = 0.925 \text{ pu} \\ V_3(F) &= V_3(0) - Z_{33}I_3(F) = 1.0 - (j0.21)(-j2.5) = 0.475 \text{ pu} \end{aligned}$$

From (9.25), the short circuit currents in the lines are

$$\begin{aligned} I_{12}(F) &= \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.925 - 0.925}{j0.75} = 0 \text{ pu} \\ I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.925 - 0.475}{j0.3} = -j1.5 \text{ pu} \\ I_{23}(F) &= \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.925 - 0.475}{j0.45} = -j1.0 \text{ pu} \end{aligned}$$

To check the result, we form the line data for the network of Problem 9.4, and we use the following commands

```
zdata=[0 1 0 0.05
        0 2 0 0.075
        1 2 0 0.75
        1 3 0 0.30
        2 3 0 0.45];
Zbus=zbuild(zdata)
symfault(zdata, Zbus)
```

The result is

Zbus =

```
0 + 0.0450i    0 + 0.0075i    0 + 0.0300i
0 + 0.0075i    0 + 0.0638i    0 + 0.0300i
0 + 0.0300i    0 + 0.0300i    0 + 0.2100i
```

Enter Faulted Bus No. -> 3

Enter Fault Impedance $Z_f = R + jX$ in
complex form (for bolted fault enter 0). $Z_f = j*.19$

Balanced three-phase fault at bus No. 3

Total fault current = 2.5000 per unit

Bus Voltages during fault in per unit

Bus No.	Voltage Magnitude	Angle degrees
1	0.9250	0.0000
2	0.9250	0.0000
3	0.4750	0.0000

Line currents for fault at bus No. 3

From Bus	To Bus	Current Magnitude	Angle degrees
G	1	1.5000	-90.0000
1	2	0.0000	-90.0000
1	3	1.5000	-90.0000
G	2	1.0000	-90.0000
2	3	1.0000	-90.0000
3	F	2.5000	-90.0000

Another fault location? Enter 'y' or 'n' within single quote->'n'