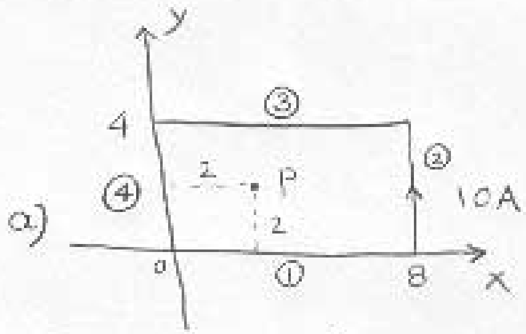
EE340 H.W. #7 Solution
3rd Edition

7.9

a) $\vec{H} = \frac{I}{4\pi\rho}\left[\cos\alpha_2 - \cos\alpha_1\right]\vec{a}_\phi$



$\vec{H} = \vec{H}_1 + \vec{H}_2 + \vec{H}_3 + \vec{H}_4$

$\vec{H}_1 = \frac{10}{4\pi(2)}\left[\cos 18.43° - \cos 135°\right]\vec{a}_z = 0.659\,\vec{a}_z$

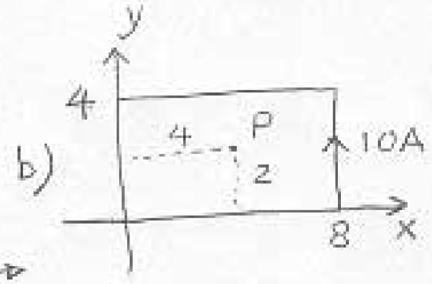
$\vec{H}_2 = \frac{10}{4\pi(6)}\left[\cos 71.57° - \cos 108.43°\right]\vec{a}_z = 0.084\,\vec{a}_z$

$\vec{H}_3 = \vec{H}_1$ (by symmetry)

$\vec{H}_4 = \frac{10}{4\pi(2)}\left[\cos 45° - \cos 135°\right]\vec{a}_z = 0.563\,\vec{a}_z$

$\vec{H} = (0.659 + 0.084 + 0.659 + 0.563)\,\vec{a}_z = 1.965\,\vec{a}_z$ A/m

b) $\vec{H}_1 = \frac{10}{4\pi(2)}\left[\cos 26.57° - \cos 153.43°\right]\vec{a}_z$



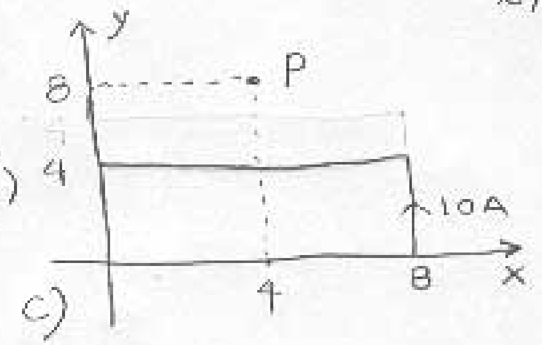
$= 0.712\,\vec{a}_z$

$\vec{H}_2 = \frac{10}{4\pi(4)}\left[\cos 63.43° - \cos 116.57°\right]\vec{a}_z$

$= 0.178\,\vec{a}_z$

$\vec{H}_3 = \vec{H}_1$, $\vec{H}_4 = \vec{H}_2$ by symmetry

$\vec{H} = 2 \times 0.712\,\vec{a}_z + 2 \times 0.178\,\vec{a}_z = 1.78\,\vec{a}_z$ A/m c)



$$\vec{H}_1 = \frac{10}{4\pi(8)}\left[\cos 63.43^\circ - \cos 116.57^\circ\right](+\vec{a}_z)$$

$$= 0.089\,\vec{a}_z$$

$$\vec{H}_2 = \frac{10}{4\pi(4)}\left[\cos 135^\circ - \cos 153.43^\circ\right]\vec{a}_z = 0.037\,\vec{a}_z$$

$$\vec{H}_3 = \frac{10}{4\pi(4)}\left[\cos 45^\circ - \cos 135^\circ\right](-\vec{a}_z) = -0.281\,\vec{a}_z$$

$$\vec{H}_4 = \vec{H}_2 \quad \text{(by symmetry)}$$

$$\vec{H} = -0.118\,\vec{a}_z \quad \text{A/m}$$

d)
$$\vec{H}_1 = \frac{10}{4\pi(2)}\left[\cos 14.04^\circ - \cos 90^\circ\right](-\vec{a}_y) = -0.386\,\vec{a}_y$$

$$\vec{H}_2 = \frac{10}{4\pi\sqrt{68}}\left[\cos 64.12^\circ - \cos 90^\circ\right]\left(\frac{2\vec{a}_x + 8\vec{a}_z}{\sqrt{68}}\right)$$

$$= 0.010\,\vec{a}_x + 0.041\,\vec{a}_z$$

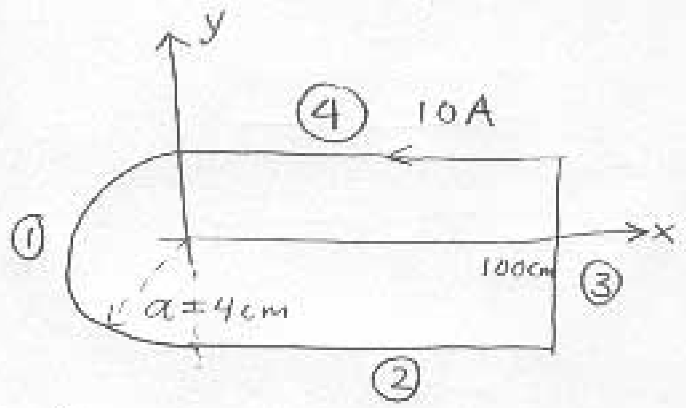
$$\vec{H}_3 = \frac{10}{4\pi\sqrt{20}}\left[\cos 90^\circ - \cos 150.79^\circ\right]\left(\frac{2\vec{a}_y + 4\vec{a}_z}{\sqrt{20}}\right)$$

$$= 0.069\,\vec{a}_y + 0.139\,\vec{a}_z$$

$$\vec{H}_4 = \frac{10}{4\pi(2)}\left[\cos 90^\circ - \cos 153.43^\circ\right](-\vec{a}_x) = -0.356\,\vec{a}_x$$

$$\vec{H} = -0.346\,\vec{a}_x - 0.317\,\vec{a}_y + 0.18\,\vec{a}_z \quad \text{A/m}$$

7.12



$$\vec{H}_1 = \left(\vec{a}_z \frac{I}{2a}\right)/2$$

$$= \vec{a}_z \frac{10}{4(4\times10^{-2})} = \vec{a}_z \, 62.5 \ \text{A/m}$$

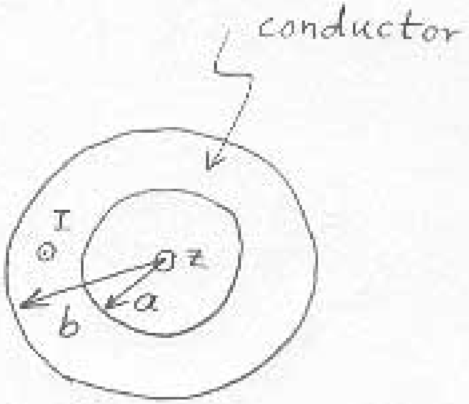
$$\vec{H}_2 = \frac{10}{4\pi(0.04)}\left[\cos 2.29^\circ - \cos 90^\circ\right]\vec{a}_z = 19.88\,\vec{a}_z \ \text{A/m}$$

$$\vec{H}_3 = \frac{10}{4\pi(1)}\left[\cos 87.71^\circ - \cos 92.29^\circ\right]\vec{a}_z = 0.064\,\vec{a}_z \ \text{A/m}$$

$$\vec{H}_4 = \vec{H}_2 = 19.88\,\vec{a}_z \ \text{A/m}$$

$$\vec{H} = 102.324\,\vec{a}_z \ \text{A/m} .$$

7.17



a) $\oint \vec{H} \cdot d\vec{l} = I$

b) $H 2\pi\rho = 0 \quad (\text{for } \rho < a)$

$\vec{H} = \vec{0} \quad (\rho < a)$

$$H 2\pi\rho = \frac{I}{\pi(b^2-a^2)}(\rho^2-a^2)\pi \quad (b>\rho>a)$$

$$\vec{H} = \frac{I(\rho^2-a^2)}{2\pi\rho(b^2-a^2)}\vec{a}_\phi \quad (b>\rho>a)$$

$$H 2\pi\rho = I \quad (\rho > b) \Rightarrow \vec{H} = \frac{I}{2\pi\rho}\vec{a}_\phi \quad (\rho > b)$$

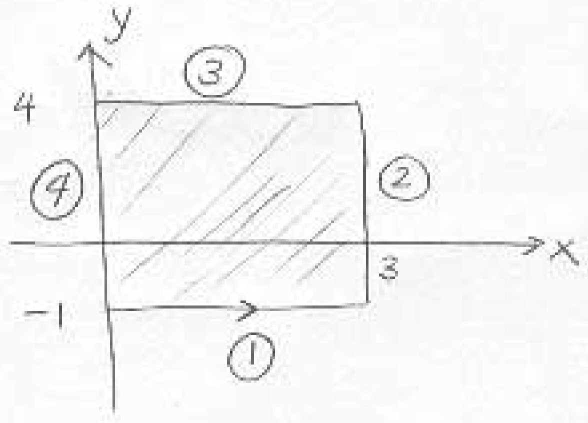
7.19

a) $\nabla \times \vec{H} = \vec{J}$

$\vec{J} = -2\,\vec{a}_z$

b) $I = \int_S \vec{J} \cdot \vec{ds} = JS$ (because $\vec{J} \perp$ area and uniform)

$= -2 \times 5 \times 3 = -30\,A$



$$\oint_\ell \vec{H} \cdot \vec{d\ell} = \int_1 + \int_2 + \int_3 + \int_4$$

$$\int_1 \vec{H} \cdot \vec{d\ell} = \int_{x=0}^{3} y\,dx \Big|_{y=-1} = -3$$

$$\int_2 \vec{H} \cdot \vec{d\ell} = \int_{y=-1}^{4} -x\,dy \Big|_{x=3} = -15$$

$$\int_3 \vec{H} \cdot \vec{d\ell} = \int_{x=3}^{0} y\,dx \Big|_{y=4} = -12$$

$$\int_4 \vec{H} \cdot \vec{d\ell} = \int_{y=4}^{-1} -x\,dy \Big|_{x=0} = 0$$

$$\therefore \oint \vec{H} \cdot \vec{d\ell} = -3 - 15 - 12 = -30\,A$$

$$\therefore \oint_\ell \vec{H} \cdot \vec{d\ell} = I = -30\,A$$

7.20

a) $\vec{J} = \nabla \times \vec{H}$

$$= \vec{a}_x \left(8x^2y + y^2x\right) + \vec{a}_y\left(y\,(x^2+y^2) - 8xy^2\right)$$

$$+ \vec{a}_z\left(-y^2z - x^2z - 3y^2z\right)$$

$$\vec{J}(5,2,-3) = \vec{a}_x\, 420 - \vec{a}_y\, 102 + \vec{a}_z\, 123 \quad A/m^2$$

b) $$I = \int_s \vec{J} \cdot \vec{ds}$$

$$= \int_{z=0}^{2}\int_{y=0}^{2} \vec{J} \cdot \left(\vec{a}_x\, dy\, dz\right)$$

$$= \int_{z=0}^{2}\int_{y=0}^{2} \left(8x^2y + xy^2\right) dy\, dz \Big|_{x=-1}$$

$$= \int_{z=0}^{2}\int_{y=0}^{2} \left(8y - y^2\right) dy\, dz = 2\int_{y=0}^{2} \left(8y - y^2\right) dy$$

$$= 2\left[8\frac{y^2}{2} - \frac{y^3}{3}\right]_0^2 = 2\left[4(2)^2 - \frac{2^3}{3}\right]$$

$$= 2\left[16 - \frac{8}{3}\right] = 2\left[\frac{48-8}{3}\right] = \frac{80}{3}$$

$$= 26.67 \text{ A}$$

c) $\nabla \vec{B} = \mu \nabla \vec{H} = \mu\left[2xyz - 2yxz + 0\right] = 0$