EE 202-Winter 2013-2014 (132)

HW5 Solution

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Due 13/4/2014

Q1 A voltage of 60 cos 4πt V appears across the terminals of a 3-mF capacitor. Calculate the current through the capacitor and the energy stored in it from t = 0 to t = 0.125 s.

$$i = C\frac{dv}{dt} = 3x10^{-3} x60(4\pi)(-\sin 4\pi\, t)$$

$$= \underline{\mathbf{-0.72\,\pi \sin (4\pi t)\ A}}$$

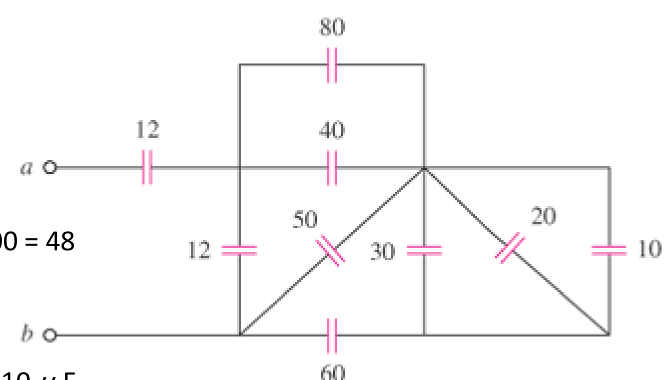
$P = vi = 60(-0.72)\pi \cos 4\pi\, t \sin (4\pi\, t) = -21.6\pi \sin (8\pi\, t)$ W

$$W = \int_o^t pdt = -\int_o^{\frac{1}{8}} 21.6\pi \sin(8\pi t)\ dt$$

$$= \frac{21.6\pi}{8\pi}\cos 8\pi \Big|_o^{1/8} = \underline{\mathbf{-5.4J}}$$

Q2 Find the equivalent capacitance between terminals *a* and *b* in the circuit of Fig. 1. All capacitances are in μF.

We combine 10-, 20-, and 30- μ F capacitors in parallel to get 60 μ F. The 60 - μ F capacitor in series with another 60- μ F capacitor gives 30 μ F.



30 + 50 = 80 μ F, 80 + 40 = 120 μ F

120- μ F capacitor in series with 80 μ F gives (80x120)/200 = 48

48 + 12 = 60

60- μ F capacitor in series with 12 μ F gives (60x12)/72 = <u>10 μ F</u>

Q3 In the circuit in Fig. 2, let $i_s = 30e^{-2t}$ mA and $v_1(0) = 50$ V, $v_2(0) = 20$ V. Determine: (a) $v_1(t)$ and $v_2(t)$, (b) the energy in each capacitor at $t = 0.5$ s.

(a) $C_{eq} = (12x60)/72 = 10\ \mu F$

$$v_1 = \frac{10^{-3}}{12x10^{-6}}\int_0^t 30e^{-2t}dt + v_1(0) = \underline{-1250e^{-2t}\ \Big|_0^t + 50} = \underline{-1250e^{-2t} + 1300}V$$

$$v_2 = \frac{10^{-3}}{60x10^{-6}}\int_0^t 30e^{-2t}dt + v_2(0) = \underline{250e^{-2t}\ \Big|_0^t} + 20 = \underline{-250e^{-2t} + 270V}$$

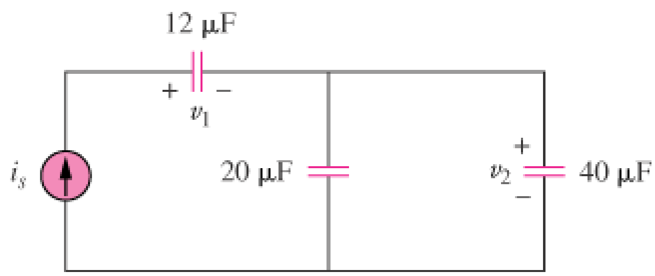
(b) At t=0.5s,

$$v_1 = -1250e^{-1} + 1300 = 840.2, \qquad v_2 = -250e^{-1} + 270 = 178.03$$

$$w_{12\mu F} = \frac{1}{2}x12x10^{-6}x(840.15)^2 = \underline{4.235\ \ J}$$

$$w_{20\mu F} = \frac{1}{2}x20x10^{-6}x(178.03)^2 = \underline{0.3169\ \ J}$$

$$w_{40\mu F} = \frac{1}{2}x40x10^{-6}x(178.03)^2 = \underline{0.6339\ J}$$



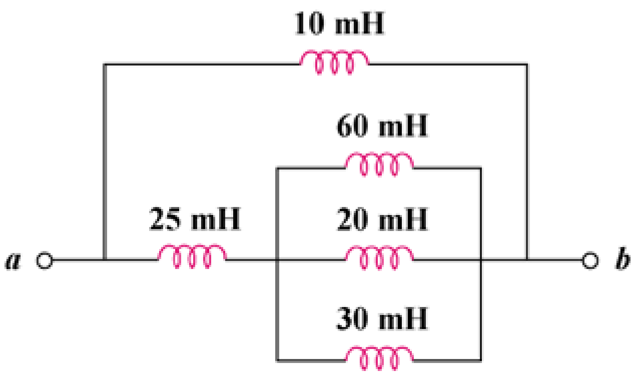
Q4 Determine L_{eq} at terminals ***a-b*** of the circuit in Fig. 3.

$$\frac{1}{L} = \frac{1}{60} + \frac{1}{20} + \frac{1}{30} = \frac{1}{10}$$

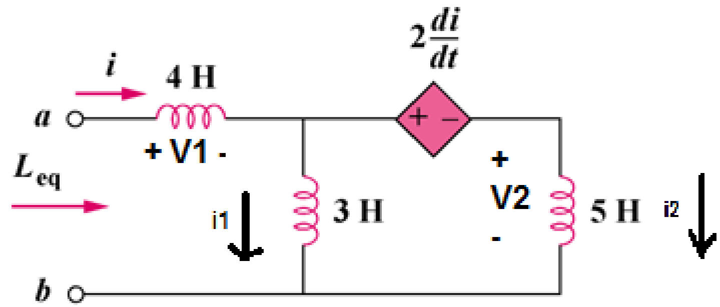
L = 10 mH

$$L_{eq} = 10 \| (25+10) = \frac{10x35}{45}$$

= **7.778 mH**



Q5 Determine L_{eq} that may be used to represent the inductive network of Fig. 4 at the terminals.



Let $v = L_{eq}\frac{di}{dt}$ (1)

$$v = v_1 + v_2 = 4\frac{di}{dt} + v_2$$

(2)

$i = i_1 + i_2 \longrightarrow i_2 = i - i_1$ (3)

$$v_2 = 3\frac{di_1}{dt} \text{ or } \frac{di_1}{dt} = \frac{v_2}{3} \quad (4)$$

and

$$-v_2 + 2\frac{di}{dt} + 5\frac{di_2}{dt} = 0$$

$$v_2 = 2\frac{di}{dt} + 5\frac{di_2}{dt} \quad (5)$$

Incorporating (3) and (4) into (5),

$$v_2 = 2\frac{di}{dt} + 5\frac{di}{dt} - 5\frac{di_1}{dt} = 7\frac{di}{dt} - 5\frac{v_2}{3}$$

$$v_2\left(1 + \frac{5}{3}\right) = 7\frac{di}{dt}$$

$$v_2 = \frac{21}{8}\frac{di}{dt}$$

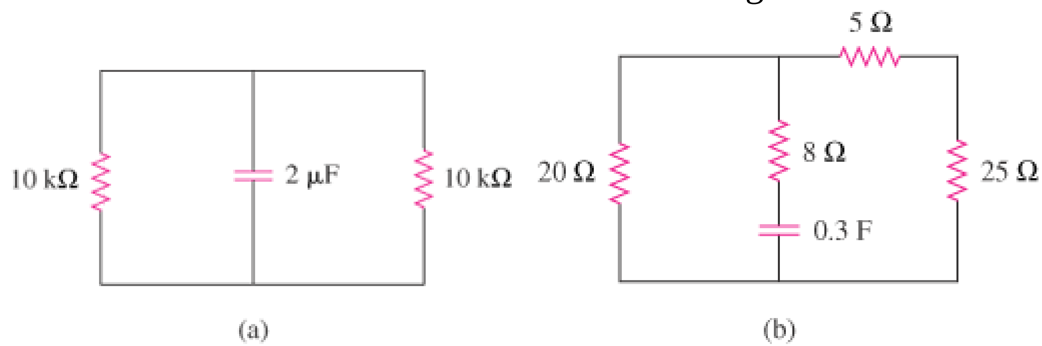
Substituting this into (2) gives

$$v = 4\frac{di}{dt} + \frac{21}{8}\frac{di}{dt} = \frac{53}{8}\frac{di}{dt}$$

Comparing this with (1),

$$L_{eq} = \frac{53}{8} = \underline{\mathbf{6.625\ H}}$$

Q6 Find the time constant of each of the circuits in Fig. 5.

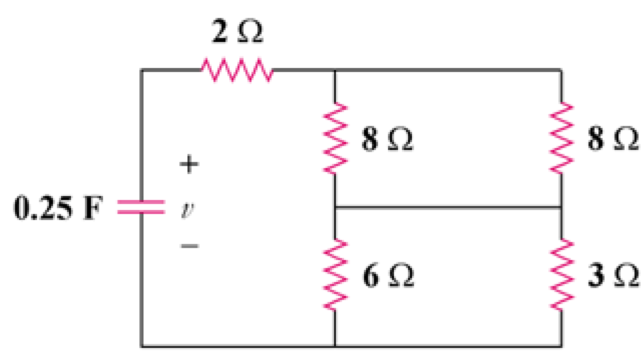


(a) $R_{Th} = 10 // 10 = 5k\Omega, \quad \tau = R_{Th}C = 5x10^3 x2x10^{-6} = \underline{10 \text{ ms}}$

(b) $R_{Th} = 20 // (5 + 25) + 8 = 20\Omega, \quad \tau = R_{Th}C = 20x0.3 = \underline{6\text{s}}$

Q7 In the circuit of Fig. 6, $v(0) = 20$ V. Find $v(t)$ for $t > 0$.

$v(t) = v(0)e^{-t/\tau}$, $\quad \tau = R_{eq}C$



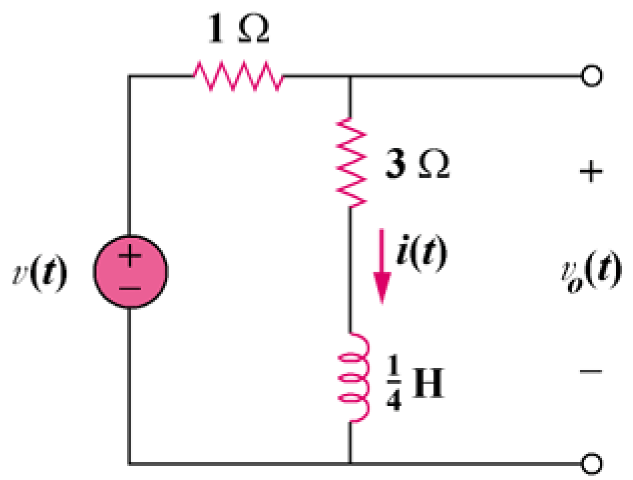
$R_{eq} = 2 + (8 \| 8) + (6 \| 3) = 2 + 4 + 2 = 8\,\Omega$

$\tau = R_{eq}C = (0.25)(8) = 2$

$v(t) = \underline{\mathbf{20e^{-t/2}\ V}}$

Q8 Consider the circuit of Fig. 7. Find $v_0(t)$ if $i(0) = 2$ A and $v(t) = 0$.

$$i(t) = i(0)\,e^{-t/\tau}, \qquad \tau = \frac{L}{R_{eq}} = \frac{1/4}{4} = \frac{1}{16}$$



$$i(t) = 2\,e^{-16t}$$

$$v_o(t) = 3i + L\frac{di}{dt} = 6e^{-16t} + (1/4)(-16)\,2\,e^{-16t}$$

$$v_o(t) = \underline{\mathbf{-\,2\,e^{-16t}\ V}}$$