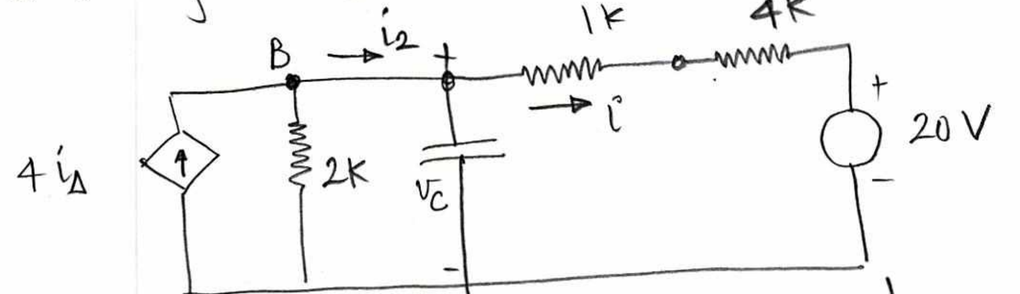
King Fahd University of Petroleum and Minerals

Electrical Engineering Department

Solution of Homework 6
EE 202 - Electric Circuits - Semester 132

Solution Problem 1.

Using source transformation, the circuit is equivalent to the following circuit:



Using (KVL) to the left loop, we have

$$-v_C + 5\times 10^{3}\, i + 20 = 0 \qquad \boxed{i = -\frac{1}{250} + \frac{1}{5000} v_C} \;(1)$$

At node A (KCL) $\boxed{i_2 = C\frac{dv_C}{dt} + i}$ (2)

KCL at node B gives $\boxed{4 i_A = \frac{v_C}{2\times 10^{3}} + i_2}$ (3)

KCL at node C gives $\boxed{i_A = i + 5\times 10^{-3}}$ (4)

From Eqs. (1), (2), (3) and (4), we write the differential equation:

$$\boxed{1.6\times 10^{-6}\frac{dv_c}{dt} - \frac{1}{10^4} v_c - \frac{1}{125} = 0}$$ 5

$$\boxed{1/\tau = -62.5}$$

Then,

$$v_c(t) = v_c(0)\,e^{-\frac{t}{\tau}} + v_c(\infty)\left(1 - e^{-\frac{t}{\tau}}\right)$$

$v_c(\infty)$ is the solution of $-10^{-4} v_c(\infty) - \frac{1}{125} = 0$

Before the switch is opened $i_\Delta = 0 \Rightarrow v_c(0) = 0$

$$v_c(t) = -80 + 80\,e^{\frac{125}{2}t}$$

$$v_c(t^*) = 14.4\times 10^{3} \Rightarrow \boxed{t^* = 0.08317\,(s)}$$

See the figure in the next page.

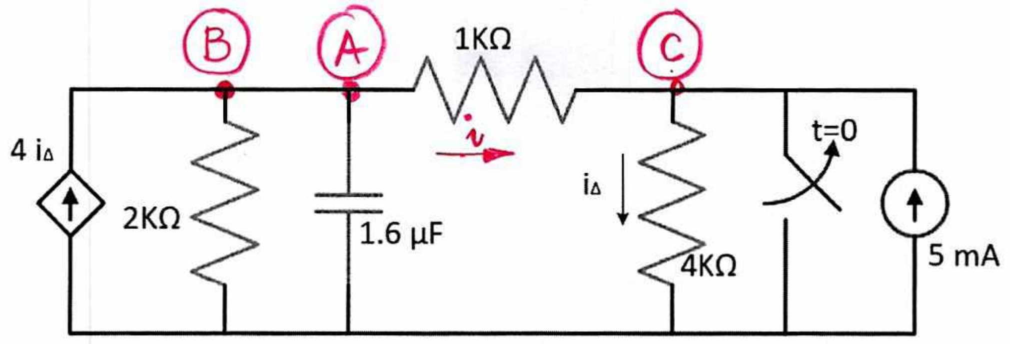


Figure 1: Circuit of Problem 1.

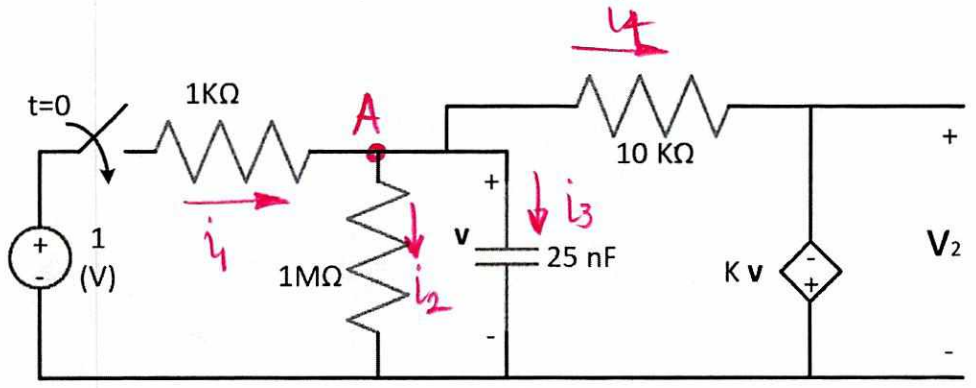


Figure 2: Circuit of Problem 2.

Solution of Problem 2.

At node A, we write

$$\frac{1-v}{10^3} = \frac{v}{10^6} + C\frac{dv}{dt} + \frac{v - v_2}{10^4}, \quad \begin{cases} C = 25\text{ nF} \\ v_2 = -Kv \end{cases}$$

This gives $25 \times 10^{-6} \frac{dv}{dt} + \left(10^2 + 1 + \frac{1}{10} + \frac{1}{10^3}\right) v = 1$

Where the solution is given by:

$$v(t) = \frac{1000}{101101} - \frac{1000}{101101} e^{-4044040t}$$

$$v_2(t) = -10^3 v(t) = -9.89 + 9.89 e^{-4.044 \times 10^6 t}$$

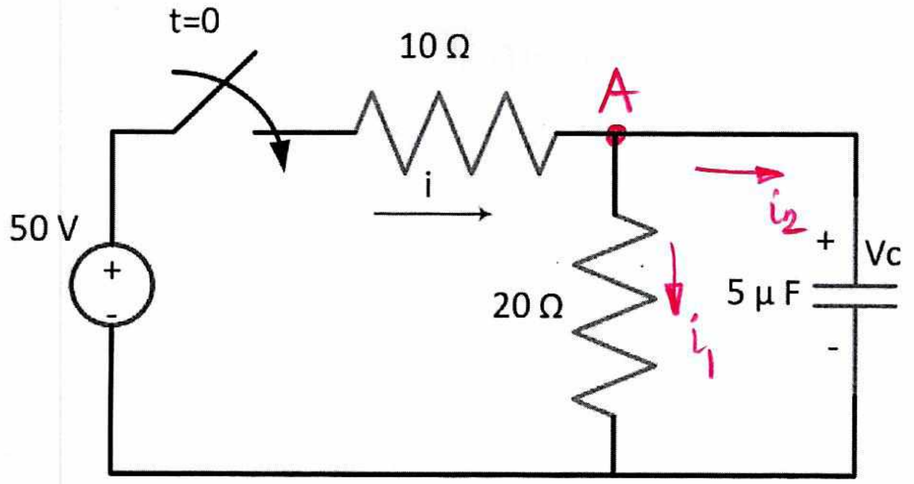


Figure 3: Circuit of Problem 3.

Solution of Problem 3.

At node A; $i = i_1 + i_2$, or

$$\frac{50 - v_C}{10} = \frac{v_C}{20} + C\frac{dv_C}{dt}$$

$$5\times10^{-6}\cdot\frac{dv_C}{dt} + \left(\frac{1}{20} + \frac{1}{10}\right)v_C = 5$$

$$\boxed{v_C(t) = \frac{100}{3}\left(1 - e^{-3\times10^{4}t}\right) \quad (V)}$$ with $v_C(0) = 0$

The current $i(t)$ is then deduced

$$\boxed{i(t) = \frac{5}{3} + \frac{10}{3}e^{-3\times10^{4}t}}$$

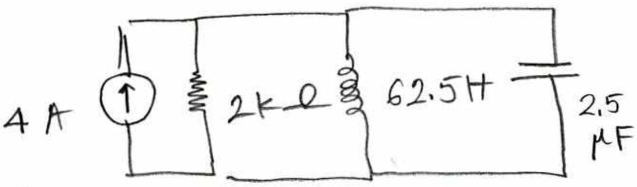
Solution of Problem 4.

Solve problem referenced (8.32, page 322) in your text book.

At $t<0$ $\quad i_L(0^-) = \frac{-15}{3000} = -5\ mA$

$v_c(0^-) = 0\ V$

The circuit is reduced to



$i_L(\infty) = 4\ mA$

$\omega_0^2 = \frac{1}{LC} = 6400$, $\omega_0 = 80$ rad/s

$\alpha = \frac{1}{2RC} = 100$

$s_1 = -40$ rad/s , $s_2 = -160$ rad/s

$i_L = I_f + A_1' e^{-40t} + A_2' e^{-160t}$; $i_L(\infty) = I_f = 4mA$

$i_L(0) = A_1' + A_2' + I_f = -5mA \longrightarrow \underline{A_1' + A_2' = -9\ mA}$

$\frac{di_L(0)}{dt} = 0 = \underline{-40A_1' - 160A_2'}$

By Solving with respect to A_1', A_2' $\Rightarrow$ $\boxed{i_L(t) = 4 - 12e^{-40t} + 3e^{-160t}\ mA}$

Solution of Problem 5.

Solve problem referenced (8.33, page 322) in your text book.

$v_c(0^+) = 120\text{ V}$; $i_L(0^+) = 60\text{ mA}$, $i_L(\infty) = 48\text{ mA}$

$\alpha = \frac{1}{2RC} = 40$, $\omega_0^2 = \frac{1}{LC} = 2500$

$\alpha^2 = 1600$; $\alpha^2 < \omega_0^2$ solution under damped

$S_{1,2} = -40 \pm j30$ rad/s

$$i_L = I_f + B_1 e^{-\alpha t}\cos(\omega_d t) + B_2' e^{-\alpha t}\sin(\omega_d t)$$

$$\boxed{i_L(t) = 48 + B_1 e^{-40t}\cos(30t) + B_2 e^{-40t}\sin(30t)}$$

The constants B_1 and B_2 are determined by:

$i_L(0) = 60\text{ mA}$ and $\frac{di_L}{dt}(0) = 1.5$

$$\boxed{i_L(t) = 48 + 12 e^{-40t}\cos(30t) + 66 e^{-40t}\sin(30t)}$$