EE 315-Winter 2014(132)
QZ6

Sec	Ser	ID	Name **KEY**

Let random variables X and Y have the joint density function

$$f_{XY}(x,y)=\begin{cases}3x\sqrt{y} & 0<x<1 \quad 0<y<1\\ 0 & otherwise\end{cases}$$

(a) Find the correlation $E[XY]$?

(b) Find the covariance $\text{Cov}(X,Y)=E[XY]-E[X]E[Y]$?

Solution

(a) $$E[XY]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy f_{XY}(x,y)dxdy=\int_0^1\int_0^1 xy\left(3x\sqrt{y}\right)dxdy$$

$$=3\left(\int_0^1 x^2dx\right)\left(\int_0^1 y\sqrt{y}dx\right)=\left[3\left(\frac{x^3}{3}\right)\Bigg|_0^1\right]\left[\left(\frac{2}{5}y^{5/2}\right)\Bigg|_0^1\right]=\frac{2}{5}$$

(b) To find $E[X]$ and $E[Y]$ we need the marginals $f_X(x)$ and $f_Y(y)$

$$f_X(x)=\int_{-\infty}^{\infty} f_{XY}(x,y)dy=\int_0^1 3x\sqrt{y}dy=2x \quad 0<x<1$$

$$\Rightarrow \quad E[X]=\int_{-\infty}^{\infty} xf_X(x)dx=\int_0^1 x(2x)dx=\frac{2}{3}$$

Similarly

$$f_Y(x)=\int_{-\infty}^{\infty} f_{XY}(x,y)dx=\int_0^1 3x\sqrt{y}dx=\frac{3}{2}\sqrt{y} \quad 0<y<1$$

$$\Rightarrow \quad E[Y]=\int_{-\infty}^{\infty} yf_y(y)dy=\int_0^1 y\left(\frac{3}{2}\sqrt{y}\right)dy=\frac{3}{5}$$

$$\Rightarrow \text{Cov}(X,Y)=E[XY]-E[X]E[Y]$$

$$=\frac{2}{5}-\left(\frac{2}{3}\right)\left(\frac{3}{5}\right)=0$$ (**uncorrelated**)