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Real-Time Secondary Spectrum Sharing with QoS Provisioning

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***Abstract*— We study the quality of service (QoS) issue in secondary spectrum sharing subject to an interference temperature constraint. A non-linear optimization problem with the objective to maximize the total transmitting rate of the secondary users is formulated. The non-linear optimization is solved efficiently using geometric programming techniques. When not all the secondary links can be supported with their QoS requirement, a reduced complexity searching algorithm is introduced to find the optimal subset of links which contains the maximum number of links with both QoS and interference temperature constraints satisfied. We also defined a secondary spectrum sharing potential game. The Nash equilibria of this potential game are reached by distributed sequential play. The efficiency of the Nash equilibria solutions is characterized. Finally, the performances of both the reduced complexity algorithm and the sequential play are examined through simulations.**

I. Introduction

Enhancing spectrum efficiency and use is a significant task of regulatory authorities worldwide. A number of measurement studies of spectrum utilization have indicated that spectrum is sporadically used in many geographical areas and times. Low utilization and increased demand for the radio resource suggests the notion of secondary use, which allows unused parts of spectrum to become available temporarily. The secondary use of spectrum is one of the promising ideas that can mitigate unsatisfied spectrum demand, potentially without major changes to incumbents. Then, the question comes to how to share the available spectrum efficiently and fairly. The FCC spectrum Policy Task Force [1] has recommended a paradigm shift in interference assessment, that is, a shift away from largely fixed operations in the transmitter and toward real-time interactions between the transmitter and receiver in an adaptive manner. The recommendation is based on a new metric called the interference temperature, which is intended to quantify and manage the sources of interference in a radio environment. The interference temperature is defined to be the RF power measured at a receiving antenna per unit bandwidth. The key idea for this new metric is that, firstly, the interference temperature at a receiving antenna provides an accurate measure for the acceptable level of RF interference in the frequency band of interest; any transmission in that band is considered to be "harmful" if it would increase the noise floor above the interference temperature threshold. Secondly, given a particular frequency band in which the interference temperature is not exceeded, that band could be made available to secondary users. Hence, a secondary device might attempt to coexist with the primary, such that the presence of secondary devices goes unnoticed.

Related work on secondary use of radio spectrum has appeared in [2] [3] [4] [5]. Here we consider a scenario similar to [5], where secondary users wish to use a local, relatively short-term data service, and all users adopt a spread spectrum signaling format, in which the transmitted power is evenly spread across the entire available band controlled by the manager. A practical realization of this model would be secondary users with spread spectrum signaling and primary direct-sequence CDMA (DS-CDMA) systems coexist in the up-link spectrum band of the primary DS-CDMA systems. In [5], auction mechanisms for allocating the received power are studied. The logarithmic utilities which is a function of the received Signal-to-Interference Ratio (SIR) is maximized under the constraint of interference temperature. But without any constraint of the minimum received SIR or maximum transmitting power for the secondary users, the auction based mechanisms may lead to some inefficient solutions. The received SIR for some secondary links may become too low to properly accomplish transmitting but wasting energy to do endless retransmissions and causing interference to other links, or the required optimal transmitting power exceeds the maximum available transmitting power for the secondary users. An alternative choice would be to completely switch off some of the secondary links when the system is infeasible, by coordination control. In this way, the active secondary links are provisioned with QoS in the sense of a guaranteed minimum achievable SIR. And those links switched off at this time period can be awakened when it can be supported with its minimum required SIR.

In our formulation, we take these factors into consideration. We first propose a centralized solution which is a logarithmic utility maximization with constraints. This non-convex optimization problem can be transformed into a convex optimization, which can be solved by Geometric Programming efficiently [6] [7].

In this paper, when not all the secondary links can be supported with their QoS requirement, a reduced complexity searching algorithm is introduced to find the optimal subset of links which contains the maximum number of links with both QoS and interference temperature constraints satisfied. In previous work [11], we have formulated an iterative and distributed joint coordination and power control algorithm, which only requires the users to obtain limited local information in order to converge to a Nash equilibrium (NE) which will guarantee that the received power at the measuring point will not exceed the interference temperature constraint. While in this paper, we will propose a sequential play solution which

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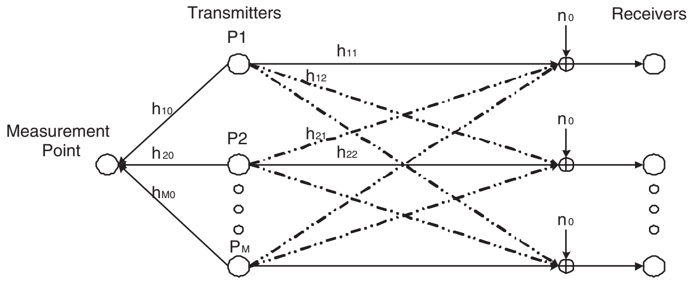

Fig. 1. System model for M transmitter-receiver pairs.

converges to the NE much faster than the joint coordination and power control algorithm. We then study the performance of both the reduced complexity algorithm and the sequential play.

II. Secondary Spectrum Sharing Model

Spectrum with bandwidth W is to be shared among M spread spectrum users, where a user refers to a transmitter and an intended receiver pair. For each i, the received SIR is given by

$$\gamma_i = \frac{y_i h_{ii}}{\frac{1}{L}(\sum_{j \neq i} y_j h_{ji}) + \sigma^2}, \quad (1)$$

where L is the normalized spreading sequence length, y_i is user $i's$ transmission power, h_{ij} is the channel gain from user $i's$ transmitter to user $j's$ receiver, and σ^2 is the background noise power that is assumed to be the same for all users. In order to satisfy an interference temperature constraint, the total received power at a specified measurement point must satisfy

$$\sum_{i=1}^{M} y_i h_{i0} \leq B, \quad (2)$$

where h_{i0} is the channel gain from user $i's$ transmitter to the measurement point, and $B > 0$ is a pre-defined threshold. We assume that all these secondary users adopt a spread spectrum signaling format, in which the transmitted power is evenly spread across the entire available band. This allows efficient multiplexing of data streams from different sources corresponding to different applications, and reduces the combined power-bandwidth allocation problem to a received power allocation problem. Hence, the interference temperature constraint is translated to a total received power threshold B at the measuring point. The system model is shown in Fig. 1.

III. Social Optimization

Secondary user $i's$ valuation of the spectrum is characterized by a utility $v_i(\gamma_i)$, where γ_i is the received SIR at user $i's$ receiver. We define the logarithmic utility $v_i(\gamma_i) = ln(\gamma_i)$. This utility function captures user's desire for higher data transmitting rate. With energy consumption and QoS provision considerations, each secondary link has a minimum SIR constraint γ_i^t. Let $\Theta = \{1, 2, ..., M\}$, be the set of transmitter and receiver link pairs, and let each transmitter's available transmitting power be $y_i \in (0, y_i^{max}], \forall i$.

We can formulate the social rate optimization problem with QoS constraint as follows

$$\begin{array}{lll} \text{maximize } \sum_i v_i(\gamma_i) & & \\ \text{subject to} & & \\ SIR_i & \geq \gamma_i^t & \forall i \\ \sum_i h_{i0} y_i & \leq B & \\ y_i & > 0 & \forall i \\ y_i & \leq y_i^{max} & \forall i. \end{array} \quad (3)$$

Maximizing $\sum_i \ln(\gamma_i)$ is equivalent to maximizing $\ln \prod_i \gamma_i$, which is then equivalent to minimizing $\prod_i \frac{1}{\gamma_i}$. Note that the objection function is posynomial. And the constraints can also be transformed into posynomial and monomial forms. So, this optimization problem is a convex optimization in geometric program form, and can be solved globally and efficiently.

We can define a normalized link gain matrix A with entries $\frac{h_{ij}}{h_{ii}}$ for $i \neq j$ and 0 for $i = j$, and let $H = \gamma^t A$, the normalized noise vector η such that $\eta_i = \frac{n_0}{h_{ii}}$, and vector $\mathbf{c}$ with $c_i = h_{i0}$. Further we define $\mathbf{y}^{max} = (y_1^{max}, \ldots, y_M^{max})$.

Theorem 1: If $\rho(H) < 1$, $(I - H)^{-1}\gamma^t\eta \leq \mathbf{y}^{max}$ and $(I - H)^{-1}\gamma^t\eta\mathbf{c} \leq B$, then there exists power vector $\mathbf{y}^* > 0$, which satisfies the above described optimization problem.

Proof $\rho(H) < 1$ and $(I-H)^{-1}\gamma^t\eta \leq \mathbf{y}^{max}$ imply there exists a positive power vector $\tilde{\mathrm{y}} = (I - H)^{-1}\gamma^t\eta$ which satisfies the SIR bound and the maximum transmitting power constraints. If further $\tilde{\mathrm{y}}\mathbf{c} < B$, each user can increase their power by a factor of $B / \sum_i y_i h_{i0}$, which increases the SIR for every user hence will increase the objective function, so we can always find another $\tilde{\mathrm{y}} \leq \mathbf{y}^* \leq \mathbf{y}^{max}$ which will satisfy the SIR constraints, make the total received power constraint tighter and maximize the objective function. □

With the total received power constraint B at the measuring point, there are some cases when not all the secondary links can achieve their minimum SIR constraint, which raise the problem of system feasibility. *Theorem 1* gives the condition under which there will be a feasible power allocation over the secondary users. When the feasibility condition is not satisfied, we can remove the secondary links one by one. Depending on the goal of optimization, the removal process will be different. If the goal is still to maximize the total utility which is proportion to the total transmitting rate, the strategy would be to exhaust all the possible active link combination, then by checking the feasibility condition and conducting optimization (3) to find the optimal feasible link set and the power allocation. An alternative removal process with more fairness would be to maximize the number of active secondary links with QoS and interference temperature constraints. This optimization needs all the system information including all the link gains and the number of secondary links to conduct the calculation. Therefore a centralized controller is needed to coordinate the access process.

IV. Optimal supported link subset searching

It can be proved that maximizing the number of active links for a QoS constrained secondary spectrum sharing problem is NP complete through the steps similar to [10]. In order to reduce the searching space and hence reduce the complexity of searching for the optimal supported link subset, we first characterize some properties of the supported link subset.

We say that a power vector $\mathbf{y}$ supports all transmitters at a SIR target γ^t, if and only if

$$\mathbf{y} \geqslant \gamma^t(A\mathbf{y} + \eta). \tag{4}$$

That is, each receiver i has a SIR $\gamma_i \geqslant \gamma_i$.

Next, we describe the power updates made by the distributed constrained power control (DCPC), when the target SIR is γ^t. The power adjustment made by the i^{th} terminal at the n^{th} time instant is given by

$$\begin{aligned} y_i(n) &= min\{y^{max}, \gamma^t \tfrac{y_i(n-1)}{\gamma_i(n-1)}\} = \\ & min\{y^{max}, \gamma^t(\eta_i + \textstyle\sum_{j\in\Theta} y_j(n-1)h_{ji})\}, 1 \le i \le M, \end{aligned} \tag{5}$$

It has been show in [9], that for any given γ^t, DCPC converges to a unique positive power vector determined by the fixed point solution to

$$\mathbf{y} = min\{\mathbf{y}^{max}, \gamma^t(A\mathbf{y} + \eta)\}. \tag{6}$$

A power vector $\mathbf{y}$ which satisfies the fixed point equations in (6), will be referred to as the stationary power vector. When all transmitters can be supported, the DCPC converge to the fixed point solution to

$$\mathbf{y} = \gamma^t(A\mathbf{y} + \eta). \tag{7}$$

It will be useful to annotate the stationary power vector with its corresponding set of transmitters. That is, for every subset of transmitters $\Theta_0 \subseteq \Theta$, $\mathbf{y}^{\Theta_0}$ will denote the stationary power vector of a system which consists only of the set Θ_0. Also, let S_{Θ_0} be the subset of transmitters which are supported (at γ^t) under the stationary power vector $\mathbf{y}^{\Theta_0}$ (i.e., in a system where DCPC runs only with the set of transmitters Θ_0). Also let $\bar{S}$ denotes the complement set of S. And let

$$y_i^{\Theta/\Theta_0} = \begin{cases} y_i^{\Theta}, & \text{if} \quad i \in \Theta_0 \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

Theorem 2: For a link set $\Theta_0 \subseteq \Theta$, if the secondary link system with DCPC consisting of link set Θ_0 is infeasible, the system consisting of set Θ will be infeasible either.

Proof Consider two cases:

1) $i \in \Theta_0$ be in the non-supported set $\bar{S}_{\Theta_0}$.
2) $\bar{S}_{\Theta_0} = \Phi$, i.e., all the links in Θ_0 are supported, but $\sum_{i\in\Theta_0} h_{i0}y_i^{\Theta_0} > B$.

For case 1)

$$y_i = y^{max}, \forall i \in \bar{S}_{\Theta_0}$$

Thus, from the fact that the h_{ij}'s are non-negative and Lemma 2 [10] ($\mathbf{y}^{\Theta_0} \le \mathbf{y}^{\Theta/\Theta_0}$).

$$y_i^{\Theta} \le y^{max} = y_i^{\Theta_0} < \gamma^t(\eta_i + \sum_{j\in\Theta_0} h_{ji}y_j^{\Theta_0}) \le \gamma^t(\eta_i + \sum_{j\in\Theta} h_{ji}y_j^{\Theta})$$

Thus, i is also in the non-supported set $\bar{S}_{\Theta}$.

For case 2) Because we have $\mathbf{y}^{\Theta_0} \le \mathbf{y}^{\Theta/\Theta_0}$ so

$$\sum_{i\in\Theta} h_{i0}y_i^{\Theta} > \sum_{i\in\Theta_0} h_{i0}y_i^{\Theta_0} > B,$$

i.e., the interference temperature bound B is violated. □

Theorem 2 establishes that the tree pruning algorithm [12] is valid for our scenario. Thus the search for optimal supported subset of links can be confined to a smaller searching space.

V. Potential Games and Secondary Spectrum Sharing

Because the nature of secondary spectrum sharing is temporary and distributed, the optimal searching gives us what is the best we can do but a practical secondary spectrum sharing scheme. In the following section, we develop a distributed algorithm which conserves the QoS provisioning properties with the objective of maximizing the secondary sharing capacity. This distributed process is composed of two phases. The coordination phase controls the optimal set of active secondary links which can access the spectrum, while the power control phase is to allocate transmitting power to support the minimum target link SIR γ_i^t given the set of active links.

A. Distributed Power Control

When there are M active links, we use the standard DCPC (5) to allocate the transmitting power. This DCPC will make the received SIR converge to the target SIR γ_i^t distributively except for cases where maximum transmitting power y_i^{max} is reached.

B. Potential Games

Suppose there are M transmitter and receiver link pairs competing for the secondary spectrum access opportunities. Let k be a time (iteration) counter and $N(k)$ be the aggregate received power at the measuring point at time k. Let

$$N(\mathbf{x}(k)) = \sum_{i=1}^{M} y_i(k)h_{i0}x_i(k), \tag{9}$$

where the $x_i(k)$ are independent Bernoulli random variable, and $x_i(k) = 1$ means the i^{th} link transmits otherwise $x_i(k) = 0$.

In order to maximize the system capacity while keeping the aggregated received power at the measuring point under the interference threshold, we define the utility function $u_i(\mathbf{x})$ shown in Fig. 2 for each link pair as follows,

$$\begin{cases} \frac{N(\mathbf{x}(k))}{3B} + \frac{2}{3}, & N(\mathbf{x}(k)) < B, \min_{j\in(j:x_j=1)} \gamma_j > \gamma^t \\ \frac{B}{3N(\mathbf{x}(k))} + \frac{1}{3}, & N(\mathbf{x}(k)) \ge B, \min_{j\in(j:x_j=1)} \gamma_j > \gamma^t \\ \frac{\min_{j\in(j:x_j=1)} \gamma_j}{3\gamma^t}, & \min_{j\in(j:x_j=1)} \gamma_j < \gamma^t \end{cases} \tag{10}$$

The objective for each user i is to maximize its utility function $u_i(\mathbf{x}(k))$. By maximizing this utility function, the system will reach an operating point where the total received interference temperature bound is tightened. This process implicitly maximizes the number of active secondary links. To emphasize that

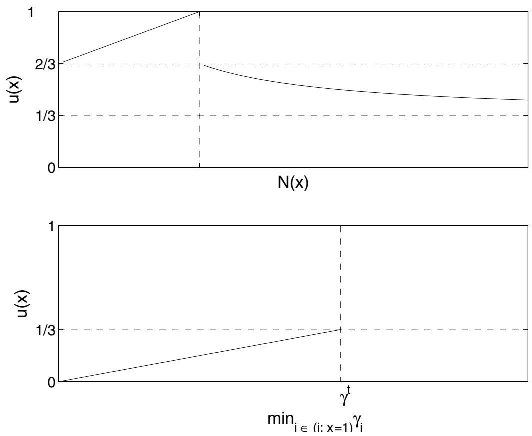

Fig. 2. utility function $u_i(N(\mathbf{x}))$.

the i^{th} user has control only over its own choice, we use an alternative notation $u_i(x_i, \mathbf{x}_{-i})$, where $\mathbf{x}_{-i}$ denotes that vector consisting of elements of $\mathbf{x}$ other than the i^{th} element.

Proposition 1: The secondary spectrum sharing game is a potential game and has a pure strategy(deterministic) equilibrium.

This proposition comes from the fact that we can define a potential function $\Phi = u(\mathbf{x})$ which satisfies $\triangle\Phi = \triangle u_i(\mathbf{x})$. Hence, we can start from an arbitrary deterministic strategy vector $\mathbf{x}$, and at each step one player increases it's utility. That means, that at each step Φ is increased identically. Since Φ can accept a finite amount of values, it will eventually reach a local maxima. At this point, no player can achieve any improvement, and we reach a Nash equilibrium. A practical method to achieve the NE would be to use sequential play where each player maximizes its own utility function sequentially while other players' strategies are fixed.

Theorem 3: The sequential play will never converge to a solution where the total received power at the measuring point exceeds the interference temperature bound.

Proof Suppose $\mathbf{x}^0$ is a Nash equilibrium solution of the game, and at this point has $N(\mathbf{x}^0) > B$, then by the definition of the utility function (10), we know that one of the link pairs with $x_i = 1$ can always increase his payoff $u_i(x_i, \mathbf{x}^0_{-i})$ by changing its strategy to $x_i = 0$, hence, this point $\mathbf{x}^0$ can never be a Nash equilibrium. We know that the sequential play will never converge to a point which is not a Nash equilibrium. Hence, we can conclude the above theorem. $\square$

Theorem 4: The sequential play will always converge to a solution where all the active links are supported with their target SIR.

Proof Similar to the previous proof, suppose $\mathbf{x}^0$ is a Nash equilibrium solution of the game, and at this point has $\gamma_i < \gamma^t$, then by the definition of the utility function (10), the link $j = \arg\min_{j\in(j:x_j=1)} \gamma_j$ can always increase his payoff by changing its strategy to $x_j = 0$, hence this point $\mathbf{x}^0$ can never be a Nash equilibrium. $\square$

To characterize the efficiency of the Nash equilibrium point achieved by the sequential play, let $\mathbf{x}^o$ be the Nash equilibrium strategy profile. This point has the property that either

$$\min_{j\in(j\ :x_j^o=0)} N(x_j = 1, \mathbf{x}^o_{-j}) > B, \tag{11}$$

which means, at the Nash equilibrium point, if any single secondary link j with $x_j = 0$ changes its choice to $x_j = 1$ unilaterally, the total received power at the measuring point would exceeds the interference temperature threshold B, or if adding one more link, some of the active link will not achieve their target SIR. So this Nash equilibrium solution tights this threshold constraint, and no new link can be added to this system with the QoS constraints. We note that multiple Nash equilibria may exist in this game.

VI. Numerical Results

In this section, we first present some numerical examples for a simple secondary sharing with only three transmitting and receiving pairs. The target SIR is selected to be $\gamma^t = 12.5$, and the noise power is $\sigma^2 = 5 \times 10^{-13}$, which approximately corresponds to the thermal noise power for a bandwidth of 1 MHz. We consider low rate data users, using a spreading gain of $L = 128$. Path gains are obtained using the simple path loss model $h_j = K/d_j^4$ where $K = 0.097$. This gives the following gain matrix:

$$H = 10^{-7}\begin{bmatrix} 0.0097 & 0.1552 & 0.0148 \\ 0.0019 & 0.0034 & 0.0066 \\ 0.0748 & 0.0237 & 0.0307 \end{bmatrix} \tag{12}$$

When the interference temperature bound and noise ratio $B/\sigma^2 = 200$, this three secondary link pair system is feasible. Using the geometric programming optimization method, we find the maximum aggregate utility is 8.3567, with $\gamma_1 = 12.5$, $\gamma_2 = 12.5$ and $\gamma_3 = 27.2541$, $y_1 = 0.0164$W, $y_2 = 0.0988$W, and $y_3 = 0.0107$W for each link. When $B/\sigma^2 = 60$, the social optimization is infeasible, but we can resort to our proposed potential game. After convergence, only link subset $\{1, 3\}$ or $\{2, 3\}$ can coexist with $\gamma = 12.5$. The optimal link subset searching results in the same optimal link subset $\{1, 3\}$ and $\{2, 3\}$.

The optimal supported link subset searching algorithm can significantly reduce the searching space which is shown in Fig. 3. Each point in the curve for the reduced complexity searching results from an averaging over 10 random secondary link geometric distribution with M link pairs, $B/\sigma^2 = 60$ and $\gamma_t = 12.5$. The naive exhaustive search needs to check 2^M subset to find the optimal supported secondary link set, while it can be seen that the searching complexity for the reduced complexity searching in this typical scenario is bounded by M^4 which is polynomial complexity. Obvious in the worst case when both the target SIR and the interference temperature bound are not taking effect, the reduced complexity searching degrades to the naive searching.

Fig. 4 depicts the performance of the sequential play results with respect to the optimal subset searching results. Each point in the sequential play solution curve represents an

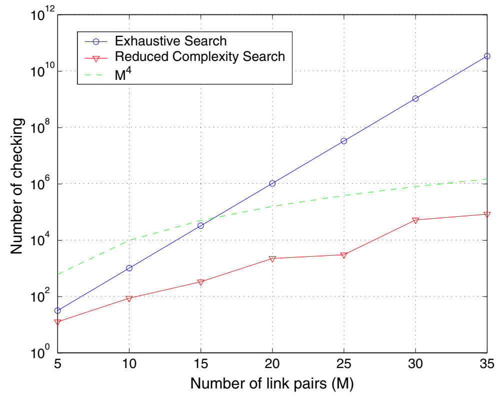

Fig. 3. The reduced complexity searching v.s. the naive exhaustive searching

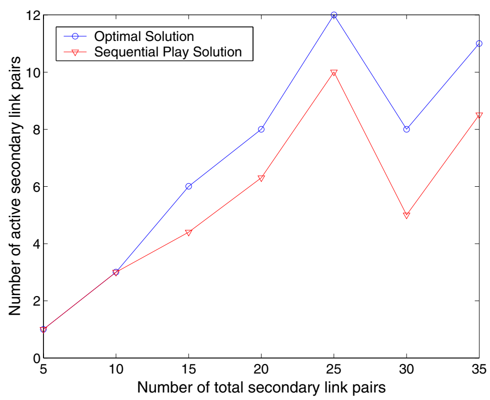

Fig. 4. Optimal subset searching v.s. sequential play

averaging over 10 random initialization strategies for each link set with $B/\sigma^2 = 60$ and $\gamma^t = 12.5$. Even though the sequential play is only eligible to converge to local optimal solutions, it can be seen from Fig. 4 that the degradation of the performance in terms of numbers of active links is not significant compared with the dramatic reduced convergence speed which is shown in Fig. 5. It can be seen that even with 35 total secondary link pairs, the sequential play converges within 100 iterations. This is compared with a 10^5 searching even with our proposed reduced complexity searching. Moreover, with simple coordination control, this sequential play can be implemented distributively.

VII. Conclusions

We have considered spectrum sharing among a group of spread spectrum users with a constraint on the total interference temperature at a particular measurement point, and a QoS constraint for each secondary link. A social optimization of this problem is formulated which is solved efficiently by using geometric programming method. There are cases, when this system with all secondary links active is infeasible. A reduced complexity optimal link subset searching is introduced, which can significantly reduce the searching space compared with naive searching. Then we define the secondary spectrum

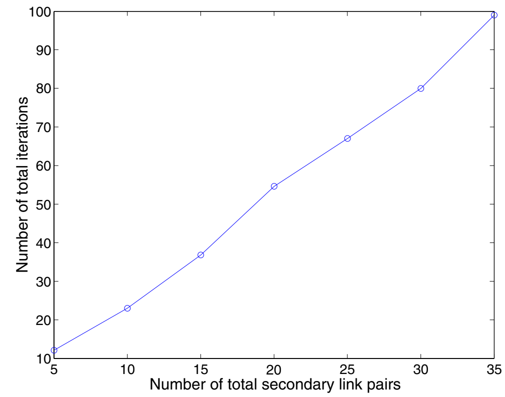

Fig. 5. Convergence of the sequential play

sharing problem as a potential game, which can be solved through sequential play. The sequential play is shown to converge to the Nash equilibria with fast speed. The achieved Nash equilibrium is characterized to be a point with a best tradeoff between the efficiency and the complexity.

Acknowledgment

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