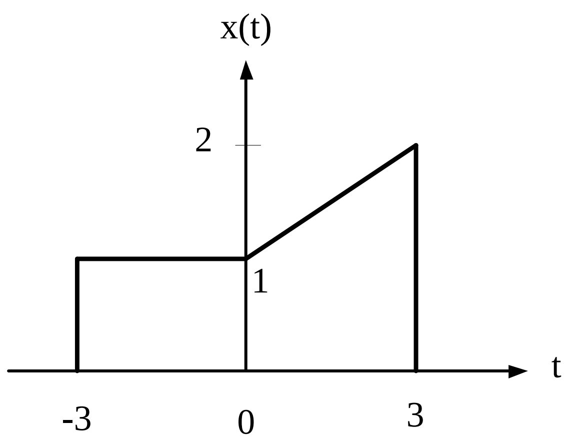
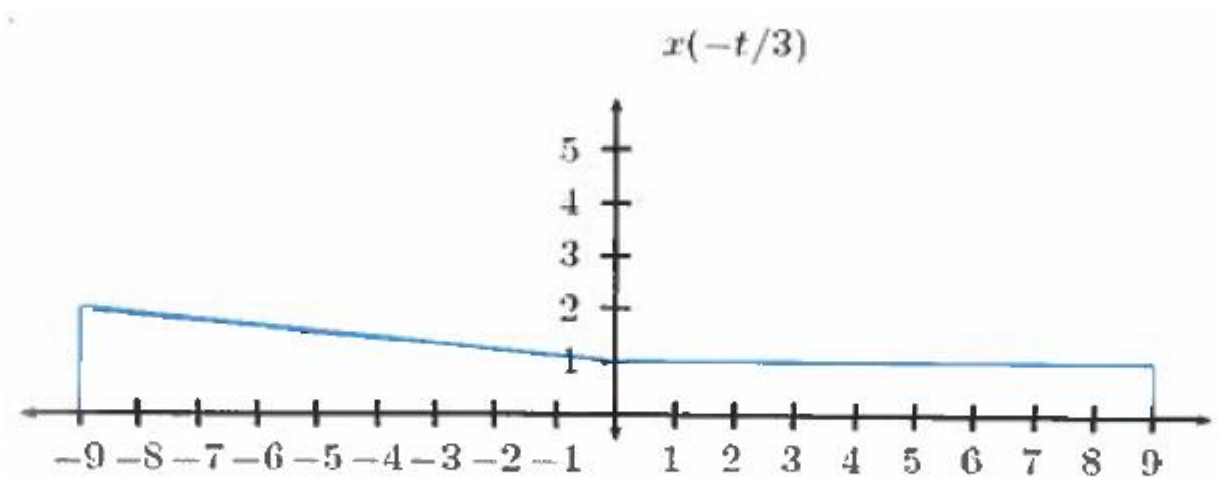
EE 207-Winter 2015(142)
Hw1
Dr. Adil Balghonaim

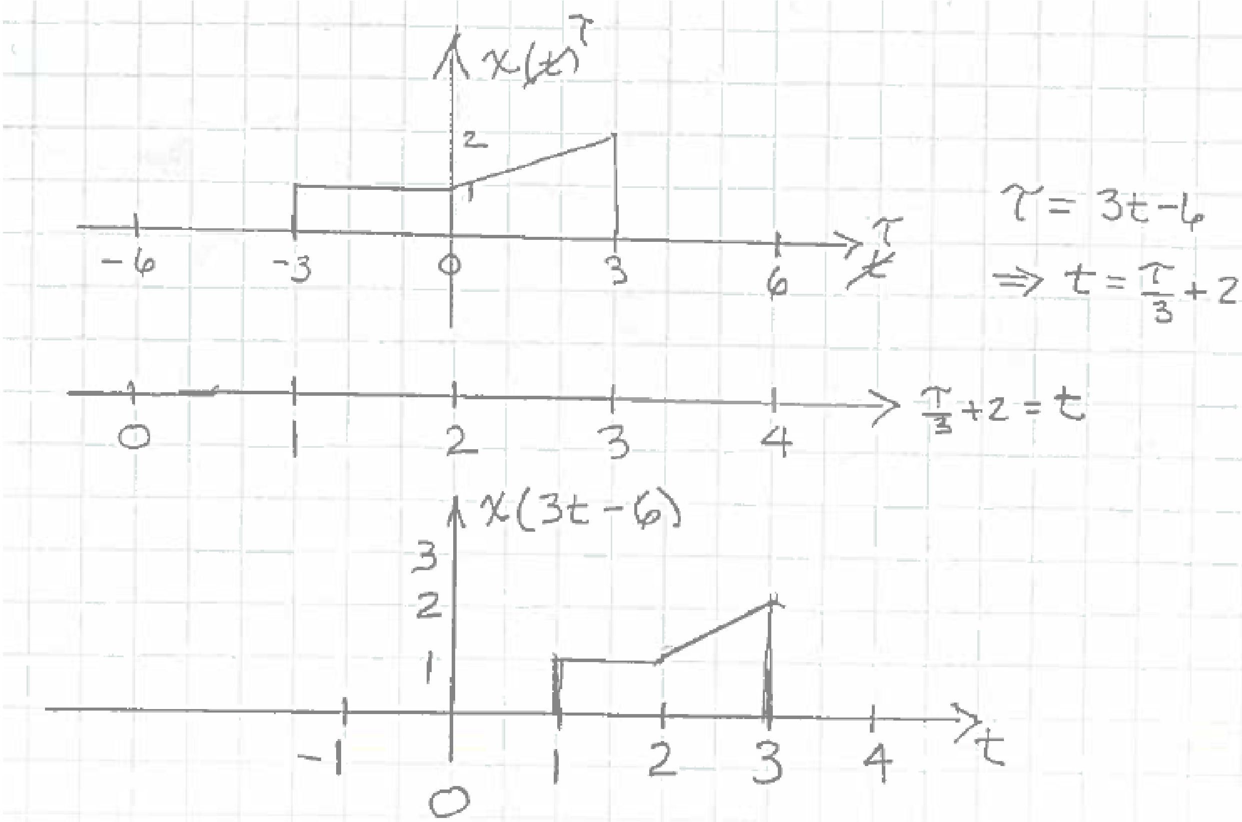
Q1 Let x(t) be the signal as shown below:



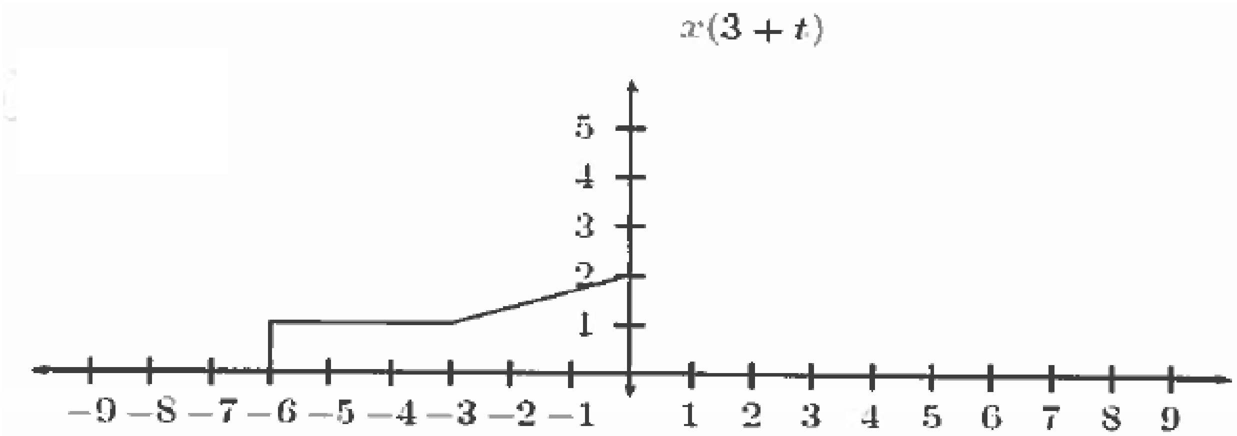
(a) Plot x(–t/3)



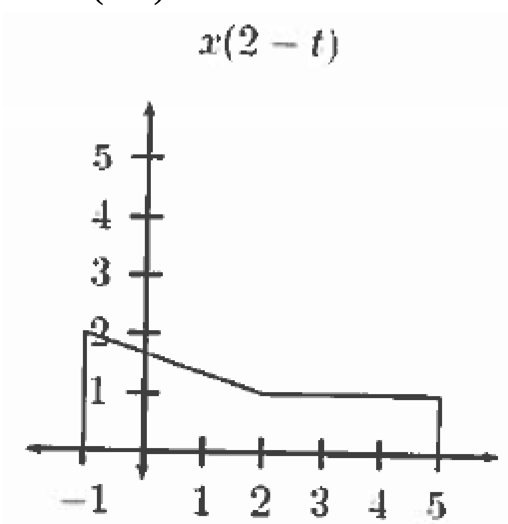
(b) Plot x(3t−6)



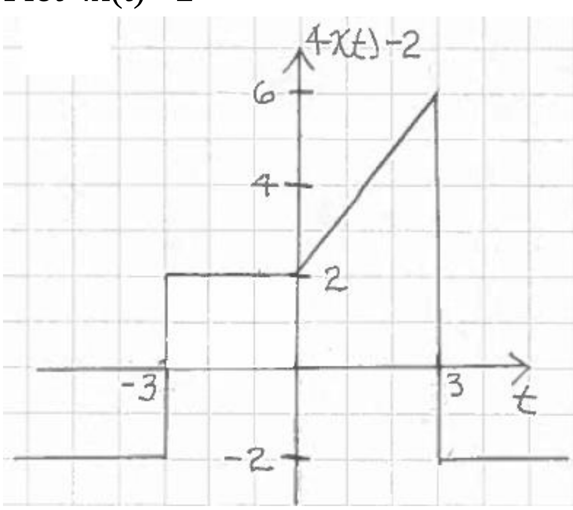
(c) Plot x(3+t)



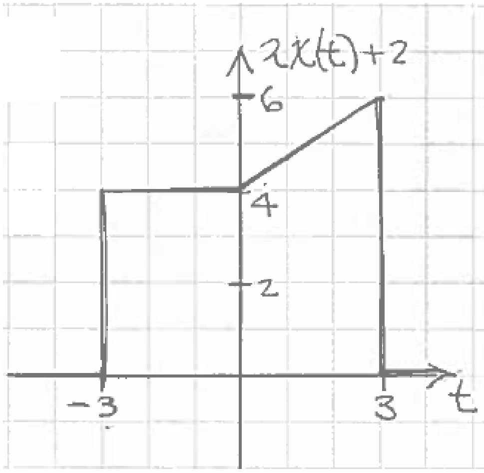
(d) Plot x(2–t)



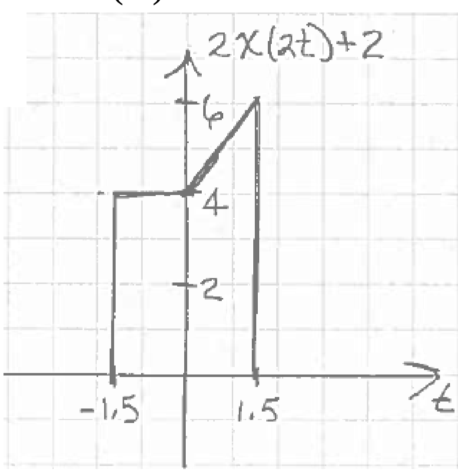
(e) Plot 4x(t) –2



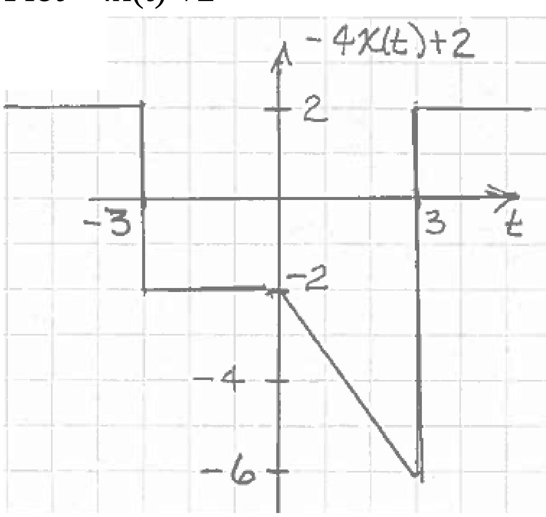
(f) Plot 2x(t) +2



(g) Plot 2x(2t) +2



(h) Plot –4x(t) +2

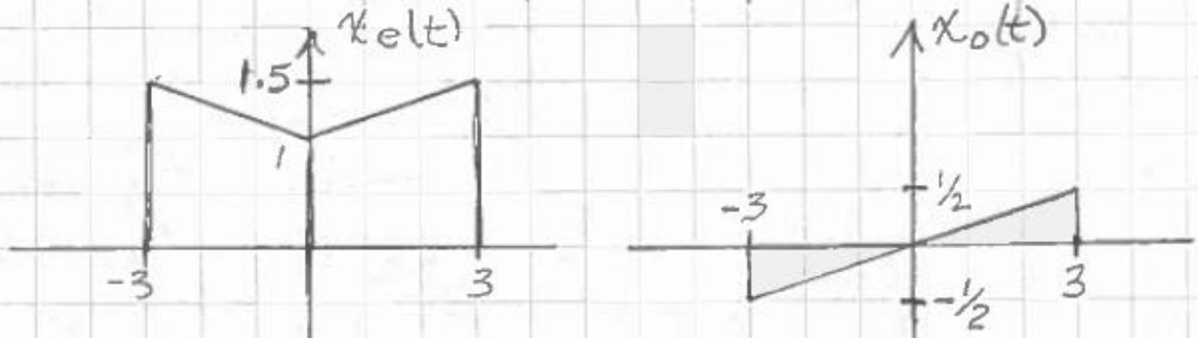


(i) Plot the even and odd part

$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$ (2.13)

$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$ (2.14)

t	$x(t)$	$x(-t)$	$x_e(t)$	$x_o(t)$
>3	0	0	0	0
3	2	1	3/2	1/2
1.5	1.5	1	1.25	0.25
0	1	1		0
−1.5		1.5	1.25	−0.25
−3		2	3/2	−1/2
<−3	0	0	0	0



(j) Express x(t) in terms of singularity functions (*impuls,step,ramp*)

$$x(t) = u(t+3) + \frac{1}{3}r(t) - \frac{1}{3}r(t-3) - 2u(t-3)$$

Q2 For each signal below , determine if the signal is periodical or not periodical . If periodical , find its fundamental period

(a) $x(t) = \cos(3t) + \sin(5t)$

$T_1 = \frac{2\pi}{3}$, $T_2 = \frac{2\pi}{5}$, $\frac{T_1}{T_2} = \frac{2\pi/3}{2\pi/5} = \frac{5}{3}$

$T_0 = 3T_1 = 2\pi$: periodic

(b) $x(t) = \cos(t) + \sin(\pi t)$

$T_1 = \frac{2\pi}{1}$, $T_2 = \frac{2\pi}{\pi} = 1$ $\frac{T_1}{T_2} = 2\pi$ not a ration of integers

∴ Not periodic

(c) $x(t) = \cos(4\pi t) + \sin(6\pi t) + e^{j5\pi t}$

$T_1 = \frac{2\pi}{4\pi} = 1/2$, $T_2 = \frac{2\pi}{6\pi} = 1/3$, $T_3 = \frac{2\pi}{5\pi} = \frac{2}{5}$

$\frac{T_1}{T_2} = \frac{1/2}{1/3} = \frac{3}{2}$, $\frac{T_1}{T_3} = \frac{1/2}{2/5} = 5/4$ both ratios of integers ∴ Sum periodic

lcm of denominators $= 4 \times 2 = 8 = k_0$

$T_0 = 8T_1 = 4$ s.

Note : Least Common Multiple (LCM)

Q3 Evaluate the following integrals:

$$(a)\ \int_{-\infty}^{\infty} \cos(2t)\delta(t)dt$$

$$(b)\ \int_{-\infty}^{\infty} \cos[(2(t-(\pi/4)]\delta(t-(\pi/4))dt$$

Recall the rules about integrating delta functions: $\delta(t)$ is nonzero only at $t = 0$, so $x(t)\delta(t) = x(0)\delta(t)$, and $\int_{-\infty}^{\infty} \delta(t)dt = 1$, so $\int_{-\infty}^{\infty} x(t)\delta(t)dt = \int_{-\infty}^{\infty} x(0)\delta(t)dt = x(0)\int_{-\infty}^{\infty} \delta(t)dt = x(0)$. We can time-shift the delta function: $\delta(t-t_0)$ is nonzero only at $t = t_0$, so $x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$ and $\int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0)$.

$$(a)\ \int_{-\infty}^{\infty} \cos(2t)\delta(t)dt$$

$$\int_{-\infty}^{\infty} \cos(2t)\delta(t)dt = \cos(2\cdot 0)\int_{-\infty}^{\infty} \delta(t)dt = 1$$

$$(b)\ \int_{-\infty}^{\infty} \cos[(2(t-(\pi/4)]\delta(t-(\pi/4))dt = \cos[(2(t-(\pi/4)]\big|_{t=\pi/4} = \cos[(2((\pi/4)-(\pi/4)] = \cos[0] = 1$$

Q4 Let the system that describe the input x(t) and output y(t) be described as

$$y(t) = \int_1^2 x(\tau - 2)d\tau$$

Determine weather the system is (explain)

(a) Memoryless

The System has memory , the output , y(t) depends on inputs x(t) over a period of time

(b) Invertible

The system is not invertible. The input at time t_0 cannot be determined from knowledge of the output at t_0 .

(c) Stable (BIBO)

The system is stable . A bounded input *x*(t) will result alwayes in a bounded output *y*(t)

(d) Time invariant

Let $y(t) = \Gamma[x(t)] = \int_1^2 x(t-2)dt \Rightarrow \Gamma[x(t-t_0)] = \int_1^2 x(t-t_0-2)dt$

Since $y(t-t_0) = \int_1^2 x(t-t_0-2)dt = \Gamma[x(t-t_0)] \Rightarrow$ **The system is Time invariant**

(e) Linear

Let $y_1(t) = \Gamma[x_1(t)] = \int_1^2 x_1(t-2)dt \qquad y_2(t) = \Gamma[x_2(t)] = \int_1^2 x_2(t-2)dt$

Then $\Gamma[\alpha_1 x_1(t) + \alpha_2 x_2(t)] = \int_1^2 [\alpha_1 x_1(t-2) + \alpha_2 x_2(t-2)]dt = \int_1^2 \alpha_1 x_1(t-2)dt + \int_1^2 \alpha_2 x_2(t-2)dt$

$$= \alpha_1 \int_1^2 x_1(t-2)dt + \alpha_2 \int_1^2 x_2(t-2)dt = \alpha_1 y_1(t) + \alpha_2 y_2(t) \Rightarrow \textbf{Linear}$$