P 9.8 $$V_{\rm rms} = \sqrt{\frac{1}{T}\int_0^{T/2} V_m^2 \sin^2 \frac{2\pi}{T} t\, dt}$$

$$\int_0^{T/2} V_m^2 \sin^2\left(\frac{2\pi}{T}t\right)\, dt = \frac{V_m^2}{2}\int_0^{T/2}\left(1 - \cos\frac{4\pi}{T}t\right)\, dt = \frac{V_m^2 T}{4}$$

Therefore $V_{\rm rms} = \sqrt{\frac{1}{T}\frac{V_m^2 T}{4}} = \frac{V_m}{2}$

P 9.12 [a] $\mathbf{V}_g = 300\angle 78^\circ; \qquad \mathbf{I}_g = 6\angle 33^\circ$

$$\therefore \quad Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = \frac{300\angle 78^\circ}{6\angle 33^\circ} = 50\angle 45^\circ\,\Omega$$

[b] i_g lags v_g by 45°:

$$2\pi f = 5000\pi; \qquad f = 2500\,\text{Hz}; \qquad T = 1/f = 400\,\mu\text{s}$$

$$\therefore \quad i_g \text{ lags } v_g \text{ by } \frac{45^\circ}{360^\circ}(400\,\mu\text{s}) = 50\,\mu\text{s}$$

P 9.13 [a] $\omega = 2\pi f = 160\pi \times 10^3 = 502.65\,\text{krad/s} = 502{,}654.82\,\text{rad/s}$

[b] $\mathbf{I} = \dfrac{25 \times 10^{-3}\underline{/0^\circ}}{1/j\omega C} = j\omega C(25 \times 10^{-3})\underline{/0^\circ} = 25 \times 10^{-3}\omega C\underline{/90^\circ}$

$\therefore \quad \theta_i = 90^\circ$

[c] $628.32 \times 10^{-6} = 25 \times 10^{-3}\,\omega C$

$$\frac{1}{\omega C} = \frac{25 \times 10^{-3}}{628.32 \times 10^{-6}} = 39.79\,\Omega, \quad \therefore \quad X_C = -39.79\,\Omega$$

[d] $C = \dfrac{1}{39.79(\omega)} = \dfrac{1}{(39.79)(160\pi \times 10^3)}$

$C = 0.05 \times 10^{-6} = 0.05\,\mu\text{F}$

[e] $Z_c = j\left(\dfrac{-1}{\omega C}\right) = -j39.79\,\Omega$

P 9.18 [a] $Y_2 = \dfrac{1}{R_2} - \dfrac{j}{\omega L_2}$

$$Y_1 = \frac{1}{R_1 + j\omega L_1} = \frac{R_1 - j\omega L_1}{R_1^2 + \omega^2 L_1^2}$$

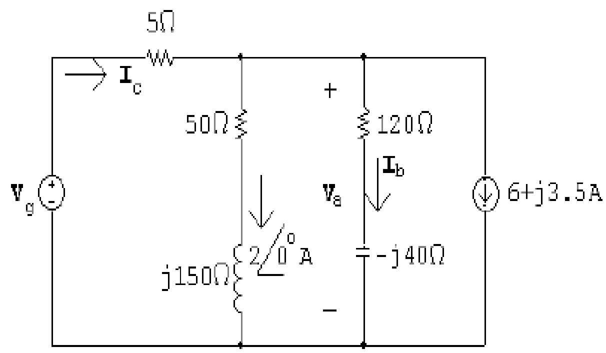
Therefore $\quad Y_2 = Y_1 \quad$ when

$$R_2 = \frac{R_1^2 + \omega^2 L_1^2}{R_1} \quad \text{and} \quad L_2 = \frac{R_1^2 + \omega^2 L_1^2}{\omega^2 L_1}$$

[b] $R_2 = \dfrac{8000^2 + 1000^2(4)^2}{8000} = 10\,\text{k}\Omega$

$$L_2 = \frac{8000^2 + 1000^2(4)^2}{1000^2(4)} = 20\,\text{H}$$

P 9.33 [a]



$$\mathbf{V}_a = (50 + j150)(2\underline{/0^\circ}) = 100 + j300\,\text{V}$$

$$\mathbf{I}_b = \frac{100 + j300}{120 - j40} = j2.5\,\text{A}$$

$$\mathbf{I}_c = 2\underline{/0^\circ} + j2.5 + 6 + j3.5 = 8 + j6\,\text{A}$$

$$\mathbf{V}_g = 5\mathbf{I}_c + \mathbf{V}_a = 5(8 + j6) + 100 + j300 = 140 + j330\,\text{V}$$

[b] $i_b = 2.5\cos(800t + 90^\circ)\,\text{A}$

$$i_c = 10\cos(800t + 36.87^\circ)\,\text{A}$$

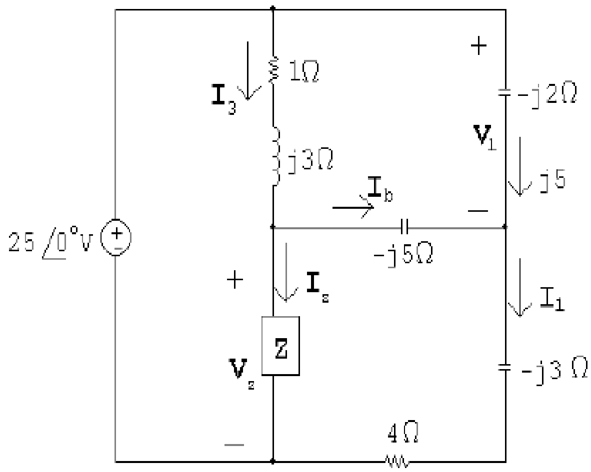
$$v_g = 358.47\cos(800t + 67.01^\circ)\,\text{V}$$

$$\mathbf{I}_b = \mathbf{I}_1 - j5 = (2.4 + j1.8) - j5 = 2.4 - j3.2\,\text{A}$$

$$\mathbf{V}_Z = -j5\mathbf{I}_2 + (4 - j3)\mathbf{I}_1 = -j5(2.4 - j3.2) + (4 - j3)(2.4 + j1.8) = -1 - j12\,\text{V}$$

$$-25 + (1 + j3)\mathbf{I}_3 + (-1 - j12) = 0 \qquad \therefore \qquad \mathbf{I}_3 = 6.2 - j6.6\,\text{A}$$

P 9.36



$$V_1 = j5(-j2) = 10\,\text{V}$$

$$-25 + 10 + (4 - j3)I_1 = 0 \quad \therefore \quad I_1 = \frac{15}{4 - j3} = 2.4 + j1.8\,\text{A}$$

$$I_b = I_1 - j5 = (2.4 + j1.8) - j5 = 2.4 - j3.2\,\text{A}$$

$$V_Z = -j5I_2 + (4 - j3)I_1 = -j5(2.4 - j3.2) + (4 - j3)(2.4 + j1.8) = -1 - j12\,\text{V}$$

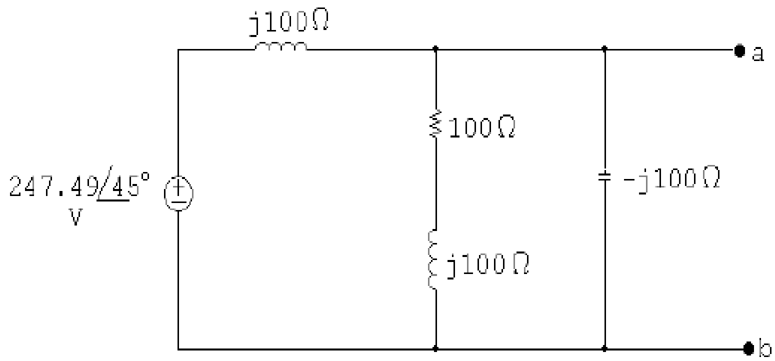
$$-25 + (1 + j3)I_3 + (-1 - j12) = 0 \quad \therefore \quad I_3 = 6.2 - j6.6\,\text{A}$$

$$I_Z = I_3 - I_2 = (6.2 - j6.6) - (2.4 - j3.2) = 3.8 - j3.4\,\text{A}$$

$$Z = \frac{V_Z}{I_Z} = \frac{-1 - j12}{3.8 - j3.4} = 1.42 - j1.88\,\Omega$$

P 9.44 [a] $j\omega L = j(1000)(100) \times 10^{-3} = j100\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{(1000)(10)} = -j100\,\Omega$$



Using voltage division,

$$\mathbf{V}_{\rm ab} = \frac{(100 + j100)\|(-j100)}{j100 + (100 + j100)\|(-j100)}(247.49\underline{/45^\circ}) = 350\underline{/0^\circ}$$

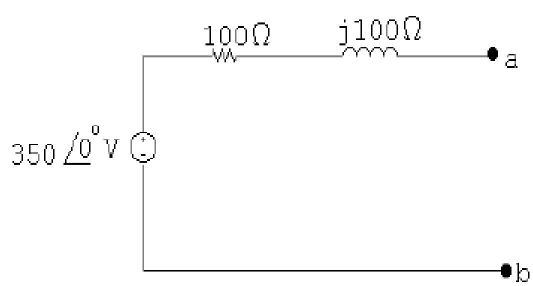
$$\mathbf{V}_{\rm Th} = \mathbf{V}_{\rm ab} = 350\underline{/0^\circ}\text{ V}$$

[b] Remove the voltage source and combine impedances in parallel to find $Z_{\rm Th} = Z_{\rm ab}$:

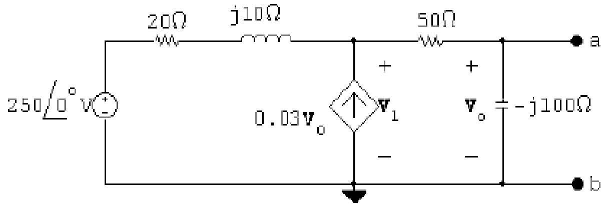
$$Y_{\rm ab} = \frac{1}{j100} + \frac{1}{100 + j100} + \frac{1}{-j100} = 5 - j5\text{ mS}$$

$$Z_{\rm Th} = Z_{\rm ab} = \frac{1}{Y_{\rm ab}} = 100 + j100\,\Omega$$

[c]



P 9.48 Open circuit voltage:



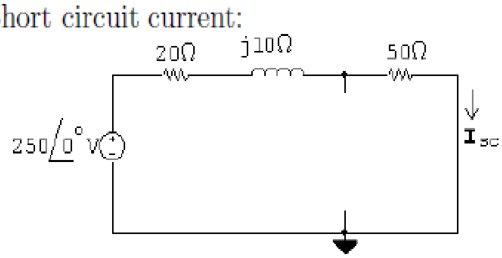
$$\frac{\mathbf{V}_1 - 250}{20 + j10} - 0.03\mathbf{V}_o + \frac{\mathbf{V}_1}{50 - j100} = 0$$

$$\therefore \quad \mathbf{V}_o = \frac{-j100}{50 - j100}\mathbf{V}_1$$

$$\frac{\mathbf{V}_1}{20 + j10} + \frac{j3\mathbf{V}_1}{50 - j100} + \frac{\mathbf{V}_1}{50 - j100} = \frac{250}{20 + j10}$$

$$\mathbf{V}_1 = 500 - j250\,\text{V}; \qquad \mathbf{V}_o = 300 - j400\,\text{V} = \mathbf{V}_{\text{Th}}$$

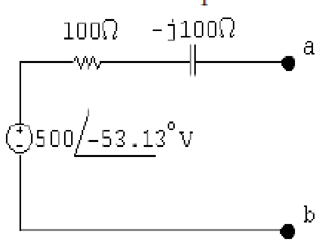
Short circuit current:



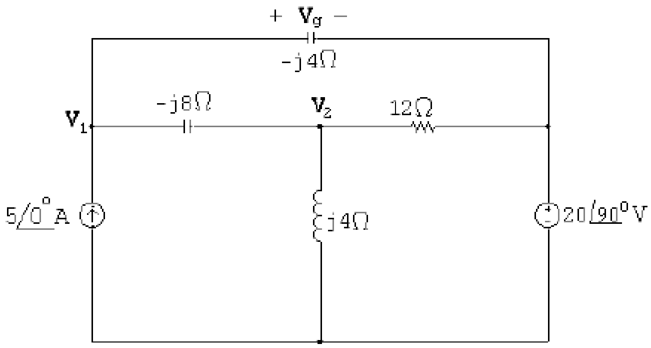
$$\mathbf{I}_{\text{sc}} = \frac{250\angle 0^\circ}{70 + j10} = 3.5 - j0.5\,\text{A}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{300 - j400}{3.5 - j0.5} = 100 - j100\,\Omega$$

The Thévenin equivalent circuit:



P 9.56 Set up the frequency domain circuit to use the node voltage method:



At $\mathbf{V}_1$: $$-5\underline{/0^\circ} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j8} + \frac{\mathbf{V}_1 - 20\underline{/90^\circ}}{-j4} = 0$$

At $\mathbf{V}_2$: $$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j8} + \frac{\mathbf{V}_2}{j4} + \frac{\mathbf{V}_2 - 20\underline{/90^\circ}}{12} = 0$$

In standard form:

$$\mathbf{V}_1\left(\frac{1}{-j8} + \frac{1}{-j4}\right) + \mathbf{V}_2\left(-\frac{1}{-j8}\right) = 5\underline{/0^\circ} + \frac{20\underline{/90^\circ}}{-j4}$$

$$\mathbf{V}_1\left(-\frac{1}{-j8}\right) + \mathbf{V}_2\left(\frac{1}{-j8} + \frac{1}{j4} + \frac{1}{12}\right) = \frac{20\underline{/90^\circ}}{12}$$

Solving on a calculator:

$$\mathbf{V}_1 = -\frac{8}{3} + j\frac{4}{3} \qquad \mathbf{V}_2 = -8 + j4$$

Thus

$$\mathbf{V}_g = \mathbf{V}_1 - 20\underline{/90^\circ} = -\frac{8}{3} - j\frac{56}{3}\text{ V}$$

P 9.59 Write a KCL equation at the top node:

$$\frac{\mathbf{V}_o}{-j8} + \frac{\mathbf{V}_o - 2.4\mathbf{I}_\Delta}{j4} + \frac{\mathbf{V}_o}{5} - (10 + j10) = 0$$

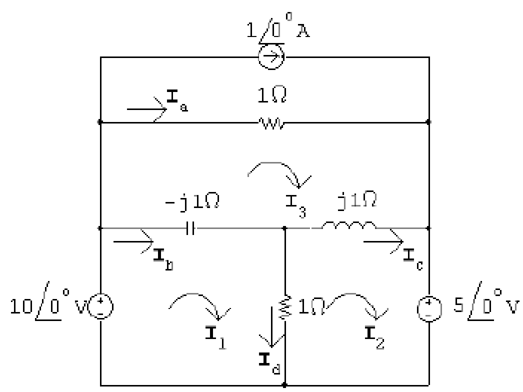
The constraint equation is:

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j8}$$

Solving,

$$\mathbf{V}_o = j80 = 80\underline{/90^\circ}\ \text{V}$$

P 9.63



$$10\underline{/0^\circ} = (1 - j1)\mathbf{I}_1 - 1\mathbf{I}_2 + j1\mathbf{I}_3$$

$$-5\underline{/0^\circ} = -1\mathbf{I}_1 + (1 + j1)\mathbf{I}_2 - j1\mathbf{I}_3$$

$$1 = j1\mathbf{I}_1 - j1\mathbf{I}_2 + \mathbf{I}_3$$

Solving,

$$\mathbf{I}_1 = 11 + j10\,\text{A}; \qquad \mathbf{I}_2 = 11 + j5\,\text{A}; \qquad \mathbf{I}_3 = 6\,\text{A}$$

$$\mathbf{I}_a = \mathbf{I}_3 - 1 = 5\,\text{A}$$

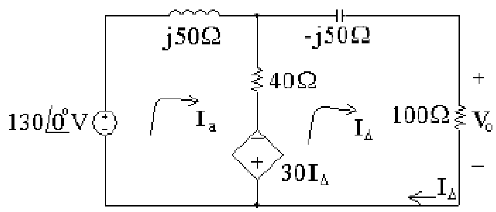
$$\mathbf{I}_b = \mathbf{I}_1 - \mathbf{I}_3 = 5 + j10\,\text{A}$$

$$\mathbf{I}_c = \mathbf{I}_2 - \mathbf{I}_3 = 5 + j5\,\text{A}$$

$$\mathbf{I}_d = \mathbf{I}_1 - \mathbf{I}_2 = j5\,\text{A}$$

P 9.64 $j\omega L = j10{,}000(5 \times 10^{-3}) = j50\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(10{,}000)(2 \times 10^{-6})} = -j50\,\Omega$$



$$130\underline{/0^\circ} = (40 + j50)\mathbf{I}_a - 40\mathbf{I}_\Delta + 30\mathbf{I}_\Delta$$

$$0 = -40\mathbf{I}_a + 30\mathbf{I}_\Delta + (140 - j50)\mathbf{I}_\Delta$$

Solving,

$$\mathbf{I}_\Delta = (400 - j400)\,\text{mA}$$

$$\mathbf{V}_o = 100\mathbf{I}_\Delta = 40 - j40 = 56.57\underline{/-45^\circ}$$

$$v_o = 56.57\cos(10{,}000t - 45^\circ)\,\text{V}$$