6.4. A fourth-order polynomial equation is given by

$$x^4 - 21x^3 + 147x^2 - 379x + 252 = 0$$

(a) Use Newton-Raphson method and hand calculations to find one of the roots of the polynomial equation. Start with the initial estimate of $x^{(0)} = 0$ and continue until $|\Delta x^{(k)}| < 0.001$.
(b) Write a *MATLAB* program to find the roots of the above polynomial by Newton-Raphson method. The program should prompt the user to input the initial estimate. Run using the initial estimates of 0, 3, 6, 10.
(c) Check your answers using the *MATLAB* function **r = roots(A)**, where **A** is a row vector containing the polynomial coefficients in descending powers.

$$\frac{df(x)}{dx} = 4x^3 - 63x^2 + 294x - 379$$

(a) for $x^{(0)} = 0$, we have

$$\Delta c^{(0)} = c - f(x^{(0)}) = 0 - (252) = -252$$

$$\left(\frac{df}{dx}\right)^{(0)} = -379$$

$$\Delta x^{(0)} = \frac{\Delta c^{(0)}}{\left(\frac{df}{dx}\right)^{(0)}} = \frac{-252}{-379} = 0.6649$$

Therefore, the result at the end of the first iteration is

$$x^{(1)} = x^{(0)} + \Delta x^{(0)} = 0 + 0.6649 = 0.6649$$

The subsequent iterations result in

$$\Delta c^{(1)} = c - f(x^{(1)}) = 0 - [(0.6649)^4 - 21(0.6649)^3 + 147(0.6649)^2 \\ -379(0.6649) + 252] = -59.0114$$

$$\left(\frac{df}{dx}\right)^{(1)} = 4(0.6649)^3 - 63(0.6649)^2 + 294(0.6649) - 379 = -210.194$$

$$\Delta x^{(1)} = \frac{\Delta c^{(1)}}{\left(\frac{df}{dx}\right)^{(1)}} = \frac{-59.0114}{-210.194} = 0.28075$$

$$x^{(2)} = x^{(1)} + \Delta x^{(1)} = 0.6649 + 0.28075 = 0.9457$$

$$\Delta c^{(2)} = c - f(x^{(2)}) = 0 - [(0.9457)^4 - 21(0.9457)^3 + 147(0.9457)^2 \\ -379(0.9457) + 252] = -8.0942$$

$$\left(\frac{df}{dx}\right)^{(2)} = 4(0.9457)^3 - 63(0.9457)^2 + 294(0.9457) - 379 = -153.9333$$

$$\Delta x^{(2)} = \frac{\Delta c^{(2)}}{\left(\frac{df}{dx}\right)^{(2)}} = \frac{-8.0942}{-153.9333} = 0.05258$$

$$x^{(3)} = x^{(2)} + \Delta x^{(2)} = 0.9457 + 0.05258 = 0.9982$$

$$\Delta c^{(3)} = c - f(x^{(3)}) = 0 - [(0.9982)^4 - 21(0.9982)^3 + 147(0.9982)^2 \\ -379(0.9982) + 252] = -0.2541$$

$$\left(\frac{df}{dx}\right)^{(3)} = 4(0.9982)^3 - 63(0.9982)^2 + 294(0.9982) - 379 = -144.3174$$

$$\Delta x^{(3)} = \frac{\Delta c^{(3)}}{\left(\frac{df}{dx}\right)^{(3)}} = \frac{-0.2541}{-144.3174} = 0.0018$$

$$x^{(4)} = x^{(3)} + \Delta x^{(3)} = 0.9982 + 0.0018 = 1.0000$$

$$\Delta c^{(4)} = c - f(x^{(4)}) = 0 - [(1.0000)^4 - 21(1.0000)^3 + 147(1.0000)^2 \\ -379(1.0000) + 252] = -0.0003$$

$$\left(\frac{df}{dx}\right)^{(4)} = 4(1.0000)^3 - 63(1.0000)^2 + 294(1.0000) - 379 = -144.0003$$

$$\Delta x^{(4)} = \frac{\Delta c^{(4)}}{\left(\frac{df}{dx}\right)^{(4)}} = \frac{-0.0003}{-144.0003} = 0.0000$$

$$x^{(5)} = x^{(4)} + \Delta x^{(4)} = 1.0000 + 0.0000 = 1.0000$$

The following commands show the procedure for the solution of the given equation by the Newton-Raphson method.

```
dx=1;              % Change in variable is set to a high value
x=input('Enter the initial estimate -> ');  % Initial estimate
iter = 0;                                    % Iteration counter
disp('iter   Dc        J         dx       x')%Heading for result
while abs(dx) >= 0.001 & iter < 100     % Test for convergence
iter = iter + 1;                             % No. of iterations
Dc=0 - (x^4-21*x^3+147*x^2-379*x+252);                % Residual
J = 4*x^3-63*x^2+ 294*x-379;                        % Derivative
dx= Dc/J;                                    %Change in variable
x=x+dx;                                    % Successive solution
fprintf('%g', iter), disp([Dc, J, dx, x])
```

The result is

```
Enter the initial estimate -> 0

iter   Dc         J          dx        x
1 -252.0000 -379.0000    0.6649    0.6649

2  -59.0114 -210.1938    0.2807    0.9457

3   -8.0942 -153.9333    0.0526    0.9982

4   -0.2541 -144.3174    0.0018    1.0000

5   -0.0003 -144.0003    0.0000    1.0000
```

Repeating for $x^{(0)} = 3$, we have

```
Enter the initial estimate -> 3
iter   Dc         J          dx        x
1   48.0000   44.0000    1.0909    4.0909

2   -4.0128   43.2427   -0.0928    3.9981

3    0.0850   45.0339    0.0019    4.0000

4    0.0000   45.0000    0.0000    4.0000
```

Repeating for $x^{(0)} = 6$, we have

```
Enter the initial estimate -> 6
iter   Dc         J          dx        x
1  -30.0000  -19.0000    1.5789    7.5789

2   19.3714  -28.1850   -0.6873    6.8917

3   -3.8917  -35.7586    0.1088    7.0005

4    0.0175  -36.0000   -0.0005    7.0000
```

Repeating for $x^{(0)} = 10$, we have

```
Enter the initial estimate -> 10
```

```
iter   Dc          J          dx          x
1 -162.0000  261.0000   -0.6207     9.3793

2  -40.6800  136.7617   -0.2975     9.0819

3   -6.9992   91.1091   -0.0768     9.0050

4   -0.4046   80.6659   -0.0050     9.0000

5   -0.0017   80.0027    0.0000     9.0000
```

The above initial estimates have converged to the values of 1, 4, 7, and 9. As we can see from the *MATLAB* **roots** function, these are the roots of the above fourth order polynomial.

```
A = [1 -21  147  -379  252];
r = roots(A)

r =
    9.0000
    7.0000
    4.0000
    1.0000
```

6.7. Figure 6.6 shows the one-line diagram of a simple three-bus power system with generation at bus 1. The voltage at bus 1 is $V_1 = 1.0\angle 0°$ per unit. The scheduled loads on buses 2 and 3 are marked on the diagram. Line impedances are marked in per unit on a 100 MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.

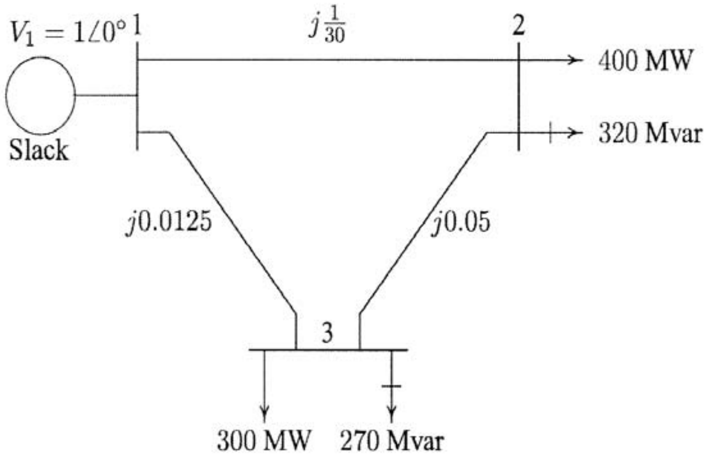
(a) Using Gauss-Seidel method and initial estimates of $V_2^{(0)} = 1.0 + j0$ and $V_3^{(0)} = 1.0 + j0$, determine V_2 and V_3. Perform two iterations.

(b) If after several iterations the bus voltages converge to

$$V_2 = 0.90 - j0.10 \text{ pu}$$
$$V_3 = 0.95 - j0.05 \text{ pu}$$

determine the line flows and line losses and the slack bus real and reactive power. Construct a power flow diagram and show the direction of the line flows.

(c) Check the power flow solution using the **lfgauss** and other required programs. (Refer to Example 6.9.) Use a power accuracy of 0.00001 and an acceleration factor of 1.0.



(a) Line impedances are converted to admittances

$$y_{12} = -j30$$
$$y_{13} = \frac{1}{j0.0125} = -j80$$
$$y_{23} = \frac{1}{j0.05} = -j20$$

At the P-Q buses, the complex loads expressed in per units are

$$S_2^{sch} = -\frac{(400 + j320)}{100} = -4.0 - j3.2 \text{ pu}$$
$$S_3^{sch} = -\frac{(300 + j270)}{100} = -3.0 - j2.7 \text{ pu}$$

For hand calculation, we use (6.28). Bus 1 is taken as reference bus (slack bus). Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.0 + j0.0$, V_2 and V_3 are computed from (6.28) as follows

$$\begin{aligned} V_2^{(1)} &= \frac{\frac{S_2^{sch^*}}{V_2^{(0)^*}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}} \\ &= \frac{\frac{-4.0+j3.2}{1.0-j0} + (-j30)(1.0 + j0) + (-j20)(1.0 + j0)}{-j50} \\ &= 0.936 - j0.08 \end{aligned}$$

and

$$\begin{aligned} V_3^{(1)} &= \frac{\frac{S_3^{sch^*}}{V_3^{(0)^*}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}} \\ &= \frac{\frac{-3.0+j2.7}{1-j0} + (-j80)(1.0 + j0) + (-j20)(0.936 - j0.08)}{-j100} \\ &= 0.9602 - j0.046 \end{aligned}$$

For the second iteration we have

$$\begin{aligned} V_2^{(2)} &= \frac{\frac{-4.0+j3.2}{0.936+j0.08} + (-j30)(1.0 + j0) + (-j20)(0.9602 - j0.046)}{-j50} \\ &= 0.9089 - j0.0974 \end{aligned}$$

and

$$\begin{aligned} V_3^{(2)} &= \frac{\frac{-3.0+j2.7}{0.9602+j0.046} + (-j80)(1.0 + j0) + (-j20)(0.9089 - j0.0974)}{(-j100)} \\ &= 0.9522 - j0.0493 \end{aligned}$$

The process is continued and a solution is converged with an accuracy of 5×10^{-5} per unit in seven iterations as given below.

$$
\begin{aligned}
V_2^{(3)} &= 0.9020 - j0.0993 & V_3^{(3)} &= 0.9505 - j0.0498 \\
V_2^{(4)} &= 0.9004 - j0.0998 & V_3^{(4)} &= 0.9501 - j0.0500 \\
V_2^{(5)} &= 0.9001 - j0.1000 & V_3^{(5)} &= 0.9500 - j0.0500 \\
V_2^{(6)} &= 0.9000 - j0.1000 & V_3^{(6)} &= 0.9500 - j0.0500 \\
V_2^{(7)} &= 0.9000 - j0.1000 & V_3^{(7)} &= 0.9500 - j0.0500
\end{aligned}
$$

The final solution is

$$
\begin{aligned}
V_2 &= 0.90 - j0.10 = 0.905554\angle{-6.34^\circ} \quad \text{pu} \\
V_3 &= 0.95 - j0.05 = 0.9513\angle{-3.0128^\circ} \quad \text{pu}
\end{aligned}
$$

(b) With the knowledge of all bus voltages, the slack bus power is obtained from (6.27)

$$
\begin{aligned}
P_1 - jQ_1 &= V_1^*[V_1(y_{12} + y_{13}) - (y_{12}V_2 + y_{13}V_3)] \\
&= 1.0[1.0(-j30 - j80) - (-j30)(0.9 - j0.1) - \\
&\quad (-j80)(0.95 - j0.05)] \\
&= 7.0 - j7.0
\end{aligned}
$$

or the slack bus real and reactive powers are $P_1 = 7.0$ pu = 700 MW and $Q_1 = 7.0$ pu = 700 Mvar.

To find the line flows, first the line currents are computed. With line charging capacitors neglected, the line currents are

$$
\begin{aligned}
I_{12} &= y_{12}(V_1 - V_2) = (-j30)[(1.0 + j0) - (0.90 - j0.10)] = 3.0 - j3.0 \\
I_{21} &= -I_{12} = -3.0 + j3.0 \\
I_{13} &= y_{13}(V_1 - V_3) = (-j80)[(1.0 + j0) - (0.95 - j.05)] = 4.0 - j4.0 \\
I_{31} &= -I_{13} = -4.0 + j4.0 \\
I_{23} &= y_{23}(V_2 - V_3) = (-j20)[(0.90 - j0.10) - (0.95 - j.05)] = -1.0 + j1.0 \\
I_{32} &= -I_{23} = 1.0 - j1.0
\end{aligned}
$$

The line flows are

$$
\begin{aligned}
S_{12} = V_1 I_{12}^* &= (1.0 + j0.0)(3.0 + j3) = 3.0 + j3.0 \ \text{pu} \\
&= 300 \ \text{MW} + j300 \ \text{Mvar}
\end{aligned}
$$

$$
\begin{aligned}
S_{21} = V_2 I_{21}^* &= (0.90 - j0.10)(-3 - j3) = -3.0 - j2.4 \text{ pu} \\
&= -300 \text{ MW} - j240 \text{ Mvar} \\
S_{13} = V_1 I_{13}^* &= (1.0 + j0.0)(4.0 + j4.0) = 4.0 + j4.0 \text{ pu} \\
&= 400 \text{ MW} + j400 \text{ Mvar} \\
S_{31} = V_3 I_{31}^* &= (0.95 - j0.05)(-4.0 - j4.0) = -4.0 - j3.6 \text{ pu} \\
&= -400 \text{ MW} - j360 \text{ Mvar} \\
S_{23} = V_2 I_{23}^* &= (0.90 - j0.10)(-1.0 - j1.0) = -1.0 - j0.80 \text{ pu} \\
&= -100 \text{ MW} - j80 \text{ Mvar} \\
S_{32} = V_3 I_{32}^* &= (0.95 - j0.05)(1 + j1) = 1.0 + j0.9 \text{ pu} \\
&= 100 \text{ MW} + j90 \text{ Mvar}
\end{aligned}
$$

and the line losses are

$$
\begin{aligned}
S_{L\,12} &= S_{12} + S_{21} = 0.0 \text{ MW} + j60 \text{ Mvar} \\
S_{L\,13} &= S_{13} + S_{31} = 0.0 \text{ MW} + j40 \text{ Mvar} \\
S_{L\,23} &= S_{23} + S_{32} = 0.0 \text{ MW} + j10 \text{ Mvar}
\end{aligned}
$$

The power flow diagram is shown in Figure 6.7, where real power direction is indicated by $\rightarrow$ and the reactive power direction is indicated by $\mapsto$. The values within parentheses are the real and reactive losses in the line.

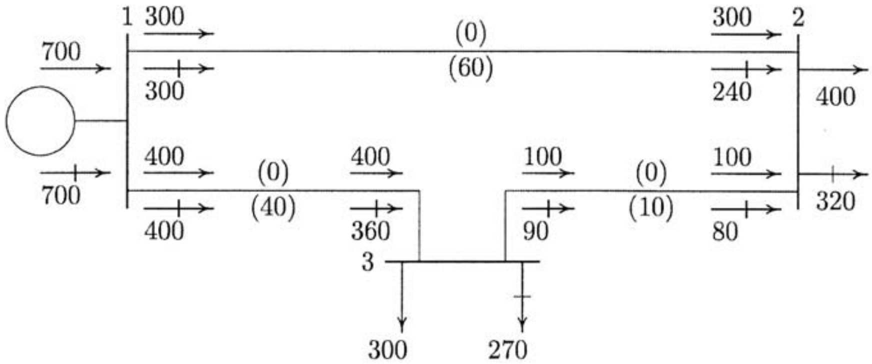


FIGURE 53
Power flow diagram of Problem 6.7 (powers in MW and Mvar).

(c) The power flow program **lfgauss** is used to obtain the solution, with the following statements:

```
clear
basemva = 100;  accuracy = 0.000001; accel = 1.1; maxiter = 100;
```

```
%       Problem   6.7(c)
%       Bus  Bus Voltage Angle -Load---   -Generator-- Injected
%        No code Mag. Degree MW   MVAR   MW  MVAR Qmin Qmax  Mvar
busdata=[1    1  1.0    0.0  0.0  0.0    0.0  0.0    0    0     0
         2    0  1.0    0.0  400  320    0.0  0.0    0    0     0
         3    0  1.0    0.0  300  270    0.0  0.0    0    0     0];

%                                       Line code
%         Bus bus   R     X    1/2 B    = 1 for lines
%          nl  nr pu    pu      pu      >1 or <1 tr. tap at bus nl
linedata=[1   2  0.0  1/30    0.0          1
          1   3  0.0  0.0125  0.0          1
          2   3  0.0  0.050   0.0          1];
disp('Problem 6.7(c)')
lfybus                            % form the bus admittance matrix
lfgauss              % Load flow solution by Gauss-Seidel method
busout            % Prints the power flow solution on the screen
lineflow        % Computes and displays the line flow and losses
```

The above statements are saved in the file **ch6p7c.m**. Run the program to obtain the solution.

6.8. Figure 54 shows the one-line diagram of a simple three-bus power system with generation at buses 1 and 3. The voltage at bus 1 is $V_1 = 1.025\angle 0^\circ$ per unit. Voltage magnitude at bus 3 is fixed at 1.03 pu with a real power generation of 300 MW. A load consisting of 400 MW and 200 Mvar is taken from bus 2. Line impedances are marked in per unit on a 100 MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.

(a) Using Gauss-Seidel method and initial estimates of $V_2^{(0)} = 1.0 + j0$ and $V_3^{(0)} = 1.03 + j0$ and keeping $|V_3| = 1.03$ pu, determine the phasor values of V_2 and V_3. Perform two iterations.
(b) If after several iterations the bus voltages converge to

$$V_2 = 1.001243\angle -2.1^\circ = 1.000571 - j0.0366898 \text{ pu}$$
$$V_3 = 1.03\angle 1.36851^\circ = 1.029706 + j.0246 \text{ pu}$$

determine the line flows and line losses and the slack bus real and reactive power. Construct a power flow diagram and show the direction of the line flows.
(c) Check the power flow solution using the **lfgauss** and other required programs. (Refer to Example 6.9.)

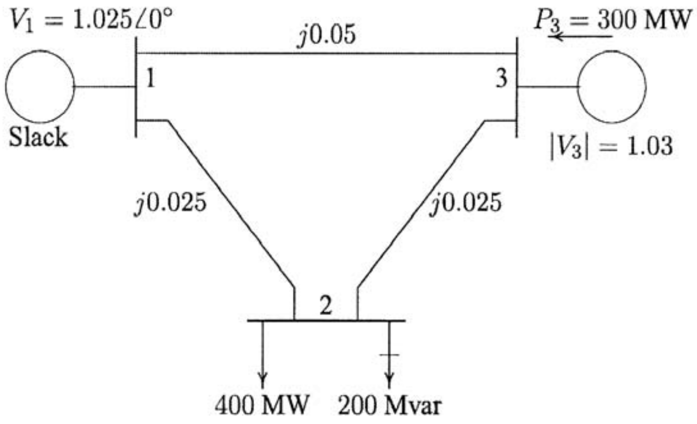


FIGURE 54
One-line diagram of Problem 6.8.

(a) Line impedances converted to admittances are $y_{12} = -j40$, $y_{13} = -j20$ and $y_{23} = -j40$. The load and generation expressed in per units are

$$S_2^{sch} = -\frac{(400 + j200)}{100} = -4.0 - j2.0 \text{ pu}$$

$$P_3^{sch} = \frac{300}{100} = 3.0 \text{ pu}$$

Bus 1 is taken as the reference bus (slack bus). Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.03 + j0.0$, V_2 and V_3 are computed from (6.28).

$$\begin{aligned} V_2^{(1)} &= \frac{\frac{S_2^{sch*}}{V_2^{(0)*}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}} \\ &= \frac{\frac{-4.0+j2.0}{1.0-j0} + (-j40)(1.025 + j0) + (-j40)(1.03 + j0)}{(-j80)} \\ &= 1.0025 - j0.05 \end{aligned}$$

Bus 3 is a regulated bus where voltage magnitude and real power are specified. For the voltage-controlled bus, first the reactive power is computed from (6.30)

$$\begin{aligned} Q_3^{(1)} &= -\Im\{V_3^{(0)*}[V_3^{(0)}(y_{13} + y_{23}) - y_{13}V_1 - y_{23}V_2^{(1)}]\} \\ &= -\Im\{(1.03 - j0)[(1.03 + j0)(-j60) - (-j20)(1.025 + j0) \\ &\quad -(-j40)(1.0025 - j0.05)]\} \\ &= 1.236 \end{aligned}$$

The value of $Q_3^{(1)}$ is used as Q_3^{sch} for the computation of voltage at bus 3. The complex voltage at bus 3, denoted by $V_{c3}^{(1)}$, is calculated

$$
\begin{aligned}
V_{c3}^{(1)} &= \frac{\frac{S_3^{sch*}}{V_3^{(0)*}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}} \\
&= \frac{\frac{3.0 - j1.236}{1.03 - j0} + (-j20)(1.025 + j0) + (-j40)(1.0025 - j0.05)}{(-j60)} \\
&= 1.0300 + j0.0152
\end{aligned}
$$

Since $|V_3|$ is held constant at 1.03 pu, only the imaginary part of $V_{c3}^{(1)}$ is retained, i.e, $f_3^{(1)} = 0.0152$, and its real part is obtained from

$$e_3^{(1)} = \sqrt{(1.03)^2 - (0.0152)^2} = 1.0299$$

Thus

$$V_3^{(1)} = 1.0299 + j0.0152$$

For the second iteration, we have

$$
\begin{aligned}
V_2^{(2)} &= \frac{\frac{S_2^{sch*}}{V_2^{(1)*}} + y_{12}V_1 + y_{23}V_3^{(1)}}{y_{12} + y_{23}} \\
&= \frac{\frac{-4.0 + j2.0}{1.0025 + j.05} + (-j40)(1.025) + (-j40)(1.0299 + j0.0152)}{(-j80)} \\
&= 1.0001 - j0.0409
\end{aligned}
$$

$$
\begin{aligned}
Q_3^{(2)} &= -\Im\{V_3^{(1)^*}[V_3^{(1)}(y_{13} + y_{23}) - y_{13}V_1 - y_{23}V_2^{(2)}]\} \\
&= -\Im\{(1.0299 - j0.0152)[(1.0299 + j0.0152)(-j60) \\
&\quad -(-j20)(1.025 + j0) - (-j40)(1.0001 - j0.0409)]\} \\
&= 1.3671
\end{aligned}
$$

$$
\begin{aligned}
V_{c3}^{(2)} &= \frac{\frac{S_3^{sch*}}{V_3^{(1)*}} + y_{13}V_1 + y_{23}V_2^{(2)}}{y_{13} + y_{23}} \\
&= \frac{\frac{3.0 - j1.3671}{1.0299 - j0.0152} + (-j20)(1.025) + (-j40)(1.0001 - j.0409)}{(-j60)} \\
&= 1.0298 + j0.0216
\end{aligned}
$$

Since $|V_3|$ is held constant at 1.03 pu, only the imaginary part of $V_{c3}^{(2)}$ is retained, i.e, $f_3^{(2)} = 0.0216$, and its real part is obtained from

$$e_3^{(2)} = \sqrt{(1.03)^2 - (0.0216)^2} = 1.0298$$

or

$$V_3^{(2)} = 1.0298 + j0.0216$$

The process is continued and a solution is converged with an accuracy of 5×10^{-5} pu to

$$\begin{aligned} V_2 &= 1.001243\angle{-2.1^\circ} = 1.000571 - j.0366898 \ \text{ pu} \\ S_3 &= 3.0 + j1.3694 \ \text{ pu} = 300 \ \text{ MW} + j136.94 \ \text{ Mvar} \\ V_3 &= 1.03\angle 1.36851^\circ \ \text{ pu} = 1.029706 + j0.0246 \end{aligned}$$

(b) Line flows and line losses are computed as in Problem 6.7, and the results expressed in MW and Mvar are

$$\begin{array}{lll} S_{12} = 150.43 + j100.16 & S_{21} = -150.43 - j92.39 & S_{L\,12} = 0.0 + j7.77 \\ S_{13} = -50.43 - j9.65 & S_{31} = \ \ 50.43 + j10.90 & S_{L\,13} = 0.0 + j1.25 \\ S_{23} = -249.58 - j107.61 & S_{32} = 249.58 + j126.03 & S_{L\,23} = 0.0 + j18.42 \end{array}$$

The slack bus real and reactive powers are

$$\begin{aligned} S_1 &= S_{12} + S_{13} = (150.43 + j100.16) + (-50.43 - j9.65) \\ &= 100 \text{ MW} + j90.51 \text{ Mvar} \end{aligned}$$

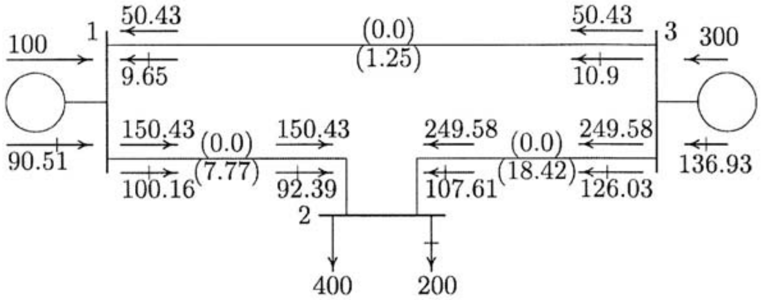


FIGURE 55
Power flow diagram of Problem 6.8 (powers in MW and Mvar).

The power flow diagram is shown in Figure 55, where real power direction is indicated by $\longrightarrow$ and the reactive power direction is indicated by $\longmapsto$. The values within parenthesis are the real and reactive losses in the line.