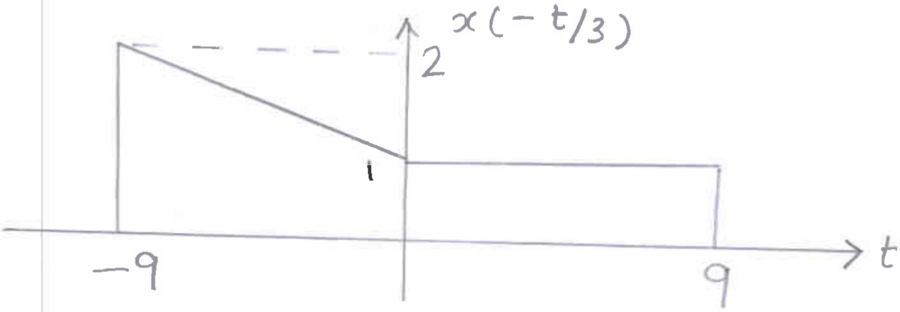
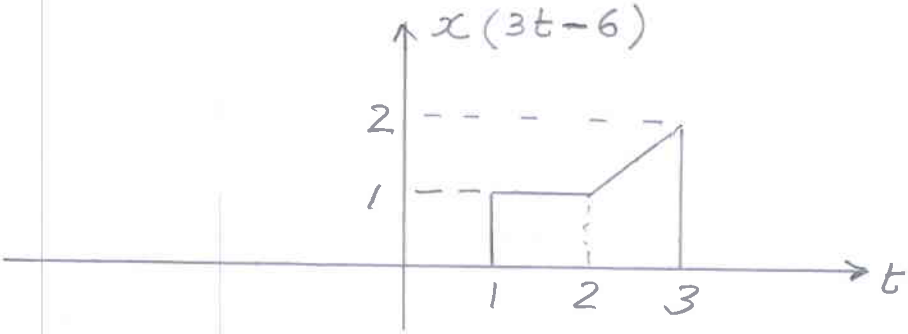
HW 1

2.1 (a)

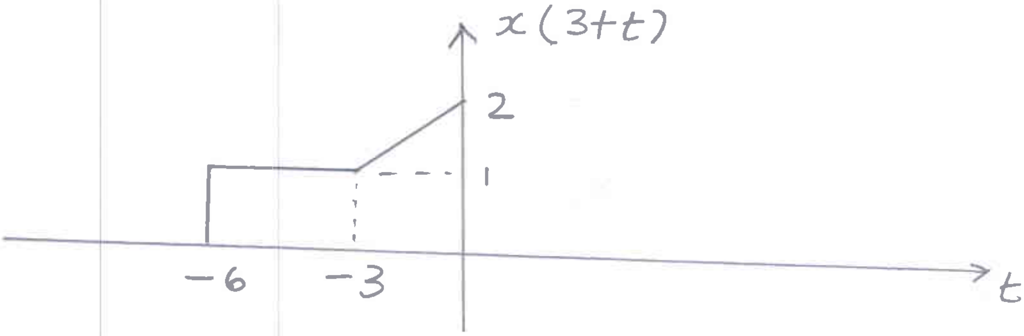
i)

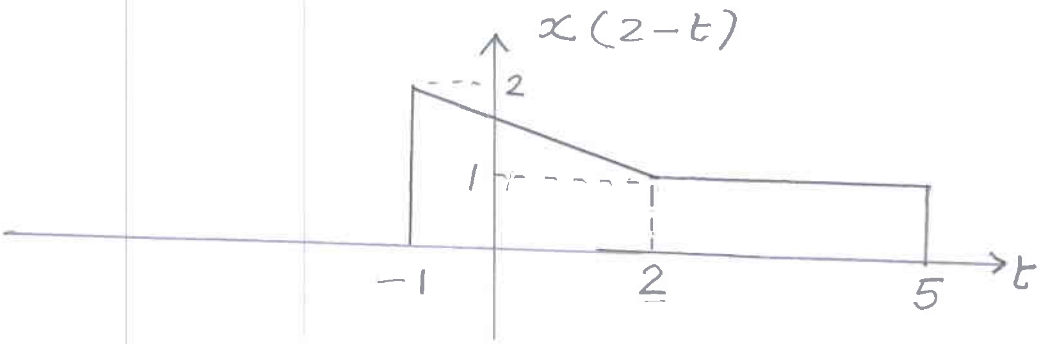


ii)



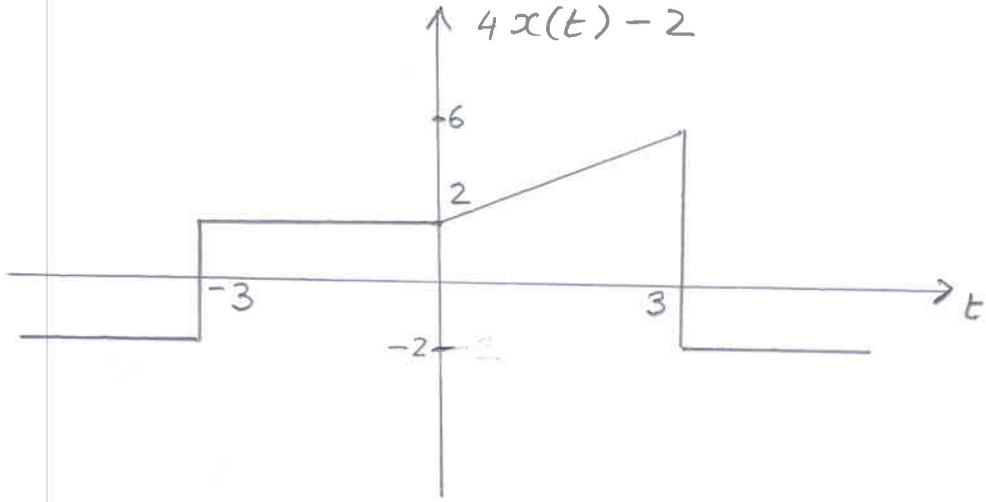
iii)



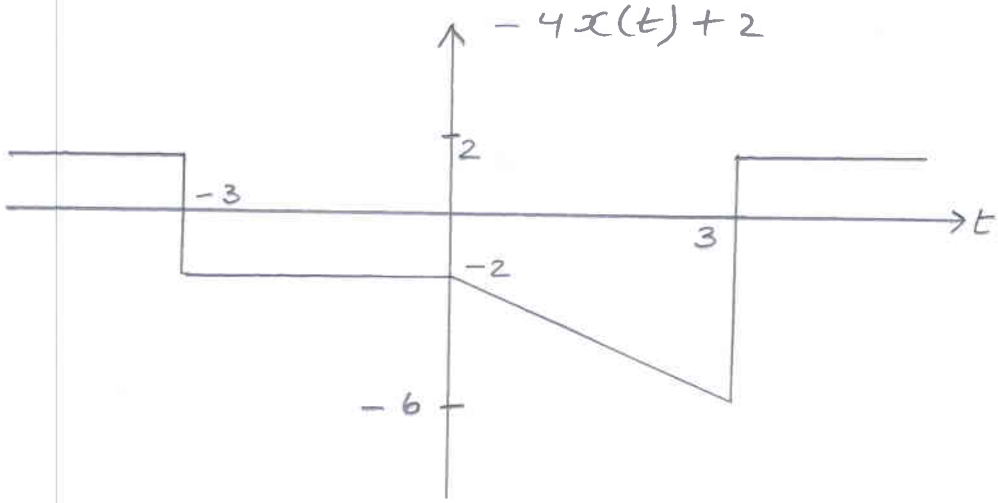


2.2(a)

i)

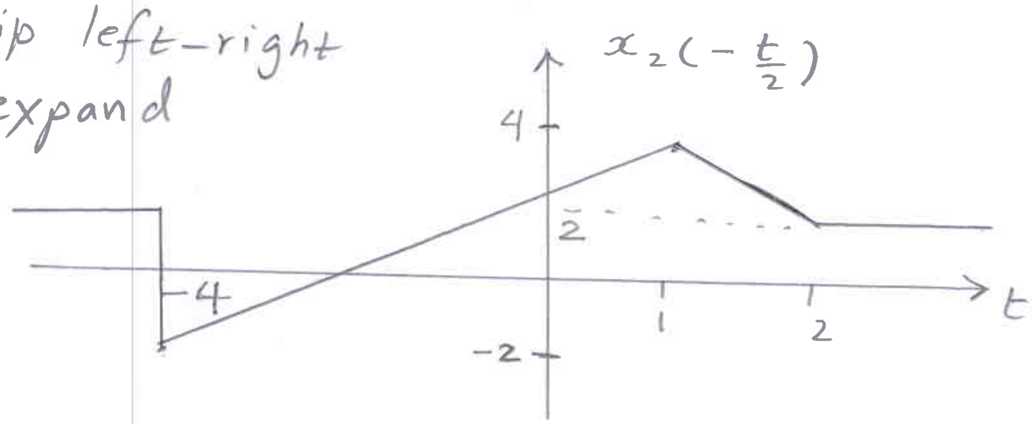


iv)

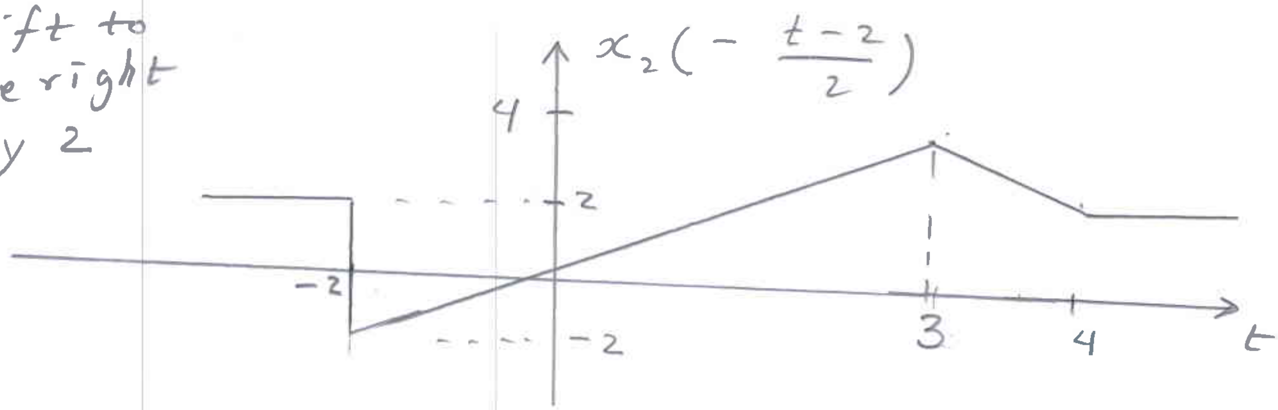


2.3 a)

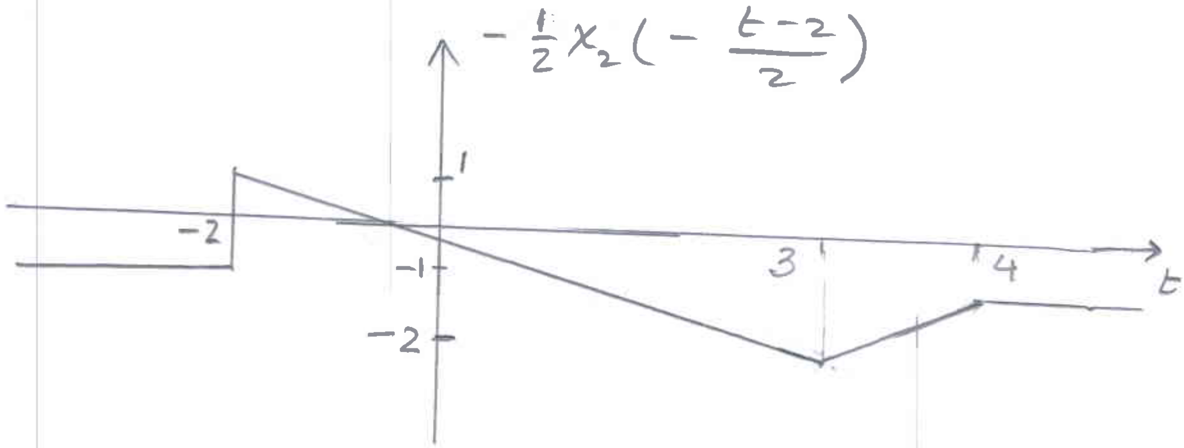
flip left-right & expand



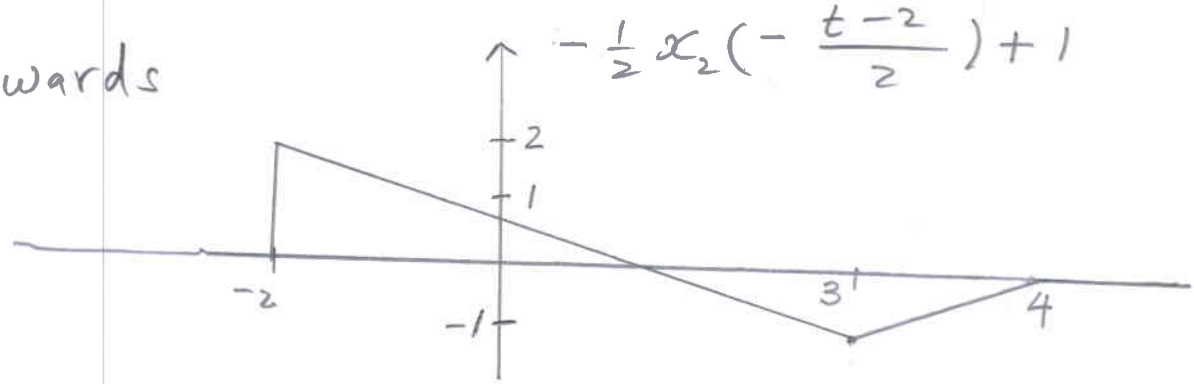
shift to the right by 2



Multiply by $-\frac{1}{2}$



Raise upwards by 1



$\therefore \quad x_1(t) = -0.5\, x_2(-0.5t+1)+1$

2.3 (b)

verification.

$$x_1(t) = -0.5\,x_2(-0.5t+1)+1$$

At $t=3$

$x_1(3) = -1$ (from figure)

$$x_1(3) = -0.5\,x_2(-0.5\times 3+1)+1$$

$$= -0.5\,x_2(-1.5+1)+1$$

$$= -0.5\,x_2(-0.5)+1$$

$$= -0.5(4)+1 \quad \text{(from figure)}$$

$$= -2+1$$

$$= -1$$

Checks.

$t = -2^+$

$x(-2^+) = 2$

$x_1(-2^+) = -0.5\, x_2(-0.5 \times (-2^+) + 1) + 1$

$= -0.5\, x_2(2^-) + 1$

$= -0.5(-2) + 1 = 2$

checks.

$t = 4$

$x_1(4) = 0$

$x_1(4) = -0.5\, x_2(-0.5 \times 4 + 1) + 1$

$= -0.5\, x_2(-1) + 1$

$= -0.5(2) + 1 = 0$

checks.

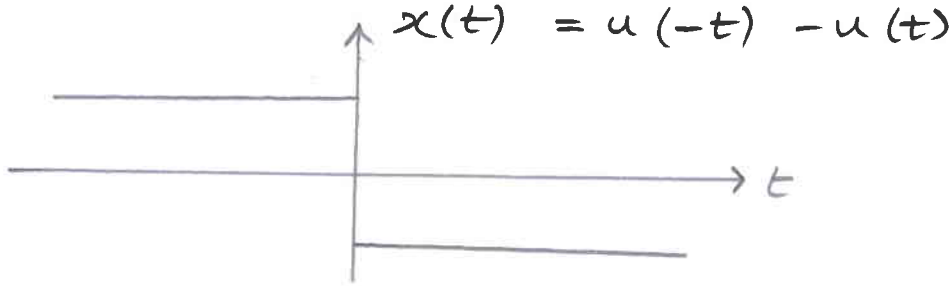
2.6

a) $x(t) = -2t$ (odd)

b) $x(t) = e^{-|4t|}$ (even)

c) $x(t) = 5\cos 6t$ (even)

d) $x(t) = u(-t) - u(t)$



(odd)

you can also show that it is odd without plotting it.

$$x(-t) = u(t) - u(-t) = -x(-t)$$

$\therefore$ odd

e) $x(t) = \sin\left(6t - \frac{\pi}{2}\right) = -\cos 6t$

($\therefore$ even)

f) $x(t) = u(-t) + u(t)$

$$x(-t) = u(t) + u(-t) \implies x(t) = x(-t)$$

($\therefore$ even)

comment: Actually $x(t) = u(-t) + u(t) = 1$

2.11

a) $x(t) = \cos 3t + \sin 5t$

$\frac{\omega_1}{\omega_2} = \frac{3}{5} \Rightarrow$ periodic

$T_1 = \frac{2\pi}{3}$, $T_2 = \frac{2\pi}{5}$

$\frac{T_1}{T_2} = \frac{5}{3} \Rightarrow T_0 = 5T_2 = 2\pi$

or $T_0 = 3T_1 = 2\pi$

$\therefore \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2\pi} = 1$ rad/s

b) $x(t) = \cos t + \sin \pi t$

$\frac{\omega_1}{\omega_2} = \frac{1}{\pi} \Rightarrow$ not periodic, because $\frac{\omega_1}{\omega_2}$ is not a rational number.

c) $x(t) = \cos 3t + \sin 9t$

$\frac{\omega_1}{\omega_2} = \frac{3}{9} \Rightarrow$ periodic

$T_1 = \frac{2\pi}{3}$ & $T_2 = \frac{2\pi}{9}$

$\frac{T_1}{T_2} = \frac{9}{3} = \frac{3}{1} \Rightarrow T_0 = 3T_2 = \frac{2\pi}{3}$

or $T_0 = 1T_1 = \frac{2\pi}{3}$

$$\therefore \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2\pi/3} = 3 \text{ rad/s.}$$

d) $x(t) = \cos 3\pi t + \sin 4\pi t + \cos t$

Not periodic

e) $x(t) = \cos 4\pi t + \sin 6\pi t + e^{j5\pi t}$

periodic

$\frac{\omega_1}{\omega_2}$ & $\frac{\omega_2}{\omega_3}$ are rational numbers.

It is easy to conclude that $\omega_0 = \pi$

(ω_0 is the "largest" common divisor of

4π, 6π, 5π).

$$\therefore T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\pi} = 2$$

Alternative Method: $T_1 = \frac{2\pi}{4\pi} = \frac{1}{2}$

$$T_2 = \frac{2\pi}{6\pi} = \frac{1}{3}$$

$$T_3 = \frac{2\pi}{5\pi} = \frac{2}{5}$$

$\frac{T_1}{T_2} = \frac{3}{2} \Rightarrow T_0' = 3 \times T_2 = 1$ (T_0' is the fundamental period associated with T_1 & T_2)

$$\frac{T_0'}{T_3} = \frac{5}{2} \Longrightarrow T_0 = 5\,T_3 = 2$$

$$\text{or} \quad T_0 = 2\,T_0' = 2$$

f) $x(t) = \cos(3t + 30^\circ) + e^{j2t} + \sin 3\pi t$

$\Longrightarrow$ Not periodic

2.19 (c)

i) $$\int_{-\infty}^{\infty} \cos(2t)\,\delta(t)\,dt = \int_{-\infty}^{\infty} \cos(0)\,\delta(t)\,dt$$

$$= \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

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ii) $$\int_{-\infty}^{\infty} \cos\left[2\left(t - \frac{\pi}{4}\right)\right] \delta\left(t - \frac{\pi}{4}\right) dt$$

$$= \int_{-\infty}^{\infty} \cos\left[2\left(\frac{\pi}{4} - \frac{\pi}{4}\right)\right] \delta\left(t - \frac{\pi}{4}\right) dt = \int_{-\infty}^{\infty} \delta\left(t - \frac{\pi}{4}\right) dt$$

$$= 1$$

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iii) $$\int_{-\infty}^{\infty} \sin(2t)\,\delta\left(t - \frac{\pi}{6}\right) dt$$

$$= \int_{-\infty}^{\infty} \sin\left(\frac{\pi}{3}\right) \delta\left(t - \frac{\pi}{6}\right) dt = \frac{\sqrt{3}}{2} \int_{-\infty}^{\infty} \delta\left(t - \frac{\pi}{6}\right) dt$$

$$= \frac{\sqrt{3}}{2} = 0{,}866$$

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iv) $$\int_{-\infty}^{\infty} \sin\left[\left(t - \frac{\pi}{4}\right)\right] \delta\left(t - \frac{\pi}{2}\right) dt$$

$$= \int_{-\infty}^{\infty} \sin\left[\left(\frac{\pi}{2} - \frac{\pi}{4}\right)\right] \delta\left(t - \frac{\pi}{2}\right) dt$$

$$= \int_{-\infty}^{\infty} \sin\left(\frac{\pi}{4}\right) \delta\left(t - \frac{\pi}{2}\right) dt = \frac{1}{\sqrt{2}} = 0.707$$

2.19 (c) continued

v) $\int_{-\infty}^{\infty} \sin\left[\left(t - \frac{\pi}{6}\right)\right] \delta\left(2t - \frac{2\pi}{3}\right) dt$

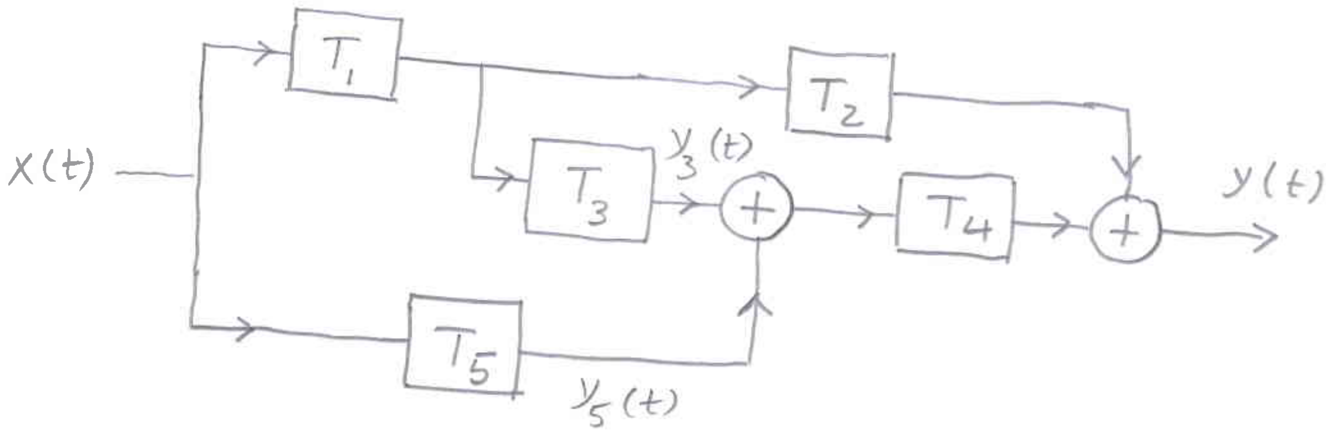
$= \int_{-\infty}^{\infty} \sin\left[\left(t - \frac{\pi}{6}\right)\right] \delta\left[2\left(t - \frac{\pi}{3}\right)\right] dt$

$= \frac{1}{2} \int_{-\infty}^{\infty} \sin\left[\left(t - \frac{\pi}{6}\right)\right] \delta\left(t - \frac{\pi}{3}\right) dt$

$= \frac{1}{2} \int_{-\infty}^{\infty} \sin\left[\left(\frac{\pi}{3} - \frac{\pi}{6}\right)\right] \delta\left(t - \frac{\pi}{3}\right) dt$

$= \frac{1}{2} \int_{-\infty}^{\infty} \sin\left(\frac{\pi}{6}\right) \delta\left(t - \frac{\pi}{3}\right) dt = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

2.27(a)



$$y(t) = T_2\left[T_1[x(t)]\right] + T_4\left[y_3(t) + y_5(t)\right]$$

$$y_3(t) = T_3\left[T_1[x(t)]\right]$$

$$y_5(t) = T_5[x(t)]$$

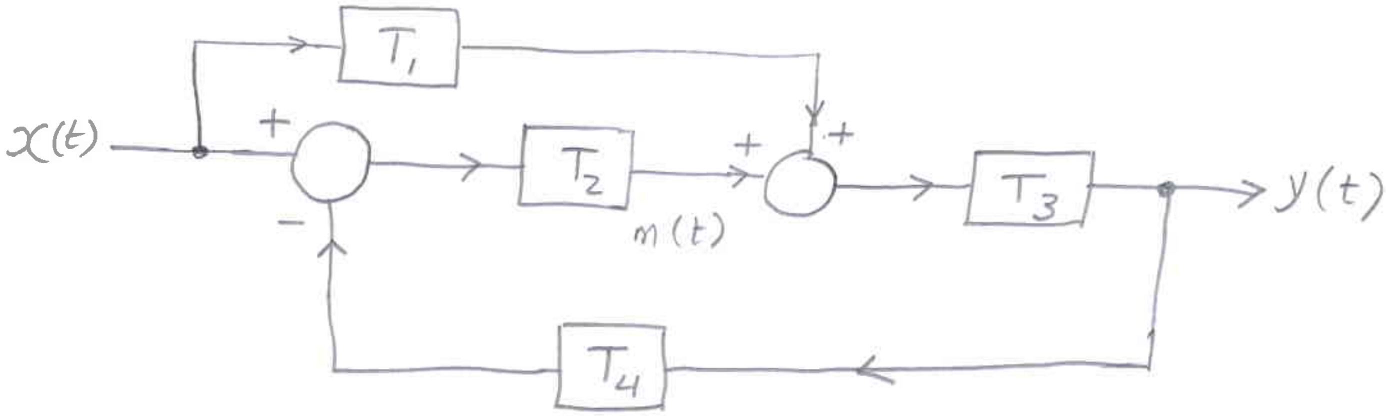
$$\therefore\ y(t) = T_2\left[T_1[x(t)]\right]$$

$$+\ T_4\left[T_3\left[T_1[x(t)]\right] + T_5[x(t)]\right]$$

$$= T_2 T_1[x(t)] + T_4\left[T_3\left[T_1[x(t)]\right]\right]$$

$$+\ T_4\left[T_5[x(t)]\right]$$

2.28



$$m(t) = T_2\left[x(t) - T_4[y(t)]\right]$$

$$y(t) = T_3\left[T_1[x(t)] + m(t)\right]$$

$$= T_3\left[T_1[x(t)] + T_2\left[x(t) - T_4[y(t)]\right]\right]$$

$$= T_3\left[T_1[x(t)]\right] + T_3\left[T_2[x(t)]\right]$$

$$- T_3\left[T_2\left[T_4[y(t)]\right]\right]$$

2.31

a) $y(t) = \cos[x(t-2)]$

i) Has memory

ii) Non-invertible

iii) Causal

iv) stable

v) Time invariant

vi) Nonlinear

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b) $y(t) = 3x(3t+3)$

i) Has memory

ii) Invertible

iii) non-causal

iv) stable

v) Time invariant

vi) Linear

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d) $y(t) = e^{t\,x(t)}$

i) Memoryless

ii) Non invertible at $t=0$

$\Longrightarrow$ Non-invertible.

iii) Causal

iv) Unstable

v) Time Varying

vi) Nonlinear

2.31 continued

e) $y(t) = 5x(t) - 3$

i) Memoryless   ii) Invertible

iii) Causal   iv) stable

v) Time Invariant   vi) nonlinear

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f) $y(t) = \int_{-\infty}^{t} x(2\tau)\,d\tau$

i) Has memory   ii) Invertible

iii) Non-causal   iv) unstable

v) Time-invariant   v) Linear

2.33

$y(t) = x(t-1)$

i) Has memory   ii) Invertible

iii) Causal   iv) Stable

v) Time-invariant   vi) Linear