بسم الله الرحمن الرحيم

King Fahd University of Petroleum & Minerals

Department of Electrical Engineering
EE 207 Signals and Systems
Second Semester (141)

Exam I
Tuesday, 28 Octobor 2014
7:00 pm – 8:30 pm

Name: KEY

ID: ______________________

Serial: ______________________

Problem	Score	Out of
1		30
2		10
3		10
4		20
5		10
6		20
Total		100

Problem 1: (MCQ)
Choose the correct answer of each question

(1) A System has the following input-output relationship: $y(t) = 3x^2(t) - 5x(t-2)$, where $x(t)$ is the input signal and $y(t)$ is the output signal. The unit step response of the system is given by:

A) $3u(t) - 5u(t-2)$

B) $5tu^2(t) - 3u(t-2)$

C) $-3u(t-2) + 5u(t)$

D) $3\delta(t) - 5u(t-2)$

E) $3u(t) - 5\delta(t-2)$

(2) Let $x(t) = e^{-t}u(t)$ and $y(t) = 2te^{-t}$. The convolution $x(t) * y(t)$ equals:

A) $t^2e^{-t}u(t)$

B) $2te^{-t}u(t)$

C) t^2e^{-t}

D) $2te^{-t}$

E) $(1 - e^{-t})u(t)$

(3) The unit step response of a linear time invariant system is given by $tu(t-4)$. The unit impulse response of the system is given by

A) $h(t) = u(t-4) + 4\delta(t-4)$

B) $h(t) = \delta(t-4)$

C) $h(t) = u(t-4) + \delta(t)$

D) $h(t) = u(t) + t\delta(t)$

E) $h(t) = tu(t-4) + \delta(t-4)$

(4) A linear time invariant system has the impulse response

$h(t) = (c+1)\delta(t) - (c^2-1)u(t),$ where c is some constant.

The possible value of the constant c such that the system will be memoryless is

A) $c = 1$

B) $c = -1$

C) $c = 2$

D) $c = 0$

E) The system will be dynamic regardless of the value of c.

(5) If $x(t) = 2u(t-2) - r(t-2) + 2r(t-4) + 2u(t-4)$, where $u(t)$ is the unit step function and $r(t) = tu(t)$ is the unit ramp function, the value of $x(5)$ is

A) -1

B) 1

C) 2

D) 5

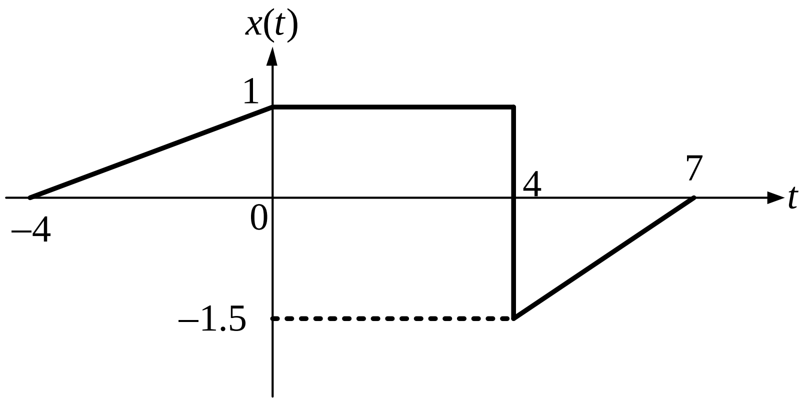
E) 0

Answer is 3 (was not included by mistake)

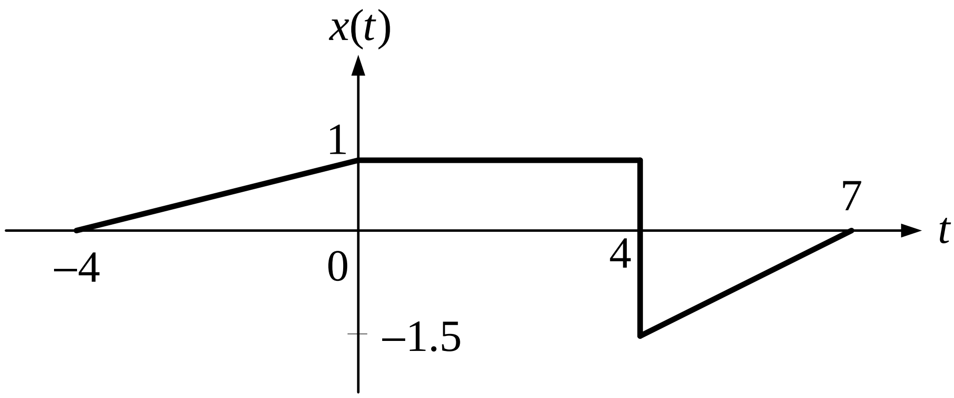
Problem 2:

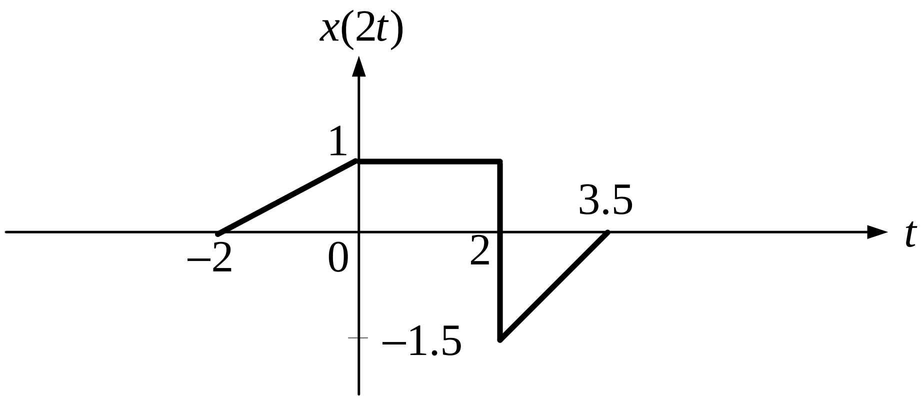
Sketch and label $y(t) = -4x(-2t+1)+2$, where x(t) is shown below.

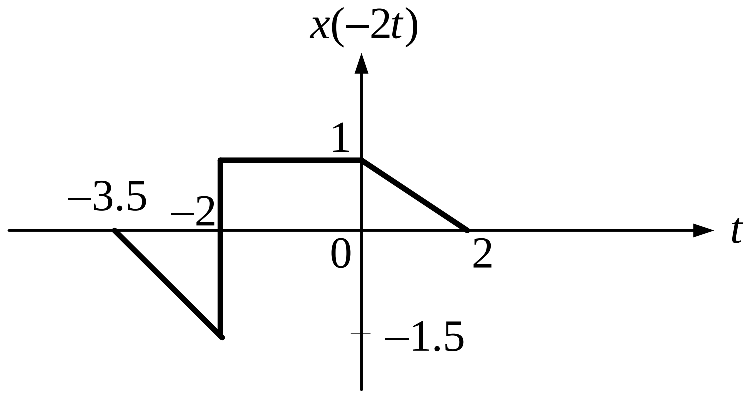
Explain all your steps.

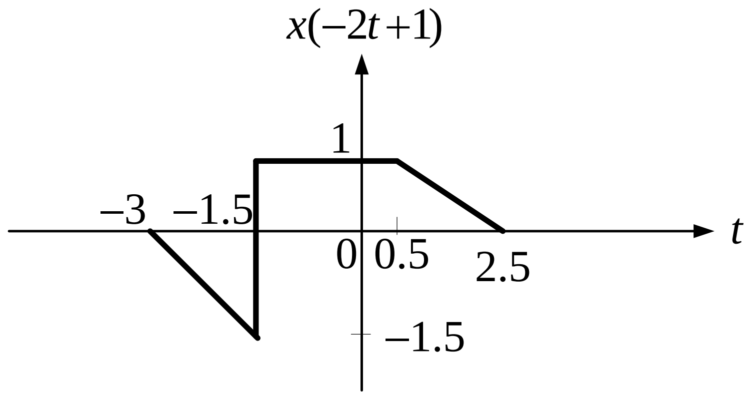


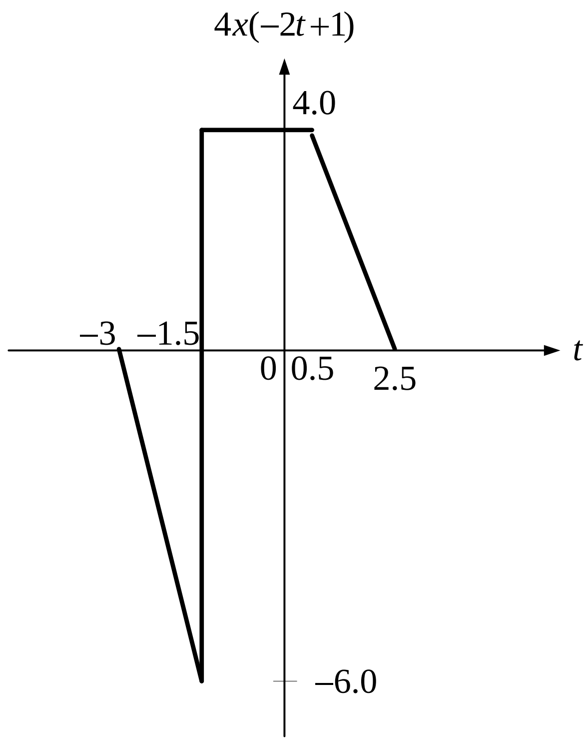
Solution

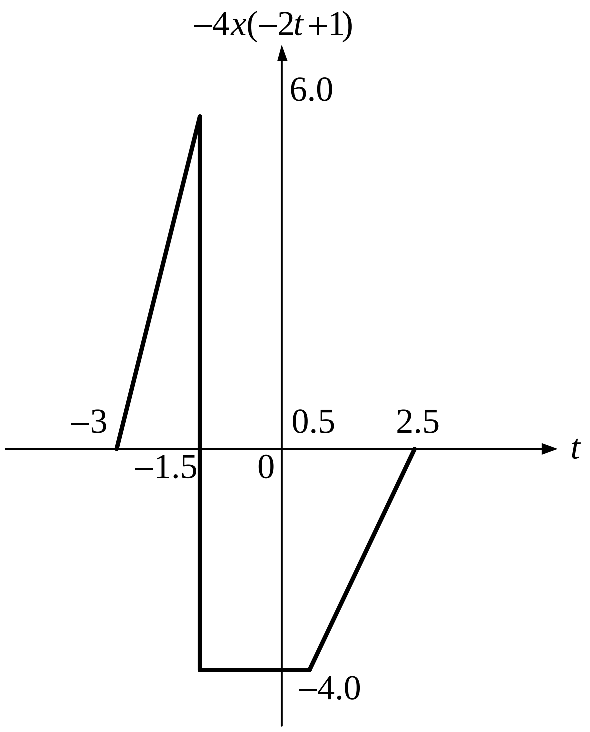


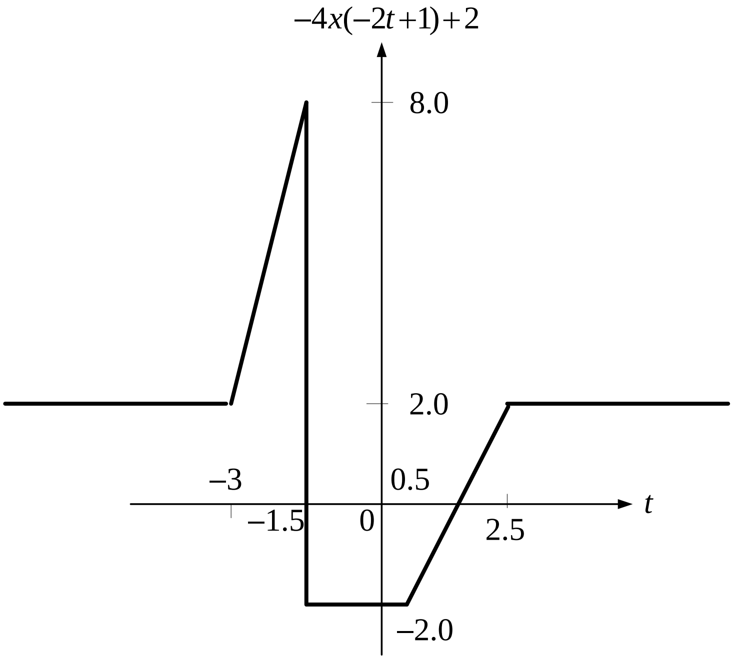



$x(-2t)$
1
-3.5
-2
0
2
t
-1.5


$x(-2t+1)$
1
-3
-1.5
0
0.5
2.5
t
-1.5


$4x(-2t+1)$
4.0
-3
-1.5
0
0.5
2.5
t
-6.0


$-4x(-2t+1)$
6.0
-3
0.5
2.5
t
-1.5
0
-4.0



−4x(−2t+1)+2
8.0
2.0
−3
0.5
−1.5
0
2.5
t
−2.0

Problem 3:

Let the input/output of a system described by $y(t) = 4\,x(t)\,u(t)$

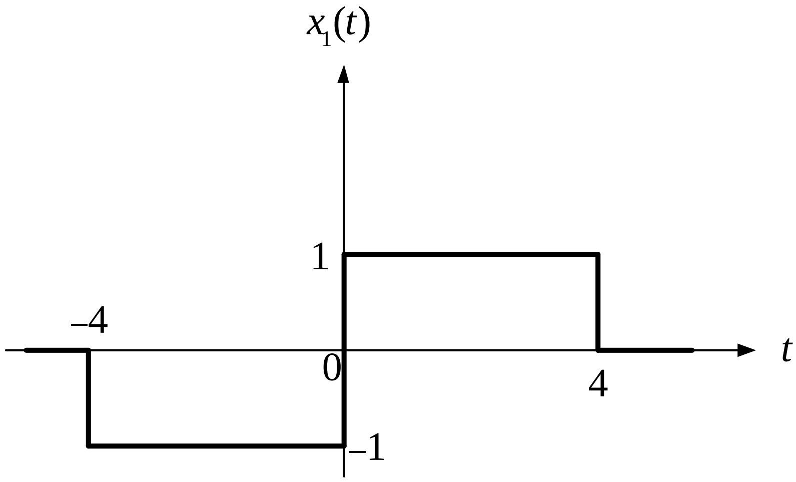
For each of the system properties listed below decide if the property is valid or not by writing **Yes** or **No** and give a brief explanation for deciding **Yes** or **No**

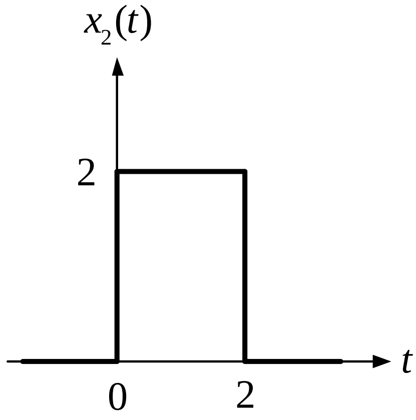
Properties	**Yes** or **No**	**Explanation**
causal	**YES**	Output do not depend on future input
BIBO Stable	**YES**	$\lvert x(t)\rvert < \infty$ and $\lvert u(t)\rvert < \infty \rightarrow \lvert y(t)\rvert < \infty$
Time-Invariant	**No**	y(t) = s[x(t)]=4x(t)u(t) → y(t-1) = 4x(t-1)u(t-1) s[x(t-1)]=4x(t-1)u(t) y(t-1) ≠ s[x(t-1)] → Time variant
Linear	**YES**	$y_1(t) = 4x_1(t)u(t) \quad y_2(t) = 4x_2(t)u(t)$ $s[\alpha_1 x_1(t) + \alpha_2 x_2(t)] = 4(\alpha_1 x_1(t) + \alpha_2 x_2(t))u(t)$ $= \alpha_1 4x_1(t)u(t) + \alpha_2 4x_2(t))u(t) = \alpha_1 y_1(t) + \alpha_2 y_2(t)$ $\Rightarrow$ Linear
Invertible	**NO**	For all t < 0 , y(t) = 0 → many x(t) (t < 0) map to y(t) = 0 → many to one mapping → Not invertible

Problem 4:

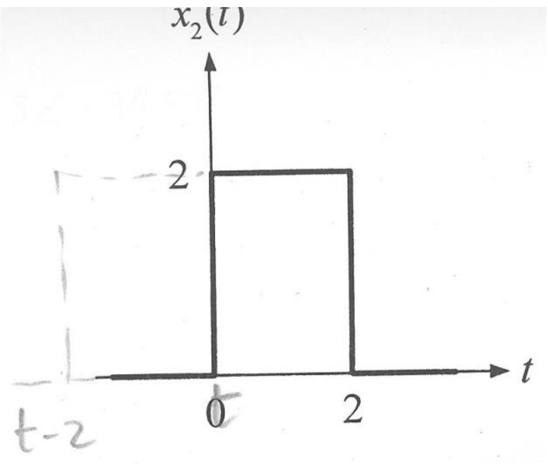
Consider the functions $x_1(t)$ and $x_2(t)$ sketched below, and let $y(t) = x_1(t) * x_2(t)$.

a) Calculate $y(0)$ before performing the convolution.
b) Perform the convolution showing all the steps and sketch $y(t)$ showing all labels and important values.





(a)



(a) $y(0) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(0-\tau)\, d\tau$

$= \int_{-2}^{0} (-1)(2)\, d\tau$

$= -2[0-(-2)] = \boxed{-4}$

(b)

(b) $t \le -4$: $y(t) = 0$

$-4 < t \le -2$: $y(t) = \int_{-4}^{t} (-1)(2)\, d\tau = -2(t+4)$

$t-2 < -4$

$-2 < t \le 0$: Full overlab with the negative box $\Rightarrow y(t) = (-1)(2)(2) = -4$

$0 < t \le 2$: Partial overlap with both boxes:

$$y(t) = \int_{t-2}^{0} (-1)(2)\, d\tau + \int_{0}^{t} (1)(2)\, d\tau$$

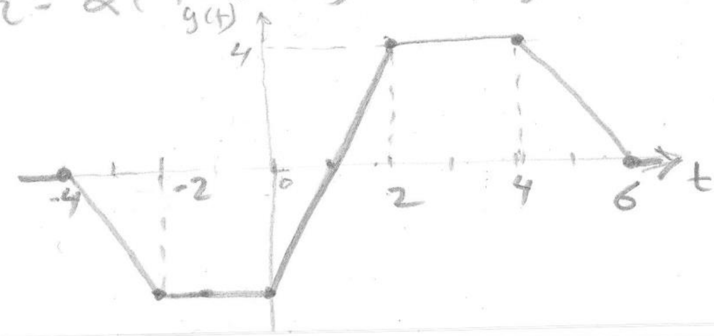
$$= +2(t-2) + 2t = 4t - 4$$

$2 < t \le 4$: Full overlab with the positive box $\Rightarrow y(t) = (1)(2)(2) = 4$

$4 < t \le 6$: $y(t) = \int_{t-2}^{4} (1)(2)\, d\tau = 2(4 - (t-2)) = 2(6-t)$

$t > 6$: $y(t) = 0$

$$\Rightarrow y(t) = \begin{cases} 0 & ; t < -4 \ \& \ t > 6 \\ -2(t+4) & ; -4 < t \le -2 \\ -4 & ; -2 < t \le 0 \\ 4t-4 & ; 0 < t < 2 \\ 4 & ; 2 < t < 4 \\ 2(6-t) & ; 4 < t \le 6 \end{cases}$$



Problem 5:

Let $x(t)$ be periodical function with period $T_o = 0.1$ s , given as

$$x(t) = 12\sin(20\pi t).\cos(40\pi t) + 8\cos(60\pi t) + 16\sin^2(20\pi t)$$

(a) Show that the complex Fourier Series coefficients C_k are

$$C_o = 8 \quad C_1 = 3\angle 90^o \quad C_2 = -4 \quad C_3 = 5\angle -36.87^o$$

Let x(t) be periodical function with period To=(1/10) s , given as

$$x(t) = 12\sin(20\pi t).\cos(40\pi t) + 8\cos(60\pi t) + 16\sin^2(20\pi t)$$

(a) Find the complex Fourier Series coefficients C_k ?

(b) Plot the magnitude and phase of C_k (i.e, line spectra)

solution using Trig identies we have

$$x(t) = 6\sin(60\pi t) - 6\sin(20\pi t) + 8\cos(60\pi t) + 8[1 - \cos(40\pi t)]$$

$$= 8 - 8\cos(40\pi t) + 8\cos(60\pi t) - 6\sin(20\pi t) + 6\sin(60\pi t)$$

$$= 8 - [4e^{(j40\pi t)} + 4e^{-(j40\pi t)}] + [4e^{(j60\pi t)} + 4e^{-(j60\pi t)}] - \left[\frac{3}{j}e^{(j20\pi t)} - \frac{3}{j}e^{-(j20\pi t)}\right]$$

$$+ \left[\frac{3}{j}e^{(j60\pi t)} - \frac{3}{j}e^{-(j60\pi t)}\right]$$

$$= \left[4 - \frac{3}{j}\right]e^{-(j60\pi t)} - 4e^{-(j40\pi t)} + \frac{3}{j}e^{-(j20\pi t)} + 8 - \frac{3}{j}e^{(j20\pi t)} - 4e^{(j40\pi t)} + \left[4 + \frac{3}{j}\right]e^{(j60\pi t)}$$

$$\Rightarrow C_o = 8 \quad C_1 = -\frac{3}{j} = 3\angle 90^o \quad C_2 = -4 \quad C_3 = \left[4 + \frac{3}{j}\right] = \left[4 - 3j\right] = 5\angle -36.87^o$$

(b) Plot the double sided ***line spectra*** (magnitude and phase) of C_k