KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

ELECTRICAL ENGINEERING DEPARTMENT

Probabilistic Methods in Electrical Engineering
EE 315

FIRST MAJOR

DATE: March 9, 2014

TIME: 6:00-7:30 pm

Name: ____________**KEY**_______________

ID :   ___________________________

Section # : ________________________

QUESTION	MARK
1	**/30**
2	**/35**
3	**/25**
4	**/10**
TOTAL	**/100**

Problem 1: [**30 points**]
Three Boxes B_1, B_2,B_3 contain the following colored balls

	Red	Green	Blue
B_1	6	2	1
B_2	4	3	2
B_3	2	4	6

The random experiment consists of selecting one box at random and then selecting one ball from that box. If the boxes have equal probabilities of selections, find the following probabilities:

(a) P(Selecting Red Ball | Box B_1 was selected)?
(b) P(Selecting Red Ball or Green Ball | Box B_2 was selected)?
(c) P(Selecting Blue Ball | Box B_2 selected or Box B_1 selected)?
(d) P(Not selecting Green Ball | Box B_3 was selected)?
(e) P(Selecting Blue Ball)?
(f) P (Box B_3 was selected | Selecting Blue Ball)?

Solution

(a) $P(R/B_1) = \frac{6}{9} = \boxed{\frac{2}{3}}$

(b) $P(\{R \text{ or } G\}|B_2) = P(\{R \cup G\}|B_2)$

$$= \frac{P(\{R \cup G\} \cap B_2)}{P(B_2)}$$

$$= \frac{P\{(R \cap B_2) \cup (G \cap B_2)\}}{P(B_2)}$$

$$= \frac{P(R \cap B_2) + P(G \cap B_2)}{P(B_2)}$$

$$= P(R|B_2) + P(G|B_2)$$

$$= \frac{4}{9} + \frac{3}{9} = \boxed{\frac{7}{9}}$$

OR $P(\{R \cup G\}/B_2) = P(\bar{B}|B_2)$

$$= 1 - P(B/B_2)$$

$$= 1 - \frac{2}{9}$$

$$= \boxed{\frac{7}{9}}$$

(c) $P(B|\{B_1 \text{ or } B_2\}) = \frac{P(B \cap \{B_1 \cup B_2\})}{P(\{B_1 \cup B_2\})}$

$$= \frac{P(\{B \cap B_1\} \cup \{B \cap B_2\})}{P(B_1 \cup B_2)}$$

$$= \frac{P(B \cap B_1) + P(B \cap B_2)}{P(B_1 \cup B_2)}$$

$$= \frac{P(B|B_2)P(B_2) + P(B|B_1)P(B_1)}{P(B_1) + P(B_2)}$$

$$= \frac{(\frac{2}{9})(\frac{1}{3}) + (\frac{1}{9})(\frac{1}{3})}{(\frac{1}{3}) + (\frac{1}{3})} = \boxed{\frac{1}{6}}$$

(d) $P(\bar{G} \mid B_3) = 1 - P(G \mid B_3)$

$= 1 - \frac{4}{12} = 1 - \frac{1}{3}$

$= \boxed{\frac{2}{3}}$

or $P(\bar{G} \mid B_3) = P(R \mid B_3) + P(B \mid B_3) = \frac{2}{12} + \frac{6}{12} = \frac{4}{6} = \boxed{\frac{2}{3}}$

(e) $P(B) = P(B \cap B_1) + P(B \cap B_2) + P(B \cap B_3)$

$= P(B \mid B_1)P(B_1) + P(B \mid B_2)P(B_2)$

$+ P(B \mid B_3)P(B_3)$

$= \left(\frac{1}{9}\right)\left(\frac{1}{3}\right) + \left(\frac{2}{9}\right)\left(\frac{1}{3}\right) + \left(\frac{6}{12}\right)\left(\frac{1}{3}\right)$

$= \boxed{\frac{5}{18}}$

(f) $P(B_3 \mid B) = \dfrac{P(B_3 \cap B)}{P(B)} = \dfrac{P(B \mid B_3)P(B_3)}{P(B)}$

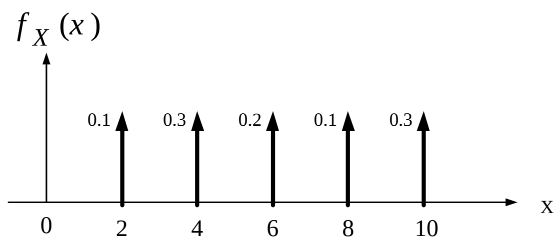
$= \dfrac{(6/12)(1/3)}{(5/18)} = \boxed{\frac{3}{5}}$

Problem 2: [**35 points**]
Let X be a discrete random variable with sample space $S_X = \{2,4,6,8,10\}$ and mass probability $P(X=x_i)$ shown below:

P(X=2) =0.1 P(X=4) =0.3 P(X=6) =0.2 P(X=8) =0.1 P(X=10) =0.3

(a) Find and plot the denity function $f_X(x)$?
(b) Find $F_X(4) - F_X(2)$? (where $F_X(x)$ is the cumulative distribution function)
(c) Find and plot the conditional density $f_X(x \mid X < 7)$?
(d) Find $E[X \mid X < 7]$?

Solution



$$(a)\ f_X(x) = \sum_{\forall x_i \in S_X = \{2,4,6,8,10\}} P(X = x_i)\delta(x - x_i)$$
$$= (0.1)\delta(x-2) + (0.3)\delta(x-4) + (0.2)\delta(x-6) + (0.1)\delta(x-8) + (0.3)\delta(x-10)$$

(b) $F_X(4) - F_X(2) = P(2 < X \le 4) = P(X=4) = 0.3$

(c) $$f_X(x \mid X<7) = \sum_{\forall x_i \in S_X = \{2,4,6,8,10\}} P(X = x_i \mid X<7)\delta(x - x_i)$$

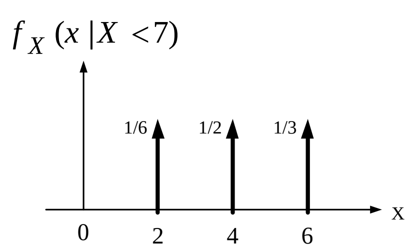
$$P(X=2 \mid X<7) = \frac{P(\{2\} \mid \overbrace{\{2,4,6\}}^{X<7})}{P(\{2,4,6\})} = \frac{P(2)}{P(2)+P(4)+P(6)} = \frac{0.1}{0.1+0.3+0.2} = \frac{0.1}{0.6} = \frac{1}{6}$$

$$P(X=4 \mid X<7) = \frac{P(\{4\} \mid \{2,4,6\})}{P(\{2,4,6\})} = \frac{P(4)}{P(2)+P(4)+P(6)} = \frac{0.3}{0.6} = \frac{1}{2}$$

$$P(X=6 \mid X<7) = \frac{P(\{6\} \mid \{2,4,6\})}{P(\{2,4,6\})} = \frac{P(6)}{P(2)+P(4)+P(6)} = \frac{0.2}{0.6} = \frac{1}{3}$$

$$P(X=8 \mid X<7) = \frac{P(\{8\} \mid \{2,4,6\})}{P(\{2,4,6\})} = \frac{P(\phi)}{P(2)+P(4)+P(6)} = \frac{0}{0.6} = 0$$

$$P(X=10 \mid X<7) = \frac{P(\{10\} \mid \{2,4,6\})}{P(\{2,4,6\})} = \frac{P(\phi)}{P(2)+P(4)+P(6)} = \frac{0}{0.6} = 0$$



Therefor

$$f_X(x \mid X<7) = \frac{1}{6}\delta(x-2) + \frac{1}{2}\delta(x-4) + \frac{1}{3}\delta(x-6)$$

(d) $$E[X \mid X < 7] = \sum_{\forall x_i \in S_X = \{2,4,6,8,10\}} x_i P(X = x_i \mid X<7)$$
$$= (2)\left(\frac{1}{6}\right) + (4)\left(\frac{1}{2}\right) + (6)\left(\frac{1}{3}\right) = \frac{13}{3} = 4.33$$

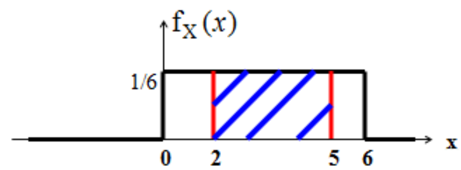
Problem 3: [**25 points**]

A random variable X has a **uniform density function** $f_X(x)$ which is distributed between **0** and **6** . Define the following events

A = { **2**< X <**5** } B= {**0**< X <**3** }

(a) Find P[A]?
(b) Find P[A U B]?
(c) Are events A and B independent? Explain ?
(d) Find $F_X(3|A) - F_x(0|A)$? (where $F_X(x)$ is the cumulative distribution function)

Solution



(a) $P[A] = \int_2^5 f_X(x)dx = \int_2^5 (\frac{1}{6})dx = (5-2)(\frac{1}{6}) = \frac{1}{2}$

(b) $A \cup B = \{2<X<5\} \cup \{0<X<3\} = \{0<X<5\}$

$\Rightarrow P[A \cup B] = P[\{0<X<5\}] = \int_0^5 f_X(x)dx = \int_0^5 (\frac{1}{6})dx = (5-0)(\frac{1}{6}) = \frac{5}{6}$

OR

$P[A \cup B] = P[A] + P[B] - P[A \cap B]$

$P[B] = P[\{0<X<3\}] = \int_0^3 f_X(x)dx = \int_0^3 (\frac{1}{6})dx = (3-0)(\frac{1}{6}) = \frac{1}{2}$

$P[A \cap B] = P[\{2<X<3\}] = \int_2^3 f_X(x)dx = \int_2^3 (\frac{1}{6})dx = (3-2)(\frac{1}{6}) = \frac{1}{6}$

$\Rightarrow P[A \cup B] = \frac{1}{2} + \frac{1}{2} - \frac{1}{6} = \frac{5}{6} = 0.8333$

(c) $P[A \cap B] \overbrace{=}^{?} P[A]P[B]$

$P[A \cap B] = \frac{1}{6}$ and $P[A]P[B] = (\frac{1}{2})(\frac{1}{2}) = \frac{1}{4}$

$\Rightarrow P[A \cap B] \neq P[A]P[B]$

$\Rightarrow$ A and B are not independent events

(d) $F_X(3|A) - F_X(0|A) = P[\{0<X<3\}|A] = P[B|A] = \frac{P[AIB]}{P[A]}$

$$= \frac{(1/6)}{(1/2)} = \frac{1}{3}$$

OR

$$F_X(3|A) = P[\{X \le 3\}|A] = \frac{P[\{X \le 3\} \cap A]}{P[A]} = \frac{P[2<X<3]}{P[A]} = \frac{(1/6)(3-2)}{(1/6)(5-2)} = \frac{1}{3}$$

$$F_X(0|A) = P[\{X \le 0\}|A] = \frac{P[\overbrace{\{X \le 0\} \cap A}^{\phi \text{ the empty set}}]}{P[A]} = \frac{0}{(1/2)} = 0$$

$$\Rightarrow F_X(3|A) - F_X(0|A) = \frac{1}{3} - 0 = \frac{1}{3}$$

<u>Problem 4:</u> [**10 points**]

Find P[2< Y<5], if the random variable Y has a Gaussian probability density function given by

$$f_Y(y) = \frac{1}{\sqrt{18\pi}} e^{-(y-2)^2/18}$$

Solution

$$P[2<X<5] = F_X(5) - F_X(2) \quad \text{were} \quad F_X(x) = \underbrace{\int_{-\infty}^{x} \frac{1}{\sqrt{18\pi}} e^{-(x-2)^2/18} dx}_{\text{No Closed Form Solution}}$$

Since the Gaussian Density is given as

$$\frac{1}{\sqrt{2\pi\sigma_X^2}} e^{-(x-a_x)^2/2\sigma_x^2} \Rightarrow a_x = 2 \quad \text{and} \quad \sigma_X = 3$$

$$\Rightarrow P[2<X<5] = F(\frac{5-2}{3}) - F(\frac{2-2}{3}) \quad \text{were} \quad F(x) = \underbrace{\int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx}_{\text{Normalized and Tabulated}}$$

$$= F(1) - F(0) = 0.8413 - 0.5000 = 0.3413$$