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Let $X(t)$ be a random process given as

$$X(t) = e^{-\alpha t}$$

where α is a random variable distributed uniformly between 0 and 1

(a) sketch one sample of $X(t)$?



(b) Find the mean of $X(t)$?

(c) Find the autocorrelation $R_{XX}(t, t+\tau)$?

(d) is $X(t)$ WSS, explain ?

$$(b) E[X(t)] = \int_{\alpha=0}^{\alpha=1} (e^{-\alpha t}) \underbrace{(1)}_{f_{\alpha}(\alpha)} d\alpha = \left. \frac{e^{-\alpha t}}{-\alpha} \right|_{\alpha=0}^{\alpha=1} = \frac{1 - e^{-t}}{t}$$

$$(c) R_{XX}(t, t+\tau) = E[X(t)X(t+\tau)] = \int_{\alpha=0}^{\alpha=1} (e^{-\alpha t}) (e^{-\alpha(t+\tau)}) (1) d\alpha$$

$$= \int_{\alpha=0}^{\alpha=1} e^{-2\alpha t - \alpha\tau} d\alpha = \int_{\alpha=0}^{\alpha=1} e^{-\alpha(2t+\tau)} d\alpha$$

$$= \left. \frac{e^{-\alpha(2t+\tau)}}{-(2t+\tau)} \right|_{\alpha=0}^{\alpha=1} = \frac{1 - e^{-(2t+\tau)}}{(2t+\tau)}$$

(d) $X(t)$ is not WSS because $X(t) \neq \text{Constant}$
 $R_{XX}(t, t+\tau)$ is function of time.