# KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS 

## ELECTRICAL ENGINEERING DEPARTMENT

SECOND SEMESTER 2008/2009

EE 340 (03) FINAL EXAM
LOCATION: 59-1001

TIME: 7:300-10:00 A.M.
DATE: THURSDAY 4-FEBRUARY-2010

Student's Name:
Student's I.D. Number: $\qquad$
Section Number: $\qquad$

|  | Maximum Score | Score |
| :---: | :---: | :---: |
| Problem 1 | 20 |  |
| Problem 2 | 20 |  |
| Problem 3 | 20 |  |
| Problem 4 | 20 |  |
| Problem 5 | 20 |  |
| Total | 100 |  |

Answer the following multiple choice problems:
a) Consider a uniform plane electromagnetic wave which propagates in a lossy medium. The electric field amplitude of the wave is found to drop from $3 \mathrm{~V} / \mathrm{m}$ to $0.5 \mathrm{~V} / \mathrm{m}$ within a distance of 15 m along the wave direction of propagation. Using the given information, it can be conclude that the depth of penetration equals:

1) 15.00 m
2) 0.119 m
3) 4.10 m
4) 8.37 m
5) 2.18 m
6) 2.50 m
b) Consider two semi-infinite media, which are separated by the $x z$ plane. Medium $1(y>0)$ has a relative permeability of 3 and medium $2(y<0)$ has a relative permeability of 15 . The magnetic field intensity vector in medium 1 , at the boundary, is given by $\vec{H}_{1}=-2 \vec{a}_{x}+5 \vec{a}_{y}+11 \vec{a}_{z} \quad A / m$. Now consider a square area of dimension $2 m \times 2 m$ which lies entirely at the boundary between the two media (i.e. the $x z$ plane). The boundary has no surface current. From the given information, it can be conclude that the magnetic flux (in $W b$ ) through the given area equals:
7) $88 \mu_{o}$
8) $20 \mu_{o}$
9) $8 \mu_{o}$
10) $60 \mu_{o}$
11) $15 \mu_{o}$
12) $30 \mu$,
c) A conductor occupies the cylindrical volume ( $\rho<a$ ) while the outside volume ( $\rho>a$ ) is occupied by air. The electrostatic field $\vec{E}=\frac{A}{\rho} \vec{a}_{\rho} \quad$ V/m exists in the volume occupied by air, where A is some constant. Using the given information, it can be concluded that the surface charge density (in $C / \mathrm{m}^{2}$ ) on the conductor/air boundary is given by:
13) $\frac{A}{a} \varepsilon_{o}$
14) $\frac{A}{\rho}$
15) $-\frac{A}{a} \varepsilon_{o}$
16) $\frac{A}{\rho} \varepsilon_{o}$
17) $-\frac{A}{\rho} \varepsilon_{o}$
18) $\frac{A}{a} \varepsilon_{o} \vec{a}_{\rho}$
d) Which one of the following expressions represents the 3D wave equation for the electric field in a source free region?
19) $\nabla^{2} \vec{E}=\mu \varepsilon \vec{E}$
20) $\nabla^{2} \vec{E}=\mu \varepsilon \frac{\partial^{2} \vec{E}}{\partial t^{2}}$
21) $\nabla^{2} \vec{E}=\frac{1}{\mu \varepsilon} \frac{\partial^{2} \vec{E}}{\partial t^{2}}$
22) $\frac{\partial^{2} \vec{E}}{\partial z^{2}}=\mu \varepsilon \frac{\partial^{2} \vec{E}}{\partial t^{2}}$
23) $\nabla^{2} \vec{E}=\frac{\mu}{\varepsilon} \frac{\partial^{2} \vec{E}}{\partial t^{2}}$
24) $\nabla^{2} \vec{E}=\frac{\partial^{2} \vec{E}}{\partial t^{2}}$
e) The figure shows a partially-filled parallelplate capacitor with a plate separation $d=0.5 \mathrm{~cm}$. The relative permittivity of the dielectric filling is $\varepsilon_{r}=5$. The electric flux density vector in the dielectric region is given by:
$\vec{D}_{1}=-\vec{a}_{z} 0.707 \mu C / m^{2}$

The resulting potential difference $V_{o}$ across the capacitor equals:

1) 16 V
2) 2000 V
3) 60 V
4) 30 V
5) 80 V
6) 400 V

## Problem 2 [20 Points]

a) Consider the infinitely long cylindrical conductor shown in the figure. The conductor has radius $a$ and it carries D.C. current $I$ in the $+z$ direction. The axis of the conductor coincides with the $z$ axis. The conductor is made of a nonmagnetic material.

Derive an expression for the magnetostatic energy stored in 1 m length of the conductor.

b) Consider the infinitely long straight filamentary conductor which is placed along the entire $z$-axis. The conductor carries DC current $I_{1}$. As shown in the figure, the coplanar N -turn closed rectangular circuit recedes from the straight conductor with the constant velocity $\vec{u}=u_{o} \vec{a}_{o}$. At $t=0$ the left hand side of the rectangular loop coincides with the straight conductor.

Assuming that the rectangular circuit has total resistance $R$, derive an expression (valid for $t>0$ ) for the induced current $i_{2}(t)$.

a) A uniform plane EM propagates in a lossless nonmagnetic medium. The wavelength of the wave equals 0.75 cm . The magnetic flux density vector associated with the wave is given by:
$\vec{B}=\vec{a}_{z} 20 \times 10^{-6} \cos \left(10^{11} t-\beta y\right) \mathrm{Wb} / \mathrm{m}^{2}$
Find an expression for the $\vec{D}$ field associated with the wave. The required expression must not contain any unknown quantities.
b) The electric field of a 50 MHz uniform plane EM wave propagating in a lossless medium is given by:
$\vec{E}_{s}=\vec{a}_{y}(1+j) e^{j 2 x}+\vec{a}_{z} \sqrt{2} e^{j 2 x-j \pi / 4}$

1) What is the direction of wave propagation?
2) Calculate the phase velocity.
3) What is the wave polarization?

Problem 4 [20 Points]
a) A uniform plane TEM wave propagates in a lossy nonmagnetic medium. The intrinsic impedance of the wave equals $110 e^{j 37^{\circ}} \Omega$. Calculate the phase velocity of the wave.
b) Consider a uniform EM wave propagating in free space. The wave has the following instantaneous Poynting's vector:
$\vec{a}_{y} 20 \cos ^{2}\left(1.5 \times 10^{8} t-0.5 y\right) W / m^{2}$

Assuming the electric is linearly polarized in the $x$ direction, find an expression for the magnetic field intensity vector of the wave.

Consider two semi-infinite media. Medium $1(x<0)$ is a lossless nonmagnetic dielectric and medium 2 $(x>0)$ is air. A 5 GHz uniform plane EM wave is normally incident from medium 1 onto medium 2. The incident and reflected electric fields are respectively given by:

$$
\begin{array}{ll}
\vec{E}_{i s}=30 e^{-j \beta_{1} x} \vec{a}_{z} & \mathrm{~V} / \mathrm{m} \\
\vec{E}_{r s}=10 e^{j \beta_{1} x} \vec{a}_{z} & \mathrm{~V} / \mathrm{m}
\end{array}
$$

Where the subscript $s$ indicates the phasor form of the field. Find:
a) $\beta_{1}$
b) $\vec{E}_{s}$
c) An expression for the reflected instantaneous magnetic field $\vec{H}_{r}$.
d) The numerical value of the reflected electric field $\vec{E}_{r}, 5 \mathrm{~cm}$ from the boundary at $t=1 \mu \mathrm{~s}$.

Electrostatics:
$\vec{E}=\sum_{i=1}^{N} \frac{Q_{i} \vec{R}_{i}}{4 \pi \varepsilon R_{i}^{3}}+\int_{l} \frac{\rho_{L} \vec{R}}{4 \pi \varepsilon R^{3}}+\int_{s} \frac{\rho_{s} \vec{R}}{4 \pi \varepsilon R^{3}}+\int_{v} \frac{\rho_{v} \vec{R}}{4 \pi \varepsilon R^{3}} \quad, \quad V=\sum_{i=1}^{N} \frac{Q_{i}}{4 \pi \varepsilon R_{i}}+\int_{l} \frac{\rho_{L}}{4 \pi \varepsilon R}+\int_{s} \frac{\rho_{s}}{4 \pi \varepsilon R}+\int_{v} \frac{\rho_{v}}{4 \pi \varepsilon R}$
$\oint_{s} \vec{D} \cdot d \vec{s}=Q, \quad \oint_{l} \vec{E} \cdot d \vec{l}=0, \quad \nabla \vec{D}=\rho_{v}, \quad \nabla \times \vec{E}=0, \quad \vec{D}=\varepsilon \vec{E} \quad, \quad V_{P}=-\int_{\infty}^{P} \vec{E} \cdot d \vec{l}$,
$V_{A B}=V_{B}-V_{A}=-\int_{A}^{B} \vec{E} \cdot d \vec{l}, \vec{E}=-\nabla V, \quad w_{e}=\frac{1}{2} \varepsilon E^{2}, \vec{E}=\frac{\rho_{L}}{2 \pi \varepsilon \rho} \vec{a}_{\rho}, \quad \vec{E}=\frac{\rho_{s}}{2 \varepsilon} \vec{a}_{N}, \quad D_{1 n}-D_{2 n}=\rho_{s}$,
$E_{1 t}=E_{2 t}, C=\frac{Q}{V_{o}}, \nabla^{2} V=-\frac{\rho_{v}}{\varepsilon}$

## Magnetostatics:

$\oint_{s} \vec{B} \cdot d \vec{s}=0, \quad \oint_{l} \vec{H} \cdot d \vec{l}=I, \quad \nabla \vec{B}=0, \quad \nabla \times \vec{H}=\vec{J}, \quad \vec{B}=\mu \vec{H}, \quad \vec{B}=\nabla \times \vec{A}, \quad \vec{A}=\frac{I}{4 \pi} \int_{l} \frac{d \vec{l} \times \vec{R}}{R}, \quad \nabla \vec{A}=0$,
$\nabla^{2} \vec{A}=-\mu \vec{J}, \quad \psi_{m}=\int_{s} \vec{B} \cdot d \vec{s}=\oint_{l} \vec{A} \cdot d \vec{l}, \quad M_{21}=\frac{\Lambda_{21}}{I_{1}}=\frac{N_{2} \psi_{21}}{I_{1}}, \quad L=\frac{\Lambda}{I}, \quad d \vec{H}=\frac{I}{4 \pi} \frac{d \vec{l} \times \vec{R}}{R^{3}}$,
$w_{m}=\frac{1}{2} \mu H^{2} \vec{H}=\vec{a}_{\phi} \frac{I}{4 \pi \rho}\left[\cos \alpha_{2}-\cos \alpha_{1}\right], \vec{H}=\vec{a}_{\phi} \frac{I}{2 \pi \rho}, \quad \vec{F}_{m}=I \int_{l} d \vec{l} \times \vec{B} \quad, \quad B_{1 n}=B_{2 n}$,
$\left(\vec{H}_{1}-\vec{H}_{2}\right) \times \vec{a}_{n 12}=\vec{K}$
EMF: $\quad e m f=-\int_{s} \frac{\partial \vec{B}}{\partial t} d \vec{s}+\oint_{l}(\vec{u} \times \vec{B}) d \vec{l}=-\frac{d \psi_{m}}{d t}$
Maxwell's Equations (General Form):
$\nabla \times \vec{E}=-\frac{\partial \vec{B}}{\partial t}, \quad \nabla \times \vec{H}=\vec{J}+\frac{\partial \vec{D}}{\partial t}, \quad \nabla \vec{D}=\rho_{v}, \quad \nabla \vec{B}=0$

## Plane TEM Waves:

$$
\begin{aligned}
& u_{p}=\frac{\omega}{\beta}, u_{g}=\frac{d \omega}{d \beta}, u_{p}=\frac{c}{\sqrt{\mu_{r} \varepsilon_{r}}}, \eta=\sqrt{\frac{\mu}{\varepsilon}}, \lambda=\frac{2 \pi}{\beta}, \eta=\frac{\sqrt{\mu / \varepsilon}}{\left[1+(\sigma / \omega \varepsilon)^{2}\right]^{1 / 4}} \exp \left[j \frac{1}{2} \tan ^{-1}(\sigma / \omega \varepsilon)\right] \\
& \gamma=\alpha+j \beta=j \omega \sqrt{\mu \varepsilon(1-j \sigma / \omega \varepsilon)}, \alpha=\omega \sqrt{\frac{\mu \varepsilon}{2}\left[\sqrt{1+(\sigma / \omega \varepsilon)^{2}}-1\right]}, \beta=\omega \sqrt{\frac{\mu \varepsilon}{2}\left[\sqrt{1+(\sigma / \omega \varepsilon)^{2}}+1\right]} \\
& \delta=1 / \alpha, \\
& \Gamma_{\perp}=\frac{\eta_{2} \cos \theta_{i}-\eta_{1} \cos \theta_{t}}{\eta_{2} \cos \theta_{i}+\eta_{1} \cos \theta_{t}}, \tau_{\perp}=\frac{2 \eta_{2} \cos \theta_{i}}{\eta_{2} \cos \theta_{i}+\eta_{1} \cos \theta_{t}}, 1+\Gamma_{\perp}=\tau_{\perp} \\
& \Gamma_{/ /}=\frac{\eta_{2} \cos \theta_{t}-\eta_{1} \cos \theta_{i}}{\eta_{2} \cos \theta_{t}+\eta_{1} \cos \theta_{i}}, \quad \tau_{/ /}=\frac{2 \eta_{2} \cos \theta_{i}}{\eta_{2} \cos \theta_{t}+\eta_{1} \cos \theta_{i}}, 1+\Gamma_{/ /}=\tau_{/ /}\left(\frac{\cos \theta_{t}}{\cos \theta_{i}}\right), \quad \beta_{1} \sin \theta_{i}=\beta_{2} \sin \theta_{t} \\
& \theta_{r}=\theta_{i}
\end{aligned}
$$

For normal incidence, $\theta_{i}=0$
Poynting's Vector $=\vec{E} \times \vec{H} \quad$, Average Poynting's Vector $=\frac{1}{2} \operatorname{Re}\left(\vec{E}_{s} \times \vec{H}_{s}^{*}\right)$

## Differentials:

$d \vec{l}=\vec{a}_{\rho} d \rho+\vec{a}_{\phi} \rho d \phi+\vec{a}_{z} d z, \quad d \vec{s}=\vec{a}_{\rho} \rho d \phi d z+\vec{a}_{\phi} d \rho d z+\vec{a}_{z} \rho d \rho d \phi, \quad d v=\rho d \rho d \phi d z$
$d \vec{l}=\vec{a}_{r} d r+\vec{a}_{\theta} r d \theta+\vec{a}_{\phi} r \sin \theta d \phi, \quad d \vec{s}=\vec{a}_{r} r^{2} \sin \theta d \theta d \phi+\vec{a}_{\theta} r \sin \theta d r d \phi+\vec{a}_{\phi} r d r d \theta, \quad d v=r^{2} \sin \theta d r d \theta d \phi$

