Opt Quant Electron (2009) 41:397–408
DOI 10.1007/s11082-010-9364-4

Analysis of ultra short pulse propagation in optical directional coupler structures

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Received: 31 August 2009 / Accepted: 25 January 2010 / Published online: 13 February 2010

Abstract We employ an efficient time-domain beam propagation technique to study the effect of the propagation of ultra short pulse duration on the behavior of directional coupler operation. The technique used is based on higher order non-paraxial formulation that takes into account the spatiotemporal coupling effect which is crucial for the proper propagation of ultra short optical pulses. In this work the characterization of pulse spread and broad frequency content interactions of short pulse propagation have been analyzed. We validate in this rigorous analysis that the intermodal dispersion of the structure changes the behavior of the well known operation and breaks up the pulse during propagation which gives rise to distortion.

Keywords Ultra short pulse propagation · Beam propagation method · Finite-difference analysis · Modeling · Numerical analysis · Optical waveguide theory · Partial differential equation · Pade approximant · Directional coupler

1 Introduction

Optical directional couplers are one of the most important elements of photonic circuits. Many important applications use these structures to perform a variety of functions such as optical power splitters, nonlinear optical switches and WDM filter devices. The structure of directional couplers consists simply of two waveguides closely adjacent to each other to create a coupling environment. For CW operation the distance at which the complete transfer of light from one waveguide to the other can be calculated precisely using the familiar coupling length (Marcuse 1991; Snyder and Love 1983). On the other hand, the behavior of pulsed optical beam propagation in linear directional couplers is much more complicated to analyze due to the involvement of a wide spectrum of frequencies that

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cause dispersion effects on the coupling behavior. It should be clear here that this dispersion effect will be more intense for ultra short pulse durations due to the increase of both frequency content and the spatiotemporal coupling effect taking place in this limit. The problem becomes even more difficult if the interaction of pulsed beams is taking place in the existence of nonlinear and dispersive material of the coupler structure. In this context, it should be mentioned that for linear and non-dispersive waveguides one may use superposition to find the response of individual linear frequency module, however, this approach fails for nonlinear systems. For complicated problems of this nature, accurate numerical techniques should be developed to understand such behavior.

It is to be mentioned that in linear directional coupler waveguide propagation there exists two types of dispersions. The first is the normal waveguide dispersion and the second, which has a stronger outcome, is the intermodal dispersion that takes place between the two dominant modes of the directional coupler structure. This dispersion causes pulse spreading which gives rise to pulse breakup and distortion, and as a result a loss of information. It is to be said that studying such complicated phenomenon requires accurate and efficient tools suitable to clarify detail nature of this effect. The pulse break up phenomenon due to intermodal dispersion was first observed and explored qualitatively by Chiang (2005, 1997), using the approximate traditional coupled-mode theory (Marcuse 1991; Snyder and Love 1983), based on simplified one dimensional coupled Time-Domain (TD) *parabolic* equations. It is to be observed at this point that TD parabolic equation showed severe limitation in modeling short pulses due to the paraxial approximation involved (Masoudi et al. 1999, 2001). As the initial pulse duration decreases the spatiotemporal coupling effect becomes more significant for the proper propagation of ultra short pulses. When paraxial approximation is used to model this class of pulses they fail to calculate accurately the behavior of this coupling due to the negligence of higher order propagation terms that contributes considerably for the fast variation of pulses in this category (Masoudi 2007a,b). For accurate investigation of pulsed optical beam propagation in directional couplers, it is very natural to use numerical techniques that can provide rigorous understanding to the complicated behavior of such analysis. In principle the classical Finite-Difference Time-Domain (FDTD) (Yee 1966; Taflove 1988) is suited to simulate both TD and continuous wave problems, but it needs huge computer resources and more importantly it is not suited for long optical pulse interaction of the type involved here, where the length of propagation that may take several hundreds or thousands of wavelengths (Shibayama et al. 2005). Recently, we proposed and tested a new non-paraxial Time-Domain Beam Propagation Method (TD-BPM) intended for ultra short pulse duration in homogenous and different waveguide environments (Masoudi 2007a,b). It is based on the full TD wave equation where the operator is formulated as a one-way equation for the propagation along the longitudinal direction *z*, and all time and spatial variations are kept intact. This arrangement allows for the numerical time window to follow the evolution of the pulse and accordingly minimizes computer resources requirements. It is to be noticed that this is well suited to the propagation of long TD interaction such as directional couplers of this work and more generally to one way long spatiotemporal interaction devices. Another strong feature of the new non-paraxial operator is the use of the rational complex coefficient approximation of Pade approximant to overcome the paraxial limitation. The square root propagation operator is formulated using the Pade quotient $\sqrt{1+X} \approx \prod_{i=1}^{p}\left(1+d_i^p X\right)/\left(1+e_i^p X\right)$, where d and e are called Pade coefficients and p being the Pade order (Milinazzo et al. 1997). Accuracy and stability analysis showed that this new operator is very accurate and has robust stability in the propagation of ultra short optical pulses in optical structures (Masoudi 2007a,b). In this work, the operator is used to model the propagation of ultra short pulse propagation in directional coupler waveguide structures. The effect of initial pulse duration on the mechanism of the coherent length of the directional coupler has been studied. In addition, the analysis shows detail characterization of the frequency content of the propagated pulse, the pulse spread due to the intermodal dispersion and the interaction of the two dominate modes of the structure. The work also shows the limit at which the pulse breakup incident starts to take place and also calculate the dispersion involved. The following section shows a brief about the non-paraxial TD technique used in the analysis, followed by the section that shows the characterization of the time-domain *GaAs* directional coupler structure.

2 Numerical method

Starting with the TD wave equation

$$\partial_z(r\partial_z\psi) + \partial_x(r\partial_x\psi) - \frac{s}{c_o^2}\partial_{tt}^2\psi = 0, \tag{1}$$

where for TE fields $r = 1$, $s = n^2$ and $\psi = E_y$ representing the electric field, for TM fields $r = 1/n^2$, $s = 1$ and $\psi = H_y$ representing the magnetic field, c_o is the wave velocity in free space, z is considered the propagation direction and $n = n(\mathbf{x})$ is the refractive index variation. In this work, linear and non-dispersive materials have been considered. First, we extract a carrier frequency ω and a propagation coefficient $k = k_o n_o$ in the direction of propagation from ψ, as $\psi = \Psi e^{-j\kappa z}e^{j\omega t} + \Psi^* e^{j\kappa z}e^{-j\omega t}$ where $k_o = \omega/c_o$, n_o is a reference refractive index. Then, the substitution of a moving time coordinate $\tau = t - v_g^{-1}z$, with arbitrary v_g that moves with the group velocity of the pulse envelope, is implemented. During propagation of a compact pulse inside a limited time window, the pulse eventually disappears after a certain number of propagation steps and this requires that the computational window to be adjusted in time at each propagation step. Then Eq. (1) can now be written for TE propagation as

$$\left(\partial_{zz}^2 - 2\mathrm{j}k\partial_\mathrm{z} + Q\right)\Psi = 0 \tag{2}$$

where

$$Q = -2\mathrm{j}n^2k_o(1/c_o - 1/v_g)\partial_\tau + \left(n^2k_o^2 - k^2\right) - \frac{n^2}{c_o^2}\partial_{\tau\tau}^2 + \partial_x^2. \tag{3}$$

The solutions for the above Eq. (2) can be split as two operators, one for the forward propagation and the other for the backward propagation. Concentrating on one-way forward propagation of ψ and write the formal solution as

$$\Psi(z) = \exp\left(jk_o n_o(1-L)z\right)\Psi(0) = \exp\left(jk_o n_o(1-\sqrt{1+X})z\right)\Psi(0) \tag{4}$$

where $\psi(0)$ is the initial field and the pseudo-differential operator is defined as $L = Q/k_o n_o$. Detail of solving the above operator can be found in Masoudi (2007a,b) and Milinazzo et al. (1997), Rao et al. (2000), Yevick and Thomson (2000).

3 Results and discussions

In this section we use the non-paraxial TD-BPM described before to study the characteristic propagation of ultra short pulse propagation in a *GaAs* directional coupler waveguide structure. The device consists of two identical single mode slab waveguides with a core refractive index of 3.6 surrounded by 3.4 materials. The waveguides are separated by a distance of

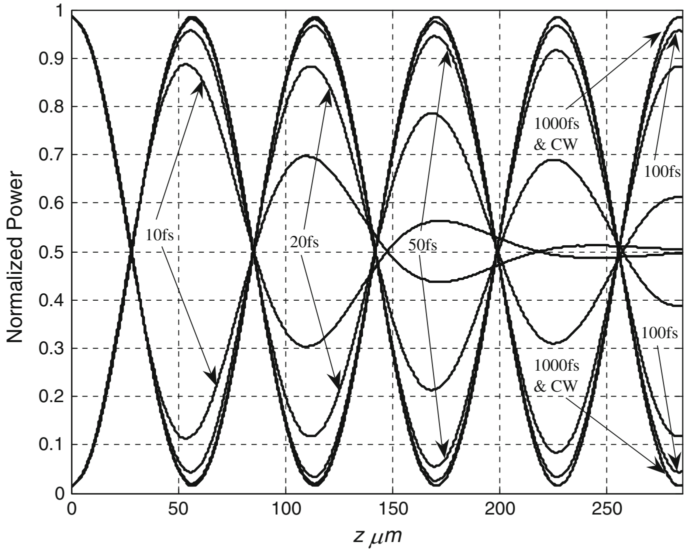

Fig. 1 The normalized power of the ultra short optical pulses in the directional coupler waveguide structure along the longitudinal direction z for several initial pulse durations. The CW results using the CW-BPM were also included

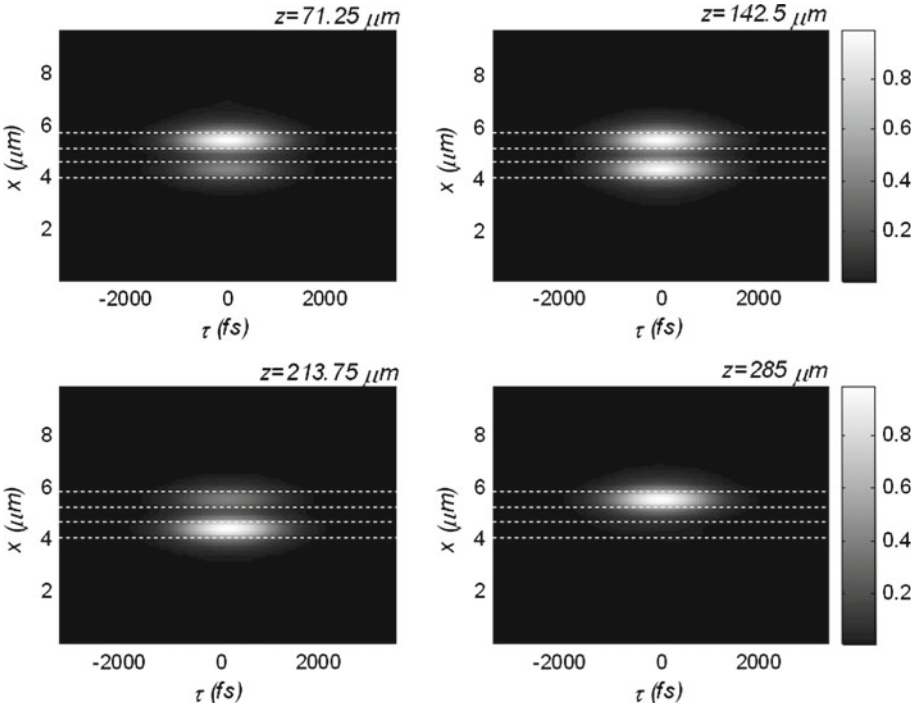

Fig. 2 Contour field plots for the evolution of the pulsed optical beam with an initial pulse beam duration of 1,000 fs inside the directional coupler structure at several distances along the longitudinal direction. The horizontal lines show the position of the two waveguide boundaries

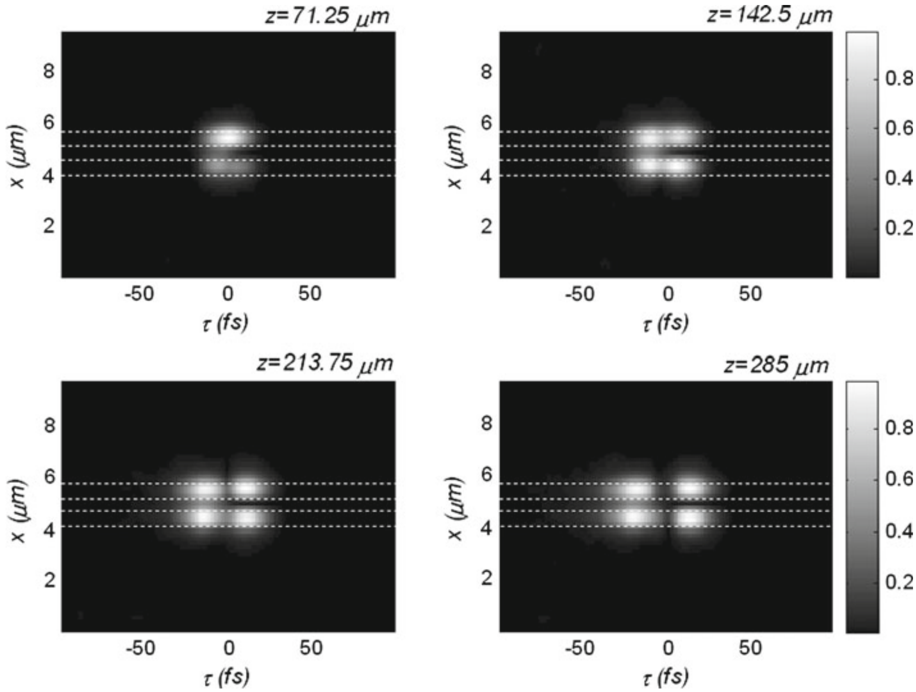

Fig. 3 The same as in Fig. 2 except that the evolution of the pulsed optical beam with initial pulse beam duration of 10 fs

0.5 μm, have core thicknesses of 0.6 μm each and with an operating central carrier wavelength of $\lambda = 1.55$ μm. In all of the following simulation results, the input field excited at $z = 0$ consists of a Gaussian pulsed beam in time of the form $\Psi(x, z = 0,\ \tau) = \Psi_0(x) \exp\left(-\tau^2/\sigma_\tau^2\right)$, where $\psi_o(x)$ is the spatial profile in the x-direction taken as the first guided mode of the single waveguide of the directional coupler structure. σ_τ is defined here as the initial pulse duration in femto second. The pulsed beam was propagated using the non-paraxial TD-BPM and the effect of reducing the initial pulse duration was studied. All numerical parameters of spatial and longitudinal step sizes, time step sizes and Pade orders were optimized to insure efficiency, convergence and accuracy. To study the effect of ultra short optical pulse on the mechanism of directional coupler operation, the length of the device was fixed to several coupling lengths of the CW case (Marcuse 1991; Snyder and Love 1983) and the initial pulse duration was varied from the short duration of 1,000 fs to the ultra short interval of 10 fs. The coupling length for CW operation of the structure at work is 56.8 μm and the total propagation distance considered in the analysis is $Z = 285$ μm. It should be implicitly understood that the coupling length of pulsed optical beams is a function of the frequency content of the pulse, where it is inversely proportional with the increase of wavelength. The proportionality is nearly linear for wide duration pulsed beams and becoming nonlinear for short pulsed beams. For ultra short pulse width of 10 fs, which has a wideband frequency spectrum and for the structure parameters of this work, the coupling length changes between 26.4 and 149.9 μm in a parabolic manner.

Figure 1 shows the normalized power of the propagation of ultra short optical pulses in the directional coupler waveguide structure for different initial pulse durations. Curves starting

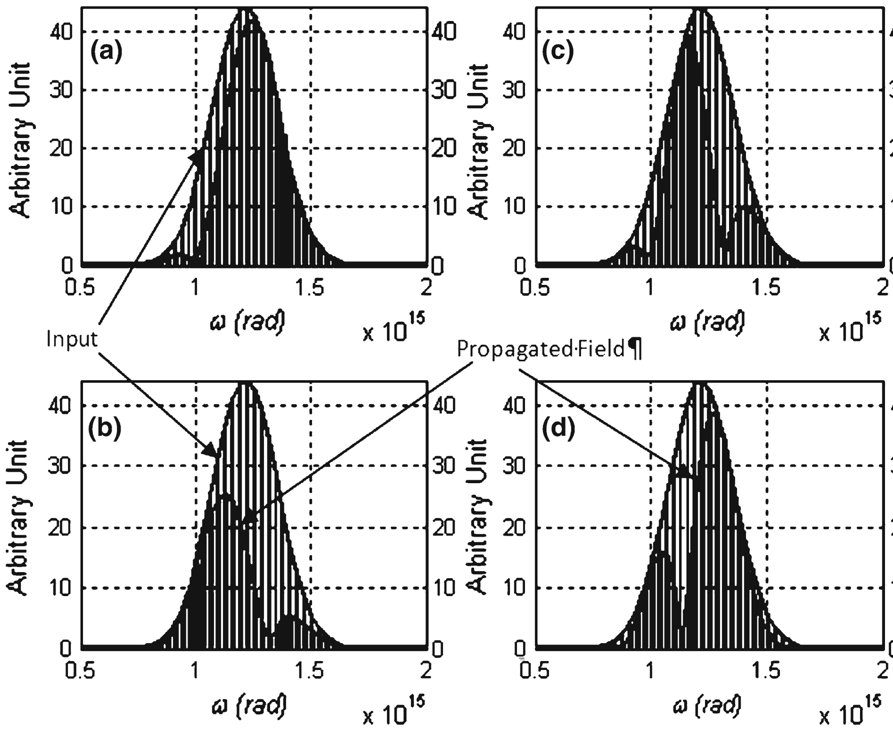

Fig. 4 The propagational spectra of the 10 fs pulse in the lower and upper waveguides at different distances. **a** and **b** for z = 71.25μm; **c** and **d** for z = 142.5 μm. **a** and **c** for upper waveguide, **b** and **d** for lower waveguide (where the input was launched)

from one belong to the lower half side of the structure where the input was launched. Curves starting from zero belong to the upper half side of the structure. In the figure, the CW results, using the CW-BPM, were included for comparison purposes. It is to be noted that the 1,000 fs results are on top of the CW curves.

The figure shows an extraordinary result from the conventional mechanism of the directional coupler operation. Two interesting observations are to be mentioned here. The first finding is that as the initial pulse duration decreases the backward coupling power recovery of the pulse decreases and this effect is diminishing rapidly for the lower scale of ultra short time segment. Another observation to be observed from the same figure is that this backward power recovery is distance dependent where the recovery of power decreases along the direction of propagation. To trace and understand the effect of the initial pulse duration from field point of view, we show the field propagation of the two farthest cases shown in Fig. 1

Figure 2 shows contour field plots for the evolution of the pulsed optical beam with initial pulse beam duration of 1,000 fs inside the directional coupler structure at several distances along the longitudinal direction, namely at $z = Z/4$, $Z/2$, $3Z/4$ and Z. It is to be noticed that a moving time window was used to follow the propagation of the pulse that moves with the group velocity of the pulse. This moving window mechanism is a primary efficiency feature in modeling long pulse propagation using the new technique. The coupling behavior between the two waveguides along the direction of propagation is clearly shown in the figure where the field is coupled spatially to the first guided mode of the waveguide with a pulse width

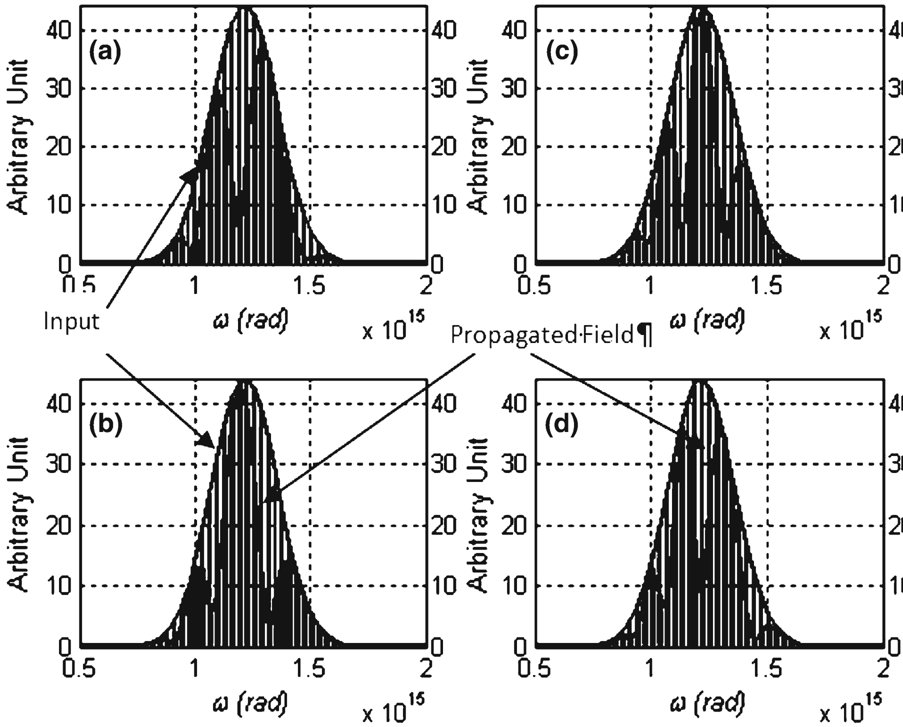

Fig. 5 The propagational spectra of the 10 fs pulse in the lower and upper waveguides at different distances. **a** and **b** for z = 213.75 µm; **c** and **d** for z = 285 µm. **a** and **c** for upper waveguide, **b** and **d** for lower waveguide (where the input was launched)

duration equal to that of the launched pulse. Figure 3 shows the same as in Fig. 2 except for an initial pulsed beam duration of 10 fs. In this figure, the pulse experiences a fundamental new behavior during the course of propagation. Pulse spread due to intermodal dispersion is shown to take place along the direction of propagation and then the pulse breaks up into nearly two equal pulses. It is to be mentioned that the pulse spread due to intermodal dispersion is seen to take place for all cases of Fig. 1 at different distances, where this dispersion effect is much stronger as the initial pulse duration is reduced. The effect of this dispersion will be shown in the following analysis.

It is known that as the initial pulse width duration decreases the frequency content will increase. It is also clear that the 10 fs launched as an input contains a wide band of frequencies around the central frequency at which the CW directional coupler was designed with, which is no longer the only dominant frequency in the operation of the time domain structure at the ultra short pulse scale. Other frequency content will have dramatic effect on the operation of the normal directional coupler device. It also understandable that the effect and the interaction of this broad frequency band are very difficult to analyze and realize using analytically based techniques. Figures 4 and 5 show the propagational spectra of the 10 fs initial pulse duration in the lower and the upper waveguides at several distances along the direction of propagation. The spectra shown in these figures correspond to pulse durations right at the middle of each waveguide. The spectra of the input pulse have been included in the figure to show the developed differences.

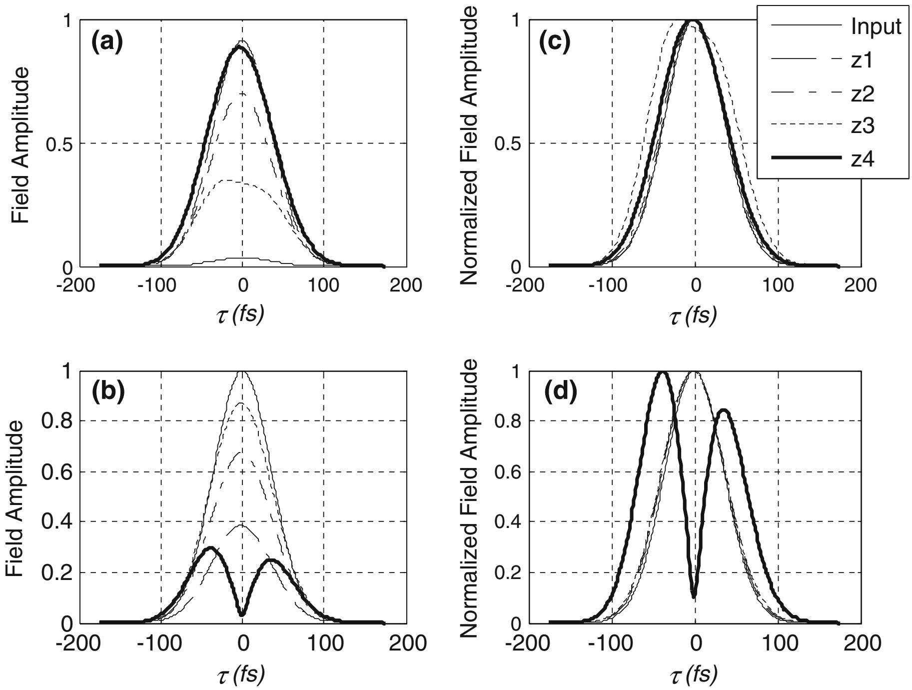

Fig. 6 The time pulse distribution in the upper and the lower waveguides at different distances in the directional coupler structure for an initial pulse width of 50 fs. z1 = 71.25 μm, z2 = 142.5 μm, z3 = 213.75 μm, and z4 = 285 μm. **a** is for the upper waveguide **b** is for the lower waveguide (where the input was launched). **c** normalized field of (**a**), and **d** normalized field of (**b**)

Following the evolution of the spectra for the lower waveguide, for example, it shows that the shift of the dominant frequency initially to the left at the first distance ($z = 71.25\ \mu m$ of Fig. 4b), then to the right at the second distance ($z = 142.5\ \mu m$ of Fig. 4d) as compared to the launched central frequency, while the formation of the two distinct peaks of frequencies are clearly shown. At $z = 213.75\ \mu m$ of Fig. 5b, the spectra shows the formation of three peaks of frequencies while at $z = 285\ \mu m$ of Fig. 5d two equal main peaks have been created. It is apparent that the creation and the interaction of several dominant frequencies during the course of propagation is the main cause of pulse spread and the breakup of the initial pulse. This action has been also observed in the spectra of wider pulse duration propagation of Fig. 1 but with moderate frequency change compared to the 10 fs pulse shown in Figs. 4 and 5. It is to be concluded that the change of the dominant frequency band content is clearly responsible for the breakup of the pulse during propagation.

In order to assess and study the effect of the change in the frequency content behavior on the pulse dispersion taking place during propagation, we compare between two initial pulse duration widths in time domain. Figures 6 and 7 show the time field pulse distribution at the middle of both waveguides, the upper and the lower waveguides (the lower where the input was excited) at several distances along the propagation and for two different initial pulse durations namely 50 and 20 fs respectively. General observations show that as the initial pulse duration decreases the pulse spread increases in both waveguides where the pulse breaks into two pulses right at the end of propagation for both cases. It is to be noticed

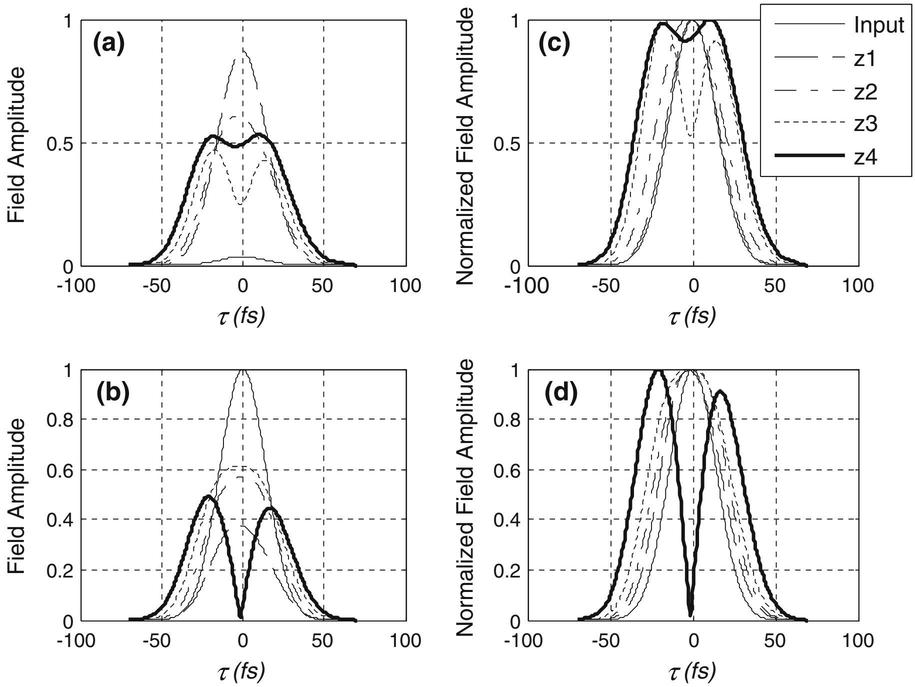

Fig. 7 The same as in Fig. 6 but for an initial pulse width of 20 fs

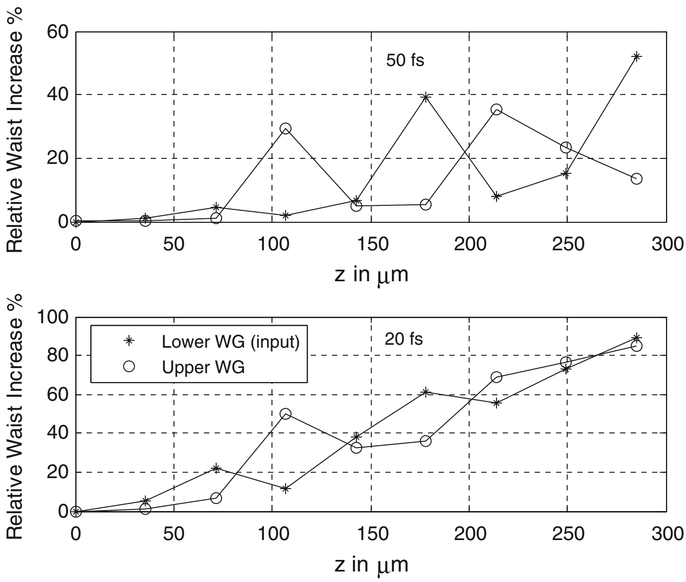

Fig. 8 The percentage relative time waist increase in both waveguides as a function longitudinal distance for two different initial pulse widths

that the pulses acutely experience both relative spread and compression during the course of propagation. Figure 8 records the change of the pulse waist as a function of propagation for the cases given in Figs. 6 and 7. The figure shows the percentage relative time waist increase in both waveguides as a function longitudinal propagational distance. The figure shows two observations, the first is that the relative pulse spread depends on the initial pulse duration and also shows the oscillatory pulse duration change (increase and decrease) behavior in both waveguides along the propagation direction. This action is linked directly to the behavior of dominant frequency swing shown previously in Figs. 4 and 5. It is to be noted as well that the pulse spread due to waveguide dispersion is very small compared to the intermodal dispersion in this study.

Figure 9 measures the relative pulse duration increase in the upper and the lower waveguides at several distances along the direction of propagations as a function of initial pulse duration. For large initial pulse duration the relative pulse change is fairly small which is around 1.8%, and as the initial pulse duration decreases the relative pulse spread is very high reaching 200% for ultra short pulse duration of 10 fs at the end of the structure. It is to be noted that the pulse changes recorded in this figure were calculated with respect to the total pulse durations taking into account the durations of the combined two splitted pulses for pulses that showed breakup.

Figure 10 shows the relative waist change in both waveguides due to intermodal dispersion for the two newly formed splitted pulses at the output of the directional coupler structure as a

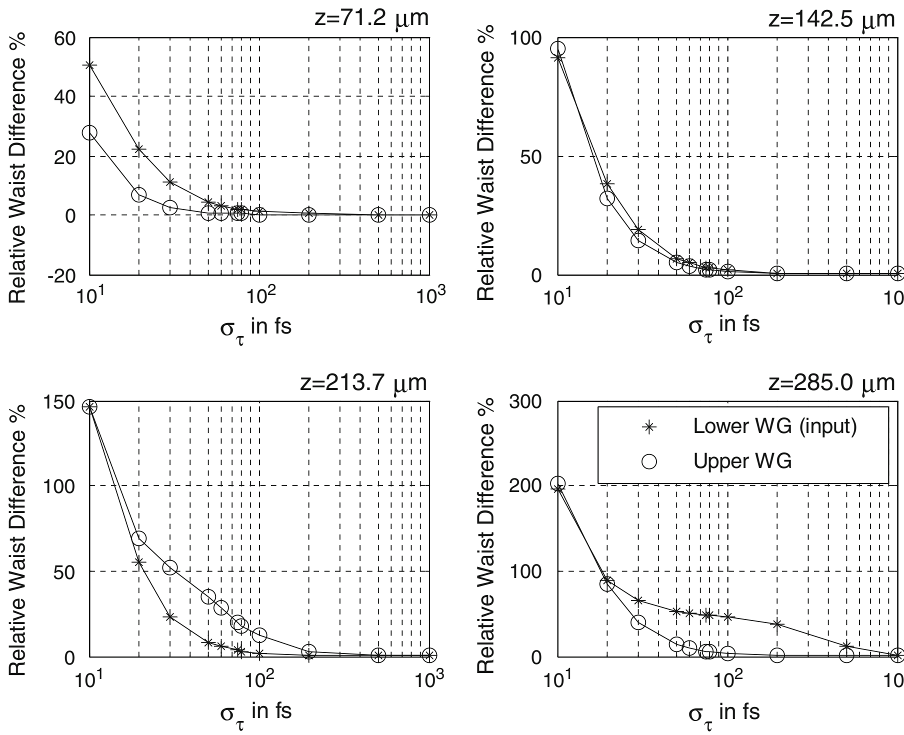

Fig. 9 The percentage relative time waist increase in both waveguides at different distances as a function of initial pulse width

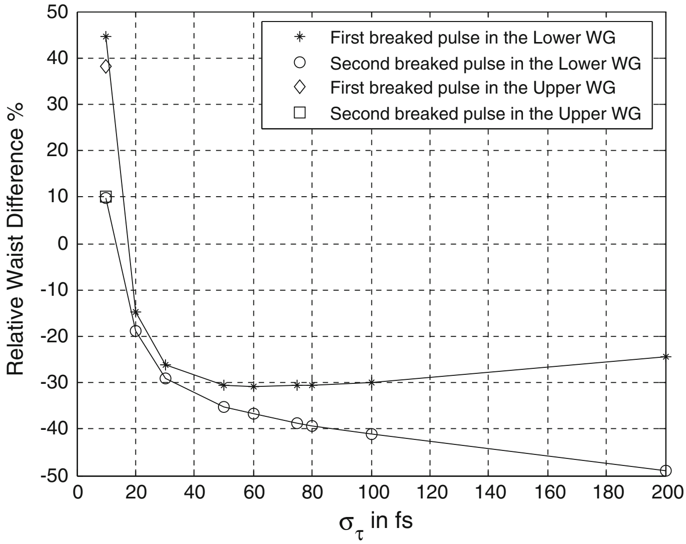

Fig. 10 The percentage relative time waist change after break up in both waveguides as a function of initial pulse duration at the output of the structure with $z = 285\,\mu m$

function of initial pulse waist duration. It is to be noticed that the changes in this figure were calculated with respect to the initial pulse duration launched at the input and that's why the curves start from negative values. The figure shows the pulse breakup behavior starts to take place as early as initial pulse duration of 200 fs.

4 Conclusion

In this work, an efficient non-paraxial beam propagation technique has been used to study the propagation of ultra short optical pulses in long directional coupler structures. The use of this technique is important in the study of ultra short pulse propagation due to the involvement of a large spectrum of frequencies and also accounts for the spatiotemporal coupling effect which increases considerably with the decrease of the initial pulse duration. These effects cannot be accounted for using usual paraxial techniques which have been used previously to predict the behavior of ultra short pulse propagation in directional couplers. Very important observations have been noted in this work. First, the normal power coupling operation of directional couplers was shown to depart drastically from the usual CW one. In addition, it has been shown that the intermodal dispersion of the structure gives rise to pulse broadening and eventually breaks up of the pulse during the course of propagation. Newly formed dominant frequencies associated with the dispersion effects were seen to change the behavior of the normal directional coupler operation. Finally, the mechanism of splitting pluses by intermodal dispersion effect taking place in directional coupler is seen as a distortion from information point of view, but this may lead to desired outcome in other applications.

Acknowledgment This work was supported by King Fahd University of Petroleum and Minerals KFUPM, Dhahran, Saudi Arabia.

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