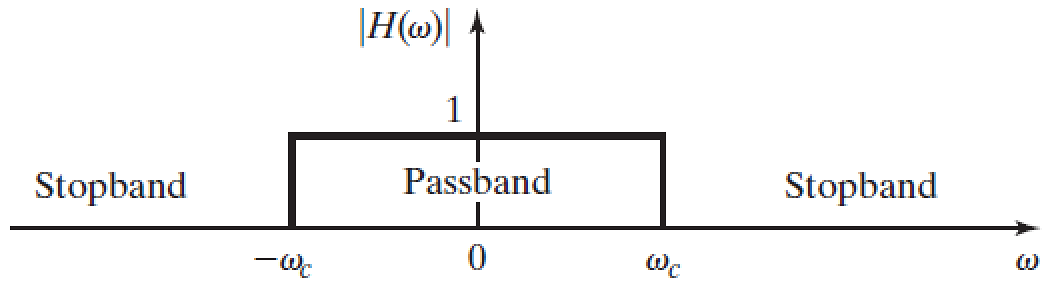
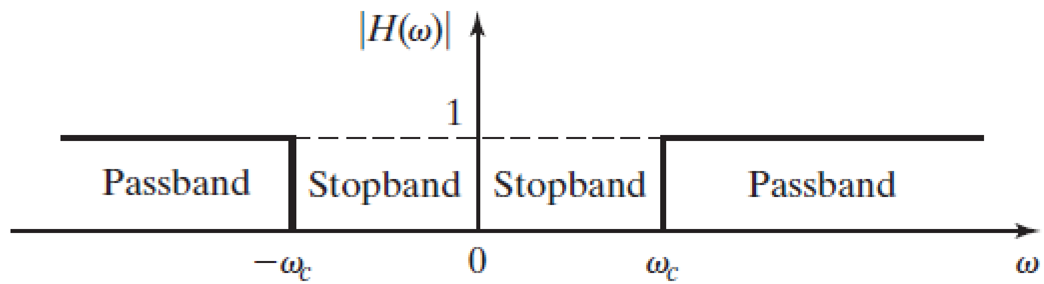
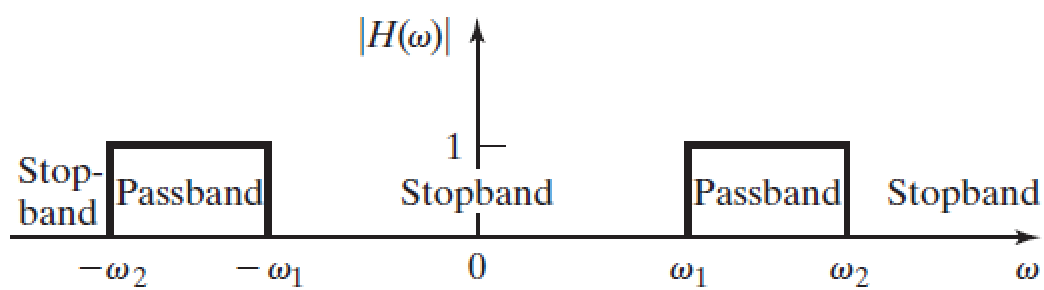
6.1 IDEAL FILTERS



ideal low-pass filter

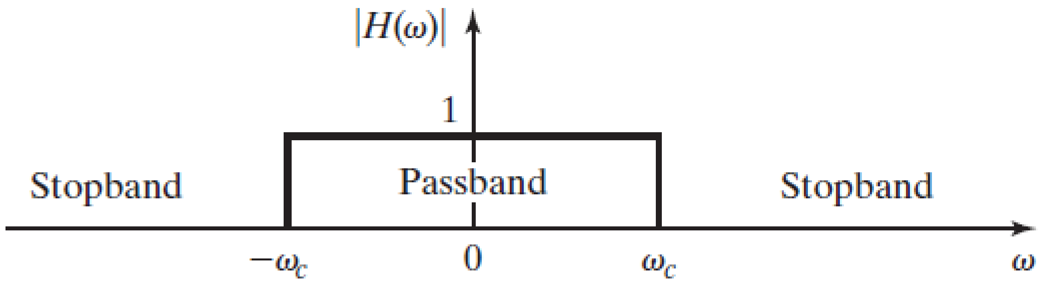


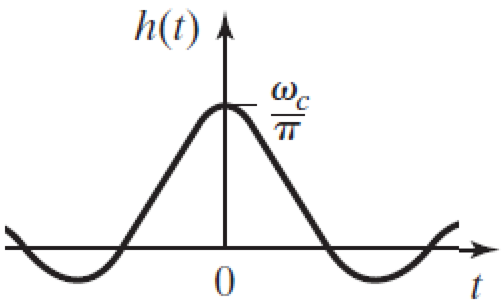
high-pass filter



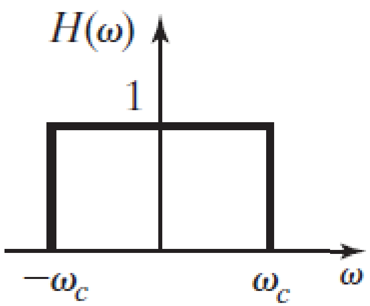
bandpass filter

Ideal **Low Pass Filter**





Sinc Function



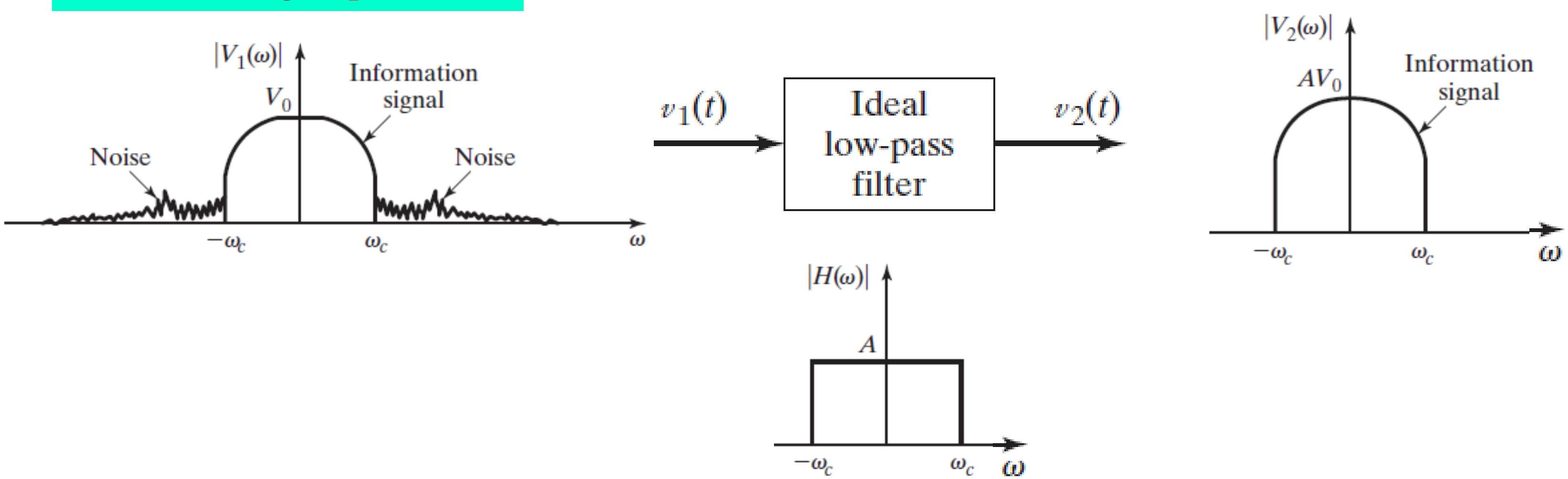
Rect Function

Example Application of **Low Pass Filter**

Suppose we have Information Signal **plus** Noise and we desire Information Signal **without (or Less)** Noise

Information Signal **plus** Noise

Information Signal **without (or Less)** Noise



EXAMPLE 6.1 **Application of an ideal high-pass filter**

Two signals $g_1(t) = 2\cos(200\pi t)$ and $g_2(t) = 5\cos(1000\pi t)$ have been multiplied together

$$g_3(t) = 5\cos(1200\pi t) + 5\cos(800\pi t)$$

$$g_4(t) = 3\cos(1200\pi t)$$

$$5\pi[\delta(\omega - 800\pi) + \delta(\omega + 800\pi)]$$

$$5\pi[\delta(\omega - 1200\pi) + \delta(\omega + 1200\pi)]$$

$$G_4(\omega) = 3\pi[\delta(\omega - 1200\pi) + \delta(\omega + 1200\pi)]$$

$$G_3(\omega) = 5\pi[\delta(\omega - 800\pi) + \delta(\omega + 800\pi)] + 5\pi[\delta(\omega - 1200\pi) + \delta(\omega + 1200\pi)]$$

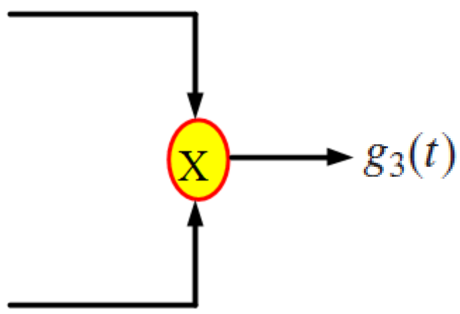
$$G_4(\omega) = G_3(\omega)H_1(\omega)$$

EXAMPLE 6.1 **Application of an ideal high-pass filter**

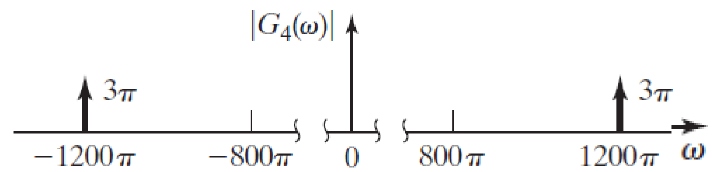
Two signals $g_1(t)$ and $g_2(t)$ have been multiplied together

$$g_1(t) = 2\cos(200\pi t)$$

$$g_2(t) = 5\cos(1000\pi t)$$



We seek a signal $g_4(t) = 3\cos(1200\pi t)$



Using Trigonometric identities

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}\cos(\alpha - \beta) + \frac{1}{2}\cos(\alpha + \beta)$$

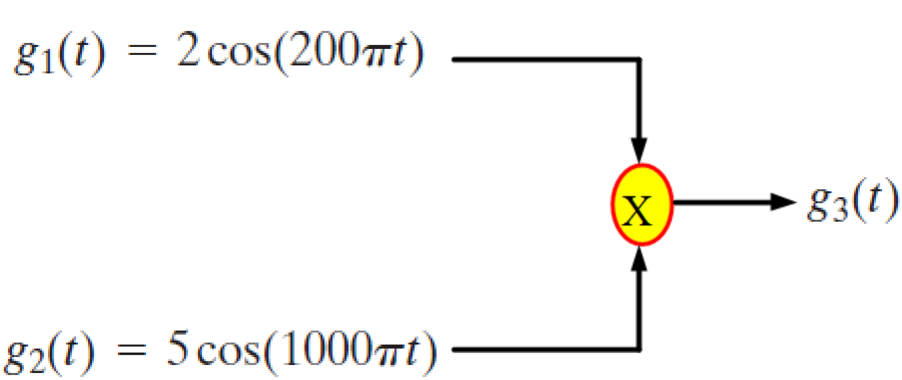
$$g_3(t) = 5\cos(1200\pi t) + 5\cos(800\pi t)$$

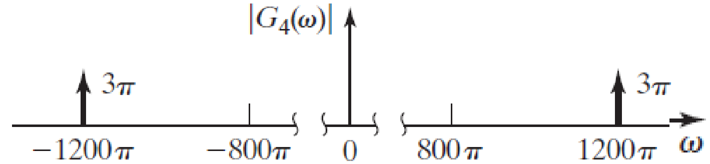
$$5\pi[\delta(\omega - 1200\pi) + \delta(\omega + 1200\pi)]$$

$$5\pi[\delta(\omega - 800\pi) + \delta(\omega + 800\pi)]$$

$$G_3(\omega) = 5\pi[\delta(\omega - 800\pi) + \delta(\omega + 800\pi)] + 5\pi[\delta(\omega - 1200\pi) + \delta(\omega + 1200\pi)]$$

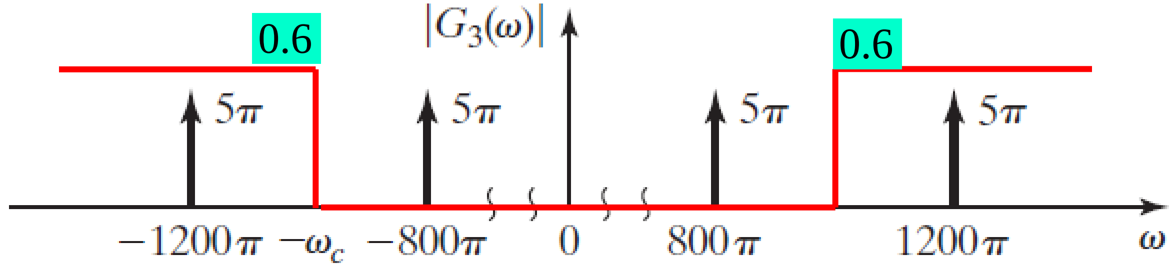
We seek a signal $g_4(t) = 3\cos(1200\pi t)$

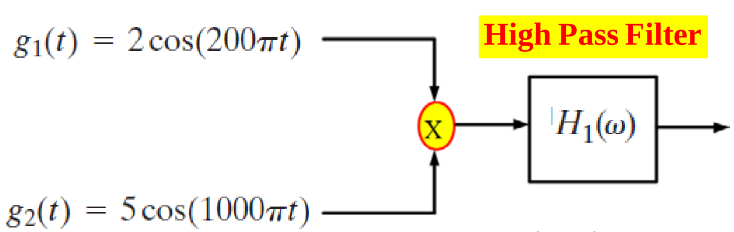


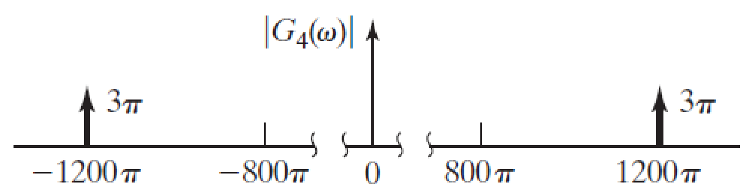


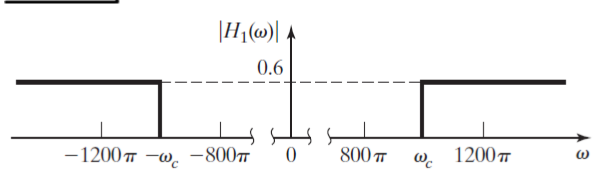
$$G_3(\omega) = 5\pi[\delta(\omega - 800\pi) + \delta(\omega + 800\pi)] + 5\pi[\delta(\omega - 1200\pi) + \delta(\omega + 1200\pi)]$$

We apply **High Pass Filter**

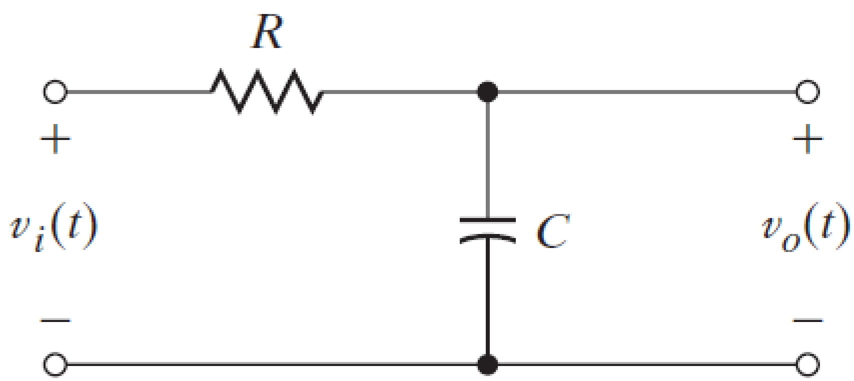








RC Low-Pass Filter



$$v_o(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$$

$$v_i(t) = Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$$

$$V_i(\omega) = RI(\omega) + \frac{1}{j\omega C}I(\omega)$$

$$V_o(\omega) = \frac{1}{j\omega C}I(\omega)$$

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} \qquad = \frac{1}{1 + j\omega RC}$$

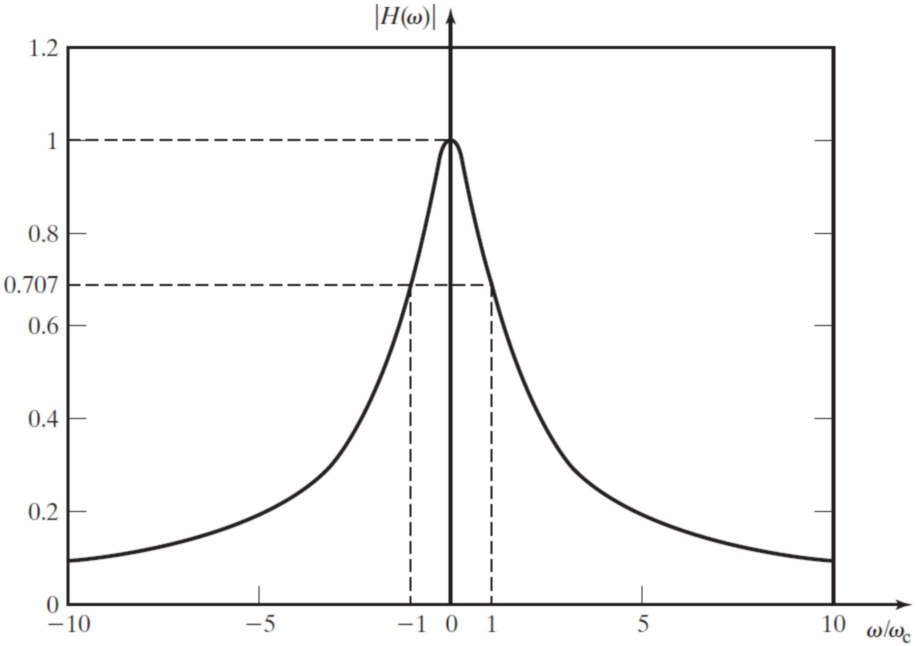
$$\omega_c = \frac{1}{RC} \qquad H(\omega) = \frac{1}{1 + j\omega/\omega_c} \qquad = |H(\omega)|e^{j\Phi(\omega)}$$

The magnitude and phase frequency spectra of the filter are described by the equations

$$H(\omega) = \frac{1}{1 + j\omega/\omega_c}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_c)^2}}$$

$$\Phi(\omega) = -\arctan(\omega/\omega_c)$$



At the frequency $\omega = \omega_c \Rightarrow \frac{\omega}{\omega_c} = 1$

$$|H(\omega_c)| = \frac{|V_o(\omega_c)|}{|V_i(\omega_c)|} = \frac{1}{\sqrt{2}}$$

The ratio of the normalized power of the **input** and **output** signals is given by

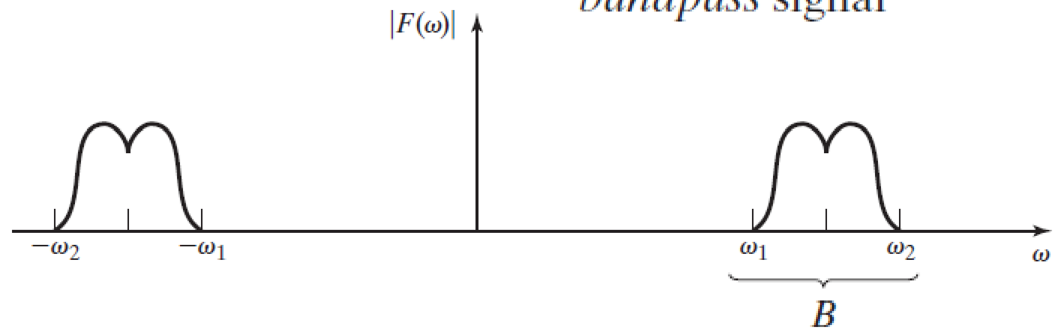
$$|H(\omega_c)|^2 = \frac{|V_o(\omega_c)|^2}{|V_i(\omega_c)|^2} = \frac{1}{2}$$

This type of filter is often called the ***half-power frequency***

6.3 BANDWIDTH RELATIONSHIPS

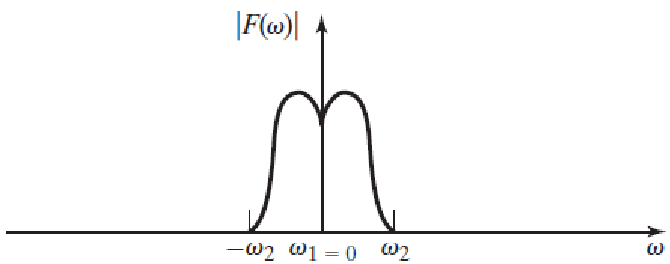
bandpass signal

$f(t)$ ⟷



Absolute bandwidth is $B = \omega_2 - \omega_1$

baseband signal



$\omega_1 = 0$ → $B = \omega_2$

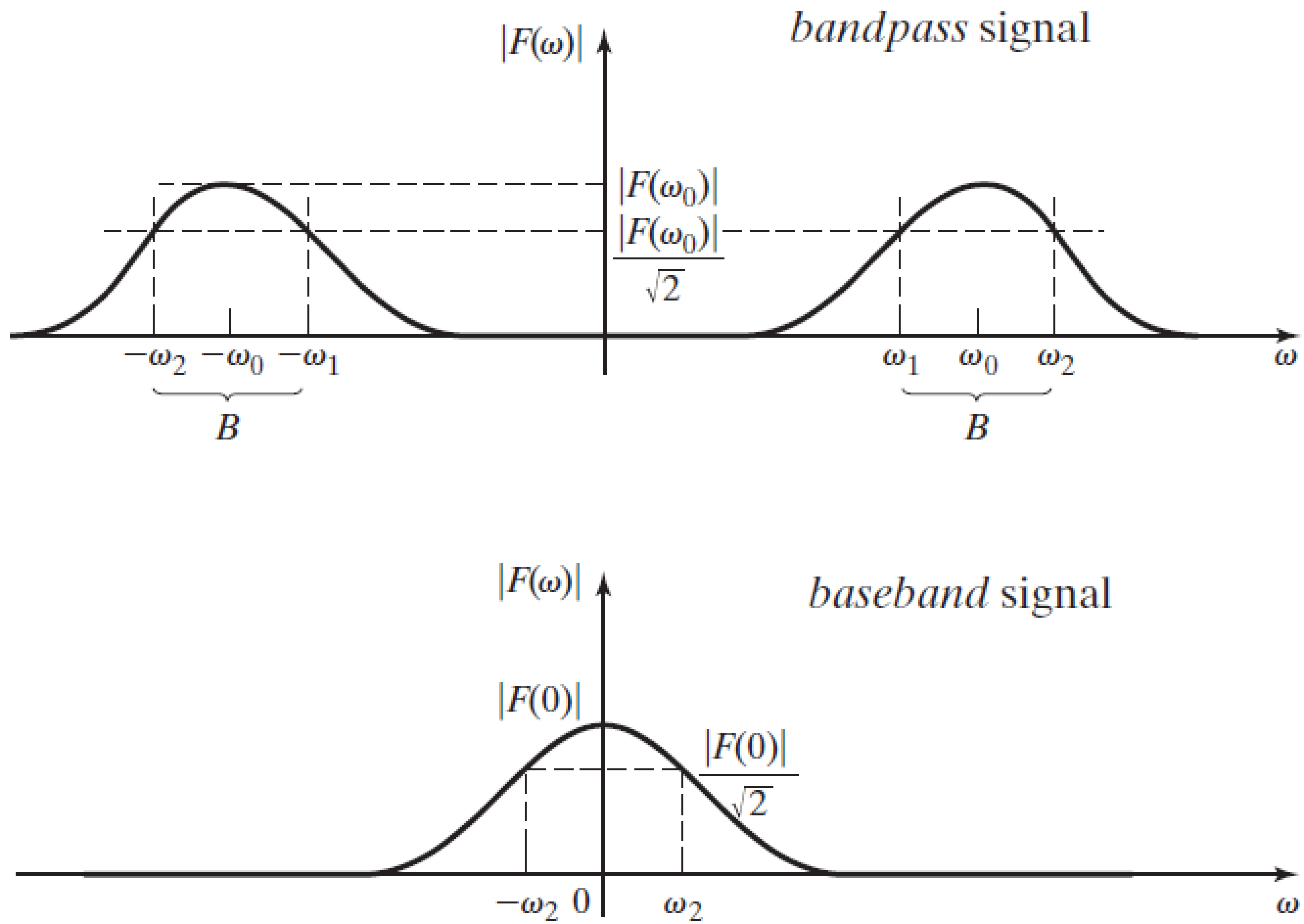
Therefore, for both bandpass and baseband signals, the bandwidth is defined by the range of positive frequencies for which the frequency spectrum of the signal is nonzero.

Three-dB bandwidth, or *half-power bandwidth*,

It is defined as the range of frequencies for which the magnitude of the frequency spectrum is no less than $1/\sqrt{2}$ times the maximum value within the range

$$20 \log_{10}\left(\frac{1}{\sqrt{2}}\right) = -3 \text{ dB}$$

The term 3-dB bandwidth comes from the relationship where dB is the abbreviation for decibel



|F(ω)|
bandpass signal
|F(ω0)|
|F(ω0)|/√2
−ω2 −ω0 −ω1
ω1 ω0 ω2
ω
B
B
|F(ω)|
baseband signal
|F(0)|
|F(0)|/√2
−ω2 0 ω2
ω