In this work, a computationally fast, numerically stable, and memory-efficient cascading and doubling algorithm is proposed within the method of lines framework to model long planar waveguide gratings having thousands of periods in the propagation direction. This algorithm can model 2^N grating periods in n calculational steps and needs N^2 for N sample points in the problem space. It can model periodic, quasi-periodic, symmetric, and asymmetric gratings efficiently. Different deep waveguide gratings are modeled using this scheme and results for the fundamental TE mode spectral reflectivity are compared with published results showing excellent agreement.