**6.9.** The one-line diagram of a four-bus power system is as shown in Figure 56. Reactances are given in per unit on a common MVA base. Transformers $T_1$ and $T_2$ have tap settings of 0.8:1, and 1.25:1 respectively. Obtain the bus admittance matrix.

From Figure 6.15 in the text, the sending end transformer $\pi$ model is

$$y_{12} = \frac{y_t}{a} = \frac{-j80}{0.8} = -j100$$
$$y_{10} = \left(\frac{1-a}{a^2}\right) y_t = \frac{1-0.8}{0.64}(-j80) = -j25$$
$$y_{30} = \left(\frac{a-1}{a}\right) y_t = \frac{0.8-1}{0.8}(-j80) = j20$$

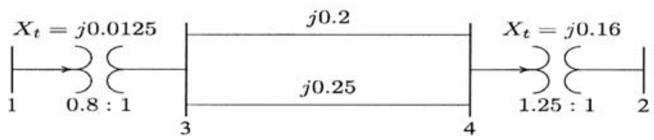


**FIGURE 56**
One-line diagram for Problem 6.9.

and the receiving end transformer $\pi$ model is

$$y_{42} = \frac{y_t}{a} = \frac{-j6.25}{1.25} = -j5$$
$$y_{40} = \left(\frac{1-a}{a^2}\right) y_t = \frac{1-1.25}{1.5625}(-j6.25) = j1$$
$$y_{20} = \left(\frac{a-1}{a}\right) y_t = \frac{1.25-1}{1.25}(-j6.25) = -j1.25$$

All line impedances are converted to admittances, and the admittance diagram is constructed as shown in Figure 57.

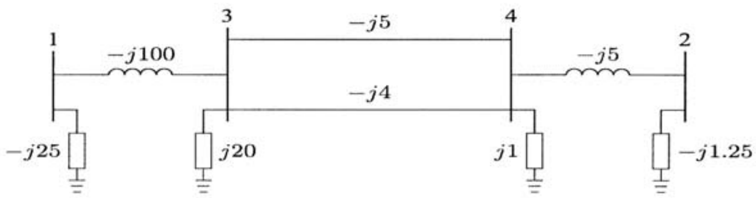


**FIGURE 57**
Admittance diagram for the system of Problem 6.9.

The bus admittance matrix for the network of Figure 57 obtained by inspection is

$$Y_{bus} = \begin{bmatrix} -j125 & 0 & j100 & 0 \\ 0 & -j6.25 & 0 & j5 \\ j100 & 0 & -j89 & j9 \\ 0 & j5 & j9 & -j13 \end{bmatrix}$$

The script file **lfybus** can be used to compute the bus admittance matrix. We use the following statements.

```
clear
%                                        Line code
%         Bus bus  R      X    1/2 B  = 1 for lines
%         nl  nr  pu     pu     pu   >1 or <1 tr. tap at bus nl
linedata=[1   3   0.0  0.0125  0.0         0.8
          3   4   0.0  0.20    0.0         1
          3   4   0.0  0.25    0.0         1
          4   2   0.0  0.16    0.0         1.25];
lfybus                          % form the bus admittance matrix
Ybus                              % displays Ybus  on the screen
```

Run **ch6p9** to obtain the bus admittance matrix.

**6.10.** In the two-bus system shown in Figure 58, bus 1 is a slack bus with $V_1 = 1.0\angle 0°$ pu. A load of 150 MW and 50 Mvar is taken from bus 2. The line admittance is $y_{12} = 10\angle -73.74°$ pu on a base of 100 MVA. The expression for real and reactive power at bus 2 is given by

$$P_2 = 10|V_2||V_1|\cos(106.26° - \delta_2 + \delta_1) + 10|V_2|^2\cos(-73.74°)$$
$$Q_2 = -10|V_2||V_1|\sin(106.26° - \delta_2 + \delta_1) - 10|V_2|^2\sin(-73.74°)$$

Using Newton-Raphson method, obtain the voltage magnitude and phase angle of bus 2. Start with an initial estimate of $|V_2|^{(0)} = 1.0$ pu and $\delta_2^{(0)} = 0°$. Perform two iterations. Partial derivatives of $P_2$, and $Q_2$ with respect to $|V_2|$, and $\delta_2$ are

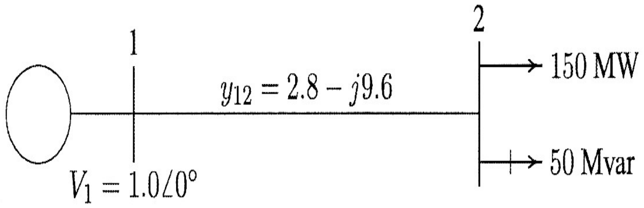


**FIGURE 58**
One-line diagram for Problem 6.10.

$$\frac{\partial P_2}{\partial \delta_2} = 10|V_2||V_1|\sin(106.26° - \delta_2 + \delta_1)$$
$$\frac{\partial P_2}{\partial |V_2|} = 10|V_1|\cos(106.26° - \delta_2 + \delta_1) + 20|V_2|\cos(-73.74°)$$
$$\frac{\partial Q_2}{\partial \delta_2} = 10|V_2||V_1|\cos(106.26° - \delta_2 + \delta_1)$$
$$\frac{\partial Q_2}{\partial |V_2|} = -10|V_1|\sin(106.26° - \delta_2 + \delta_1) - 20|V_2|\sin(-73.74°)$$

The load expressed in per unit is

$$S_2^{sch} \;=\; -\frac{(150+j50)}{100} = -1.5 - j0.5 \text{ pu}$$

The slack bus voltage is $V_1 = 1.0\angle 0$ pu. Starting with an initial estimate of $|V_2^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, the power residuals are computed from (6.63) and (6.64)

$$\begin{aligned}
\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} &= -1.5 - [10\cos(106.26°) + 10\cos(-73.74°)] \\
&= -1.5 \text{ pu} \\
\Delta Q_2^{(0)} = Q_2^{sch} - Q_2^{(0)} &= -0.5 - [-10\sin(106.26°) - 10\sin(-73.74°)] \\
&= -0.5 \text{ pu}
\end{aligned}$$

The elements of the Jacobian matrix at the initial estimate are

$$\begin{aligned}
J_1^{(0)} &= 10(1)(1)\sin(106.26°) = 9.6 \\
J_2^{(0)} &= 10(1)\cos(106.26°) + 20(1)\cos(-73.74°) = 2.8 \\
J_3^{(0)} &= 10(1)(1)\cos(106.26°) = -2.8 \\
J_4^{(0)} &= -10(1)\sin(106.26°) - 20(1)\sin(-73.74°) = 9.6
\end{aligned}$$

The set of linear equations in the first iteration becomes

$$\begin{bmatrix} -1.5 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 9.6 & 2.8 \\ -2.8 & 9.6 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta|V_2^{(0)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, voltage at bus 2 in the first iteration is

$$\begin{aligned}
\Delta\delta_2^{(0)} &= -0.13 & \delta_2^{(1)} &= 0 + (-0.13) = -0.13 \text{ radian} \\
\Delta|V_2^{(0)}| &= -0.09 & |V_2^{(1)}| &= 1 + (-0.09) = 0.91 \text{ pu}
\end{aligned}$$

For the second iteration, we have

$$\begin{aligned}
\Delta P_2^{(1)} &= P_2^{sch} - P_2^{(1)} = -1.5 - (-1.3403) = -0.1597 \text{ pu} \\
\Delta Q_2^{(1)} &= Q_2^{sch} - Q_2^{(1)} = -0.5 - (-0.3822) = -0.1178 \text{ pu}
\end{aligned}$$

Also, computing the elements of the Jacobian matrix, the set of linear equations in the second iteration becomes

$$\begin{bmatrix} -0.1597 \\ -0.1178 \end{bmatrix} = \begin{bmatrix} 8.332 & 1.0751 \\ -3.659 & 8.3160 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta|V_2^{(1)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, voltage at bus 2 in the second iteration is

$$\Delta\delta_2^{(1)} = -0.0164 \qquad \delta_2^{(2)} = -0.13 + (-0.0164) = -0.1464 \text{ radian}$$
$$\Delta|V_2^{(1)}| = -0.0214 \qquad |V_2^{(2)}| = 0.91 + (-0.0214) = 0.8886 \text{ pu}$$

**6.12.** Figure 60 shows the one-line diagram of a simple three-bus power system with generation at buses 1 and 2. The voltage at bus 1 is $V = 1.0\angle 0°$ per unit. Voltage magnitude at bus 2 is fixed at 1.05 pu with a real power generation of 400 MW. A load consisting of 500 MW and 400 Mvar is taken from bus 3. Line admittances are marked in per unit on a 100 MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.

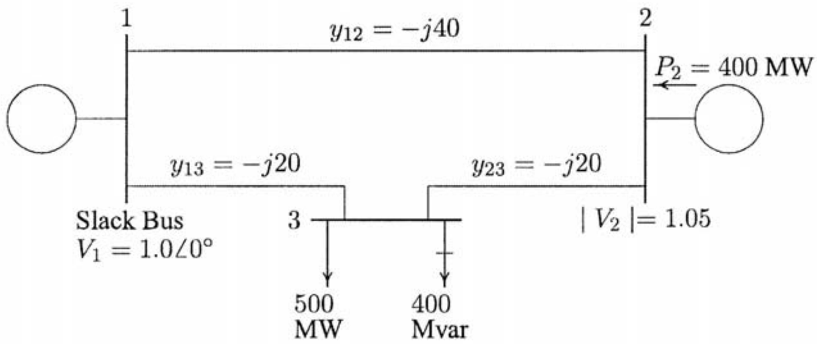


**FIGURE 60**
One-line diagram for problem 6.12.

(a) Show that the expression for the real power at bus 2 and real and reactive power at bus 3 are

$$P_2 = 40|V_2||V_1|\cos(90° - \delta_2 + \delta_1) + 20|V_2||V_3|\cos(90° - \delta_2 + \delta_3)$$
$$P_3 = 20|V_3||V_1|\cos(90° - \delta_3 + \delta_1) + 20|V_3||V_2|\cos(90° - \delta_3 + \delta_2)$$
$$Q_3 = -20|V_3||V_1|\sin(90° - \delta_3 + \delta_1) - 20|V_3||V_2|\sin(90° - \delta_3 + \delta_2) + 40|V_3|^2$$

(b) Using Newton-Raphson method, start with the initial estimates of $V_2^{(0)} = 1.05 + j0$ and $V_3^{(0)} = 1.0 + j0$, and keeping $|V_2| = 1.05$ pu, determine the phasor values of $V_2$ and $V_3$. Perform two iterations.

(c) Check the power flow solution for Problem 6.12 using the **lfnewton** and other required programs. Assume the regulated bus (bus # 2) reactive power limits are between 0 and 600 Mvar.

By inspection, the bus admittance matrix in polar form is

$$Y_{bus} = \begin{bmatrix} 60\angle-\frac{\pi}{2} & 40\angle\frac{\pi}{2} & 20\angle\frac{\pi}{2} \\ 40\angle\frac{\pi}{2} & 60\angle-\frac{\pi}{2} & 20\angle\frac{\pi}{2} \\ 20\angle\frac{\pi}{2} & 20\angle\frac{\pi}{2} & 40\angle-\frac{\pi}{2} \end{bmatrix}$$

(a) The power flow equation with voltages and admittances expressed in polar form is

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j)$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j)$$

Substituting the elements of the bus admittance matrix in the above equations for $P_2$, $P_3$, and $Q_3$ will result in the given equations.

(b) Elements of the Jacobian matrix are obtained by taking partial derivatives of the given equations with respect to $\delta_2$, $\delta_3$ and $|V_3|$.

$$\frac{\partial P_2}{\partial \delta_2} = 40|V_2||V_1|\sin(\frac{\pi}{2} - \delta_2 + \delta_1) + 20|V_2||V_3|\sin(\frac{\pi}{2} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial \delta_3} = -20|V_2||V_3|\sin(\frac{\pi}{2} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial |V_3|} = 20|V_2|\cos(\frac{\pi}{2} - \delta_2 + \delta_3)$$

$$\frac{\partial P_3}{\partial \delta_2} = -20|V_3||V_2|\sin(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial \delta_3} = 20|V_3||V_1|\sin(\frac{\pi}{2} - \delta_3 + \delta_1) + 20|V_3||V_2|\sin(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial |V_3|} = 20|V_1|\cos(\frac{\pi}{2} - \delta_3 + \delta_1) + 20|V_2|\cos(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_3}{\partial \delta_2} = -20|V_3||V_2|\cos(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_3}{\partial \delta_3} = 20|V_3||V_1|\cos(\frac{\pi}{2} - \delta_3 + \delta_1) + 20|V_3||V_2|\cos(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_3}{\partial |V_3|} = -20|V_1|\sin(\frac{\pi}{2} - \delta_3 + \delta_1) - 20|V_2|\sin(\frac{\pi}{2} - \delta_3 + \delta_2) + 80|V_3|$$

The load and generation expressed in per units are

$$P_2^{sch} = \frac{400}{100} = 4.0 \text{ pu}$$
$$S_3^{sch} = -\frac{(500 + j400)}{100} = -5.0 - j4.0 \text{ pu}$$

The slack bus voltage is $V_1 = 1.0\angle 0$ pu, and the bus 2 voltage magnitude is $|V_2| = 1.05$ pu. Starting with an initial estimate of $|V_3^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, and $\delta_3^{(0)} = 0.0$, the power residuals are

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = 4.0 - (0) = 4.0$$
$$\Delta P_3^{(0)} = P_3^{sch} - P_3^{(0)} = -5.0 - (0) = -5.0$$
$$\Delta Q_3^{(0)} = Q_3^{sch} - Q_3^{(0)} = -4.0 - (-1.0) = -3.0$$

Evaluating the elements of the Jacobian matrix with the initial estimate, the set of linear equations in the first iteration becomes

$$\begin{bmatrix} -2.8600 \\ 1.4384 \\ -0.2200 \end{bmatrix} = \begin{bmatrix} 63 & -21 & 0 \\ -21 & 41 & 0 \\ 0 & 0 & 39 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta|V_3^{(0)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, the new bus voltages in the first iteration are

$$\begin{aligned} \Delta\delta_2^{(0)} &= 0.0275 \qquad \delta_2^{(1)} = 0 + 0.0275 = 0.0275 \text{ radian} = 1.5782^\circ \\ \Delta\delta_3^{(0)} &= -0.1078 \quad \delta_3^{(1)} = 0 + (-0.1078) = -0.1078 \text{ radian} = -6.1790^\circ \\ \Delta|V_3^{(0)}| &= -0.0769 \quad |V_3^{(1)}| = 1 + (-0.0769) = 0.9231 \text{ pu} \end{aligned}$$

For the second iteration, we have

$$\begin{bmatrix} 0.2269 \\ -0.3965 \\ -0.5213 \end{bmatrix} = \begin{bmatrix} 61.1913 & -19.2072 & 2.8345 \\ -19.2072 & 37.5615 & -4.9871 \\ 2.6164 & -4.6035 & 33.1545 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta\delta_3^{(1)} \\ \Delta|V_3^{(1)}| \end{bmatrix}$$

and

$$\begin{aligned} \Delta\delta_2^{(1)} &= 0.0006 \qquad \delta_2^{(2)} = 0.0275 + 0.0006 = 0.0281 \text{ radian} = 1.61^\circ \\ \Delta\delta_3^{(1)} &= -0.0126 \quad \delta_3^{(2)} = -0.1078 + (-0.0126) = -0.1204 \text{ radian} = -6.898^\circ \\ \Delta|V_3^{(1)}| &= -0.0175 \quad |V_3^{(2)}| = 0.9231 + (-0.0175) = 0.9056 \text{ pu} \end{aligned}$$

(c) The power flow program **lfnewton** is used to obtain the solution, with the following statements:

```
clear
basemva = 100;  accuracy = 0.000001;  maxiter = 10;

%       Problem   6.12(c)
%       Bus  Bus Voltage Angle -Load---   -Generator-- Injected
%        No code Mag. Degree MW  MVAR  MW  MVAR Qmin Qmax  Mvar
busdata=[1   1  1.0   0.0  0.0  0.0   0.0  0.0   0   0     0
         2   2  1.05  0.0    0    0   400  0.0  600  0     0
         3   0  1.0   0.0  500  400   0.0  0.0   0   0     0];

%                                     Line code
%         Bus bus  R     X     1/2 B   = 1 for lines
%          nl  nr pu    pu       pu    >1 or <1 tr. tap at bus nl
linedata=[1   2  0.0  0.025    0.0         1
          1   3  0.0  0.05     0.0         1
          2   3  0.0  0.05     0.0         1];
disp('Problem 6.12(c)')
lfybus                         % form the bus admittance matrix
lfnewton            % Power flow solution by Gauss-Seidel method
busout            % Prints the power flow solution on the screen
lineflow      % Computes and displays the line flow and losses
```

The above statements are saved in the file **ch6p12c.m**. Run the program to obtain the solution.