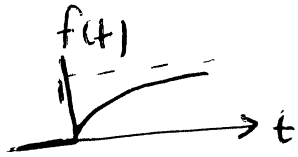
EE 207 - winter 2015 (142)

HW 3 (solution)

(Q1)

(a) $f(t) = (1 - e^{-bt})\, u(t)$



since f(t) is not absloutley integrable

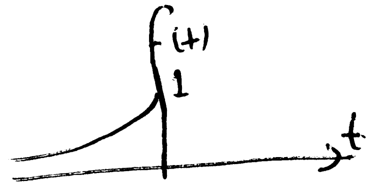
$\Rightarrow$ we can not use the integral formula

$$\Rightarrow F(w) = F.T\{u(t)\} - F.T\{e^{-bt}u(t)\}$$

$$= \left\{\pi\delta(w) + \frac{1}{jw}\right\} - \left\{\frac{1}{b+jw}\right\}$$

$$= \frac{b}{jwb - w^2} + \pi\,\delta(w)$$

(b) $f(t) = e^{at}u(t) \quad a > 0$



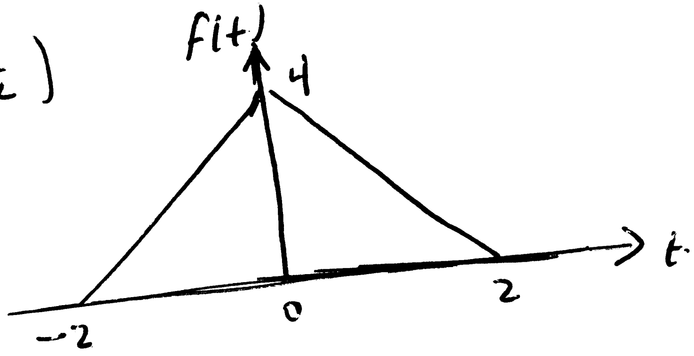
$$F(w) = \int_{-\infty}^{\infty} e^{at}u(-t)\, e^{-jwt}\, dt$$

$$= \int_{-\infty}^{\infty} e^{(a-jw)t}\, dt = \left. \frac{e^{(a-jw)t}}{(a-jw)} \right|_{-\infty}^{0}$$

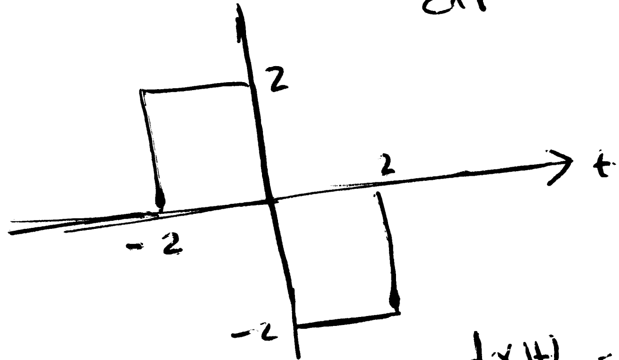
$$= \frac{1}{(a-jw)}\left[e^{(a-jw)(0)} - e^{(a-jw)(\infty)}\right]$$

$$= \frac{1}{a-jw}[1 - 0] = \frac{1}{a-jw}$$

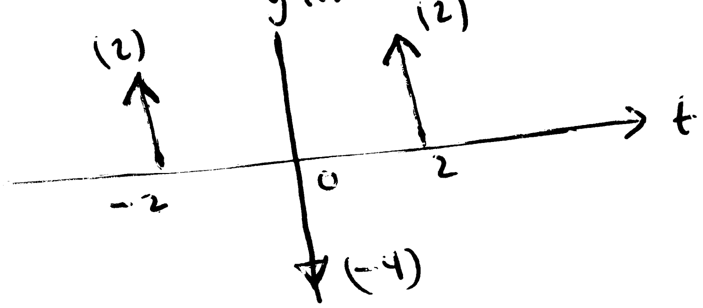
Q2 $4\,\text{tri}\left(\frac{t}{2}\right)$



$x(t) = \frac{df(t)}{dt}$



$y(t) = \frac{dx(t)}{dt} = \frac{d^2f(t)}{dt}$



$$Y(w) = 2e^{j2w} - 4 + 2e^{-j2w}$$

$$= 4(j2)\,\cos(2w) - 4$$

Since $\frac{d^2f(t)}{dt^2} = y(t) \longleftrightarrow (jw)^2 F(w) = Y(w)$

$$\Rightarrow F(w) = \frac{4(j2)\,\cos(2w) - 4}{-w^2}$$

Q3

- method ①

from the table $(-jt)^n f(t) \leftrightarrow \frac{d^n}{d\omega^n} F(\omega)$

Since, $e^{-at} u(t) \leftrightarrow \frac{1}{(a+j\omega)}$

$\Rightarrow (-jt)^2 e^{-at} u(t) \leftrightarrow \frac{d^2}{d\omega^2} \left(\frac{1}{a+j\omega} \right)$

$\Rightarrow -t^2 e^{-at} u(t) \leftrightarrow - \frac{2}{(a+j\omega)^3}$

$\Rightarrow \frac{t^2}{2} e^{-at} u(t) \leftrightarrow \frac{1}{(a+j\omega)^3}$

method ②

From the table $t^{n-1} e^{-at} u(t) \leftrightarrow \frac{(n-1)!}{(a+j\omega)^n}$

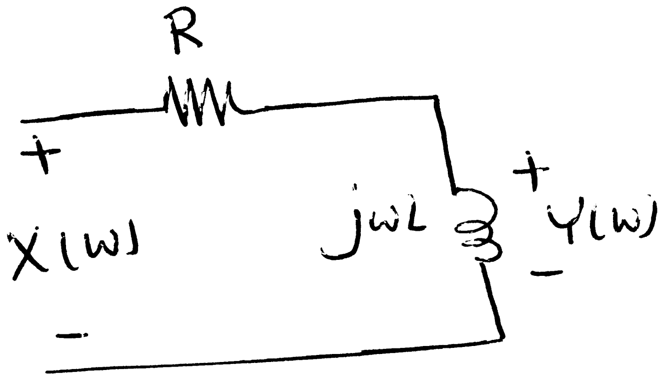
Let $n = 3$

$t^2 e^{-at} u(t) \leftrightarrow \frac{2!}{(a+j\omega)^3} = \frac{2}{(a+j\omega)^3}$

$\Rightarrow \frac{t^2}{2} e^{-at} u(t) \leftrightarrow \frac{1}{(a+j\omega)^3}$

Q4

(a)



Using voltage divison

$$H(w) = \frac{Y(w)}{X(w)} = \frac{jwL}{R + jwL}$$

(b) $$H(w) = jw \frac{L}{R + jwL} = (jw) \frac{1}{\left(\frac{R}{L} + jw\right)}$$

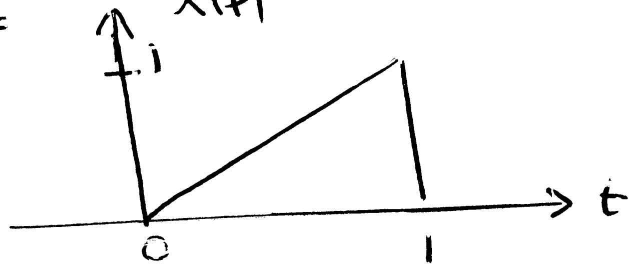
$$\Rightarrow h(t) = \frac{d}{dt} F.T^{-1}\left[\frac{1}{\frac{R}{L} + jw}\right]$$

$$= \frac{d}{dt} e^{-\frac{R}{L}t} u(t)$$

$$= -\frac{R}{L} e^{-\frac{R}{L}} u(t) + e^{-\frac{R}{L}t} \delta(t)$$

$$= \delta(t) - \frac{R}{L} e^{-\frac{R}{L}t} u(t)$$

Q5



(a) $X(s) = \int_0^1 t e^{-st} dt$

$u = t \Rightarrow du = dt$

$dv = e^{-st} dt \Rightarrow v = -\frac{e^{-st}}{s}$

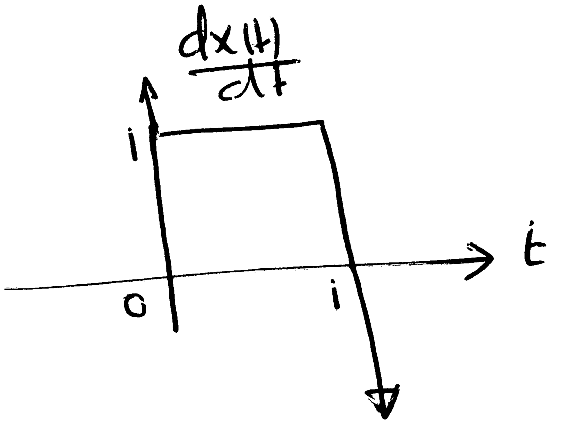
$X(s) = -\frac{t e^{-st}}{s}\Big|_0^1 + \frac{1}{s}\int_0^1 e^{-st} dt$

$= -\frac{1}{s}\left[e^{-s} - 0\right] + \frac{1}{s}\frac{e^{-st}}{(-s)}\Big|_0^1$

$= -\frac{1}{s}e^{-s} - \frac{1}{s^2}\left[e^{-s} - 1\right]$

$= \frac{1}{s^2} - \frac{1}{s}e^{-s} - \frac{1}{s^2}e^{-s}$

(b)



$$\frac{dx(t)}{dt} = u(t) - u(t-1) - \delta(t-1)$$

$$\frac{dx(t)}{dt} \leftrightarrow \frac{1}{s} - \frac{e^{-s}}{s} - e^{-s}$$

but, $$\frac{dx(t)}{dt} \leftrightarrow sX(s) - \cancel{x(0^+)}^{\,0} = sX(s)$$

$$\Rightarrow X(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}$$

(c) $$x(t) = t[u(t) - u(t-1)] \leftrightarrow -\frac{d}{ds} L.T[u(t) - u(t-1)]$$

$$L.T[u(t) - u(t-1)] = \frac{1}{s} - \frac{e^{-s}}{s} = \frac{1 - e^{-s}}{s}$$

$$-\frac{d}{ds}\left[\frac{1 - e^{-s}}{s}\right] = -\frac{e^{-s}(s) - (1 - e^{-s})}{s^2}$$

$$= \frac{1}{s^2} - \frac{e^{-s}}{s} - \frac{e^{-s}}{s^2}$$

$$\Rightarrow X(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}$$