**1.4**

a) $T=(3,-2,1)$ , $S=(4,6,2)$

b) $\overrightarrow{TS} = \vec{a}_x + 8\vec{a}_y + \vec{a}_z$

c) $TS = \sqrt{1^2+8^2+1^2} = \sqrt{66} = 8.124$

**1.9**

a) $\vec{a}_S = \dfrac{\vec{S}}{S} = \dfrac{\vec{S}}{\sqrt{6}} = \dfrac{1}{\sqrt{6}}\vec{a}_x + \dfrac{2}{\sqrt{6}}\vec{a}_y + \dfrac{1}{\sqrt{6}}\vec{a}_z$

Scalar projection of $\vec{T}$ on $\vec{S}$ is:

$\vec{T}\cdot\vec{a}_S = \dfrac{2}{\sqrt{6}} - \dfrac{12}{\sqrt{6}} + \dfrac{3}{\sqrt{6}} = -\dfrac{7}{\sqrt{6}}$

b) $\vec{a}_T = \dfrac{\vec{T}}{T} = \dfrac{\vec{T}}{7} = \dfrac{2}{7}\vec{a}_x - \dfrac{6}{7}\vec{a}_y + \dfrac{3}{7}\vec{a}_z$

Scalar projection of $\vec{S}$ on $\vec{T}$ is:

$\vec{S}\cdot\vec{a}_T = \dfrac{2}{7} - \dfrac{12}{7} + \dfrac{3}{7} = -1$

Vector projection of $\vec{S}$ on $\vec{T}$ is:

$(\vec{S}\cdot\vec{a}_T)\vec{a}_T = -\vec{a}_T = -\dfrac{2}{7}\vec{a}_x + \dfrac{6}{7}\vec{a}_y - \dfrac{3}{7}\vec{a}_z$

c) $\vec{T}\cdot\vec{S} = TS\cos\theta$

$2-12+3 = 7(\sqrt{6})\cos\theta$

$-7 = 7\sqrt{6}\cos\theta$

$\cos\theta = -\dfrac{1}{\sqrt{6}}$

$\theta = 114.09^\circ$

1.21

a) $\vec{A}(2,-1,3) = -4\vec{a}_x + 3\vec{a}_y - 9\vec{a}_z$

$A(2,-1,3) = \sqrt{16+9+81} = 10.296$

b) $\vec{a}_A(2,-1,3) = \dfrac{\vec{A}(2,-1,3)}{A(2,-1,3)} = \dfrac{-4\vec{a}_x + 3\vec{a}_y - 9\vec{a}_z}{10.296}$

$\vec{TS} = 5.6\ \vec{a}_A(2,-1,3) = 0.544(-4\vec{a}_x + 3\vec{a}_y - 9\vec{a}_y)$

$= -2.176\vec{a}_x + 1.632\vec{a}_y - 4.896\vec{a}_z$

c) $\vec{S} = \vec{TS} + \vec{T}$

$= (-2.176\vec{a}_x + 1.632\vec{a}_y - 4.896\vec{a}_z)$
$+ (2\vec{a}_x - \vec{a}_y + 3\vec{a}_z)$

$= -0.176\vec{a}_x + 0.632\vec{a}_y - 1.896\vec{a}_z$

1.22

a) $|\vec{E}(1,2,3| = E(1,2,3) = \sqrt{2^2 + 1^2 + 6^2} = 6.403$

b) $\vec{a}_F(1,2,3) = \dfrac{\vec{F}(1,2,3)}{F(1,2,3)} = \dfrac{2\vec{a}_x - 4\vec{a}_y + 6\vec{a}_z}{\sqrt{56}}$

$= 0.267\vec{a}_x - 0.535\vec{a}_y + 0.802\vec{a}_z$

$\vec{E}(1,2,3) \cdot \vec{a}_F(1,2,3) = (2\vec{a}_x + \vec{a}_y + 6\vec{a}_z) \cdot (0.267\vec{a}_x - 0.535\vec{a}_y + 0.802\vec{a}_z)$

$= 0.534 - 0.535 + 4.812 = 4.811$

$\therefore$ Scalar component $= 4.811$

vector component is :

$$4.811\ \vec{a}_F(1,2,3) = 1.285\,\vec{a}_x - 2.574\,\vec{a}_y + 3.858\,\vec{a}_z$$

c) $\vec{E}(0,1,-3) = \vec{a}_y - 3\vec{a}_z$

$\vec{F}(0,1,-3) = -\vec{a}_y$

$\vec{E}(0,1,-3) \times \vec{F}(0,1,-3) = -3\,\vec{a}_x$

Required vector $= \pm \vec{a}_x$

2.4

a) $\vec{D} = (x+z)\,\vec{a}_y$

$$\begin{bmatrix} D_\rho \\ D_\phi \\ D_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ D_y \\ 0 \end{bmatrix} = \begin{bmatrix} \sin\phi\,(x+z) \\ \cos\phi\,(x+z) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} (\rho\cos\phi + z)\sin\phi \\ (\rho\cos\phi + z)\cos\phi \\ 0 \end{bmatrix} \quad \text{(cylinderical)}$$

$$\begin{bmatrix} D_r \\ D_\theta \\ D_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ -\cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} 0 \\ D_y \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} D_y \sin\theta \sin\phi \\ D_y \cos\theta \sin\phi \\ D_y \cos\phi \end{bmatrix} = \begin{bmatrix} (x+z) \sin\theta \sin\phi \\ (x+z) \cos\theta \sin\phi \\ (x+z) \cos\phi \end{bmatrix}$$

$$\begin{bmatrix} D_r \\ D_\theta \\ D_\phi \end{bmatrix} = \begin{bmatrix} (r\sin\theta\cos\phi + r\cos\theta) \sin\theta \sin\phi \\ (r\sin\theta\cos\phi + r\cos\theta) \cos\theta \sin\phi \\ (r\sin\theta\cos\phi + r\cos\theta) \cos\phi \end{bmatrix} \quad \text{(Spherical)}$$

b) $$\begin{bmatrix} E_\rho \\ E_\phi \\ E_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$$= \begin{bmatrix} E_x \cos\phi + E_y \sin\phi \\ -E_x \sin\phi + E_y \cos\phi \\ E_z \end{bmatrix}$$

$$= \begin{bmatrix} (\rho^2\sin^2\phi - \rho^2\cos^2\phi)\cos\phi + z\rho^2\sin^2\phi\cos\phi \\ -(\rho^2\sin^2\phi - \rho^2\cos^2\phi)\sin\phi + z\rho^2\sin\phi\cos^2\phi \\ \rho^2\cos^2\phi - z^2 \end{bmatrix}$$

(Cylinderical).

$$\begin{bmatrix} E_r \\ E_\theta \\ E_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ -\cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$$\begin{aligned} E_r = {} & r^2\sin^3\theta \overset{\cos\phi}{\,}(\sin^2\phi - \cos^2\phi) + r^3\sin^3\theta\cos\theta\cos\phi\sin^2\phi \\ & + r^2\cos\theta(\sin^2\theta\cos^2\phi - \cos^2\theta) \end{aligned}$$

$$\begin{aligned} E_\theta = {} & -r^2\sin^2\theta\cos\theta\cos\phi(\sin^2\phi - \cos^2\phi) + r^3\sin^2\theta\cos^2\theta\underset{\cos\phi}{\sin^3\phi} \\ & - r^2\sin\theta(\sin^2\theta\cos^2\phi - \cos^2\theta) \end{aligned}$$

$$\begin{aligned} E_\phi = {} & -r^2\sin^2\theta(\sin^2\phi - \cos^2\phi)\sin\phi \\ & + r^3\sin^2\theta\sin\phi\cos^2\phi\cos\theta \end{aligned}$$

Which are the required components of vector $\vec{E}$ in spherical coordinates.

2.6

a)

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} A_\rho \cos\phi - A_\phi \sin\phi \\ A_\rho \sin\phi + A_\phi \cos\phi \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \rho(z^2+1)\cos\phi + \rho z \cos\phi \sin\phi \\ \rho(z^2+1)\sin\phi - \rho z \cos^2\phi \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} x\left[(z^2+1) + \frac{z\, y}{\sqrt{x^2+y^2}}\right] \\ y(z^2+1) - \frac{x^2 z}{\sqrt{x^2+y^2}} \\ 0 \end{bmatrix}$$

b)

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} B_r \\ B_\theta \\ B_\phi \end{bmatrix}$$

$$B_x = 2r\sin\theta\cos\phi\sin\theta\cos\phi + r\cos^3\theta\cos\phi - r\sin^2\phi$$

$$= 2x\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}\frac{x}{\sqrt{x^2+y^2}} + z\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^2\frac{x}{\sqrt{x^2+y^2}} - \sqrt{x^2+y^2+z^2}\left(\frac{y}{\sqrt{x^2+y^2}}\right)^2$$

$$= \frac{2x^2}{\sqrt{x^2+y^2+z^2}} + \frac{xz^3}{\sqrt{x^2+y^2}\,(x^2+y^2+z^2)} - \frac{y^2\sqrt{x^2+y^2+z^2}}{(x^2+y^2)}$$

A similar procedure can be used to find $B_y$ and $B_z$.